Research Article

Isomorphism Identification Among Kinematic Chains Of Group-Iv-D, E And F

Ali Hasan

Associate Professor, Mechanical Engineering Department, Faculty of Engineering & Technology, Jamia Millia Islamia, New Delhi, India

Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 16 April 2021

ABSTRACT : The objective of this paper is to determine identification numbers of each kinematic chains of group IV-D, E, and F. This study checks the isomorphism among 10 Links, 13 Joints, Single degree of freedom Kinematic Chains of Group IV-D, E and F. The author used a Joint-Joint [JJ] matrix methodology. In this methodology, the identification numbers 'sum of the absolute values of the characteristic polynomial coefficients' [SCPC] and 'maximum absolute value of the characteristic polynomial coefficient' [MCