Research Article

Independent Domination In Planar Graph

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Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 16 April 2021

Abstract— A dominating set D of a planar graph G is an independent dominating set Dip if D is noth independent and dominating set. The authors have generated the maximum independent dominating number of planar graph ($\square_{IP}$) and minimum independent dominating number ($\square_{IP}$). We introduced the concept of Independent domination in planar graph by integrated some results.

Keywords— independent dominating set D_{ip} , maximum independent dominating number ($\square_{IP}$), minimum independent dominating number ($\square_{IP}$).

I. INTRODUCTION

"A graph G consists of a pair (V (G), X (G)) where V(G) is a non-empty finite set whose elements are called points or vertices and □ (G) is a set of unordered pairs of distinct element of V(G).

In graph theory, planar graph is also be one of the part. A graph G is said to be a planar if it can be represent on a plane in such a fashion that the vertices are all distinct points, edges and no two edges meet one another except their terminals. A set S of vertices of G is dominating set if every vertex in V (G) is adjacent to at least one vertex in S.

In this paper, we have founded the result on independent domination in planar graph."

II. PRELIMINARIES

Definition 2.1:

"A Finite Graph is a graph $G \square (V , E)$ such that V and E are called vertices and edges finite sets.

An Infinite Graph is one with an infinite set or edges or both. Most commonly in graph theory, it is implied that the graphs discussed are finite.

If more than one edge joining two vertices is allowed, the resulting object is a Multi Graph [1]. Edges joining the same vertices are called multiple lines.

A drawing of a geometric representation of a graph on any surface such that no edges intersect is called Embedding.

Definition 2.2:

A set $I \square V$ is an independent set of G, if $\square u, v \square I$. $N(u) \square \{v\} \square \square$.

Definition 2.3:

Let $G \square (V , E)$ be a graph. A set $S \square V$ is a Dominating Set [5] of G if every vertex in V, D is adjacent to some vertex in D.

The Dominating Number $\square(G)$ [7] of G is the minimum cardinality of a dominating set. A review on $\square(G)$ is found in [2] and some recent result in [3, 4, 7].

A dominating set D is a Minimal Dominating Set [5] if no proper subset $D' \square D$ is a dominating set of G.

Definition 2.4:

An Independent Dominating Set [6] of G is a set that is both dominating and independent in G.

The Independent Domination Number of G is denoted by $i(G)$ is the minimum size of an independent dominating set. The independence number of G is denoted by $\square(G)$ is the maximum size of an independent set in G. i.e. $\square(G) \square i(G) \square \square(G)$.

A dominating set of G of size □(G) is called a □ -set, while an independent dominating set of G of $i(G)$ is called an i-set."

III.INDEPENDENT DOMINATION IN PLANAR GRAPH

"In this section, we introduce the Independent Domination in Planar Graph and derived some theorems.

Definition 2.5:

A domination set D of a planar graph G is an independent dominating D_{ip} if D is both independent set and dominating set.

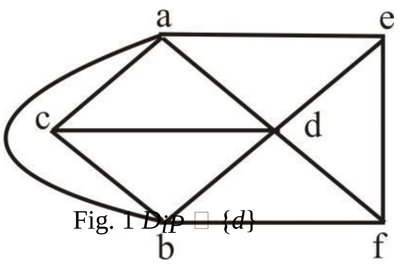

Fig. 1 D_{iP} □ {d}

Definition 2.6:

Minimum cardinality of independent dominating set is called Minimum Independent Dominating Number of Planar Graph. It is denoted by □$_{ip}$.

Maximum number of elements in a independent dominating set is called Maximum Independent Dominating Number of Planar Graph. It is denoted by □$_{IP}$.

Maximum number of independent dominating number is denoted by □$_{IP}$. Minimum number of independent dominating number is denoted by □$_{ip}$."

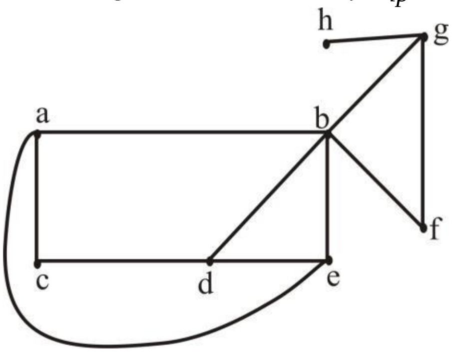

Fig. 1 D_{iP} □ {b,c,h}, □$_{ip}$ □ 3, D_{IP} □ {a,d,f,h}, □ $_{IP}$ □ 4

Theorem:

If a complete planar graph G then □$_{ip}$ □ 1 .

Proof:

"If G be a complete planar graph then there is only K_3 and K_4 be a complete planar graph. By the concept of complete planar graph. Every point should be incident with other point of a graph" from the concept. The independent dominating set will be 1.

Therefore □$_{ip}$ □ 1.

Hence the Proof.

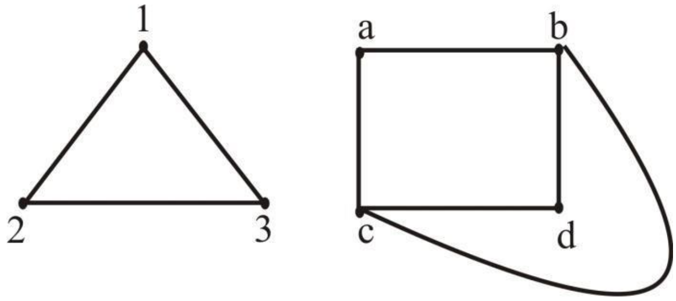

Fig. 3 V □ {1,2,3}, D_{iP} □ 1, $□_{ip}$ □ 1,V □ {a,b,c,d}, D_{ip} □ {a}, $□_{ip}$ □ 1

Theorem:

"Any planar graph G, □ □ $□_{ip}$ □ $□_{IP}$.

Proof:

Let G be a planar graph. We have to prove that: □ □ $□_{ip}$ □ $□_{IP}$

□ □ Minimum dominating number.

$□_{ip}$ □ Minimum independent dominating number."

□ $_{IP}$ □ Maximum independent dominating number. We know that,

"Every minimum independent dominating number is less than or equal to maximum independent dominating number".

Hence, $□_{ip}$ □ $□_{IP}$. (1)

□ be a minimum dominating number which is not necessary to be independent.

Hence, □ □ $□_{ip}$. (2)

From 1 and 2, □ □ $□_{ip}$ □ $□_{IP}$.

Hence proved.

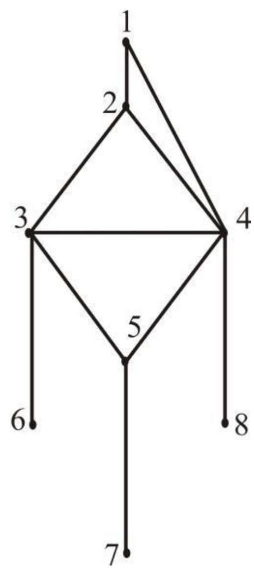

Fig. 4 V □ {1,2,3,4,5,6,7,8}, □ □ $□_{ip}$ □ □ $_{IP}$,3 □ 3 □ 4, D_{ip} □ {4,6,7}, $□_{ip}$ □ 3, D_{IP} □ {2,6,7,8}, □ $_{IP}$ □ 4

Theorem:

If a graph G be a bipartite planar graph *Proof*: $K(m, n)$ and m > n then

□$_{ip}$ □ n and □ $_{IP}$ □ m .

A graph G be a bipartite planar graph $K(m, n)$ and m > n.

In vertex set V have two partitions V_1 and V_2. m number of points □V_1 and n number of points □V_2. If G be a bipartite graph then every lines of G joins a point of V_1 to a point of V_2.
Here G □ $K(m, n)$ and m > n. Members of V_1 is greater than the members of V_2. Here every element of V_1 are independent within its set. Also every elements of V_2 are independent within its set.

Therefore □$_{ip}$ □ n and □ $_{IP}$ □ m .

Hence proved.

Theorem:

□$_{ip}$ □ 2 if G be a complete bipartite planar graph.

Proof:

Let G be a complete bipartite planar graph.

There is $K_{(1,1)}$, $K_{(2,2)}$ only be the complete bipartite planar graph.

(i) $K_{(1,1)}$

Let G □ $K_{(1,1)}$ complete bipartite planar graph. □$_{ip}$ □ 1 (1)

(ii) $K_{(2,2)}$

Let G □ $K(2,2)$ From 1 and 2 complet e bipar tite planar graph. $\square_{ip} \square 1$ (2)

(iii) $K(1,n)$ and $K(m,1)$ (3)

$\square_{ip} \square 1$. $\square_{ip} \square 2$.

From 1, 2 and 3,

Theorem:

"Independent domination numbers are equal for isomorphic planar graph of G.

Proof:

Let G_1 be a planar graph and G_2 be an isomorphic to G_1. Therefore G_2 also be a planar graph.

Two isomorphic graphs have the same number of points and the same number of lines. Hence dominating number also same number of isomorphic graphs.

Therefore $\square_{ip} \square \square_{ip} \square 1$.

1 2

Hence "Independent dominating numbers are equal for isomorphic planar graph of G.

Theorem:

Let G be a planar graph with the cut point as a independent dominating set then $V \square D_{ip}$ becomes only with the vertices.

Proof:

Let G be a planar graph with independent set D_{ip}. i.e., $D_{ip} \square \{v\}$.

Assume that, v is not a cut point. Then it will be affect the concept of independence. Hence our assumption is wrong.

Hence v is a cut point and also independent dominating set. Therefore $V \square D_{ip}$ becomes only with the vertices. Hence proved."

Theorem:

If p be an D_i

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s no cut edge in the set

independent dominating set of a planar graph G then there i

Dip

.

Proof:

Let G be a planar graph. D_{ip} be an independent dominating set of G.

D_{ip} □ {u, v} and (u, v) be a cut edge of G. If (u; v) is a cut edge then u independents on v

"I assume that, and v depends on u.

Which is a contradicts to our concept of independent dominating set. Hence, there is a no cut edge in independent dominating set."

Hence proved.

IV. CONCLUSION

I exhibited the □$_{ip}$ for complete planar graph, bipartite planar graph, complete bipartite planar graph. I derived the results on relation between □, □$_{ip}$, □$_{IP}$ and cut points, etc.

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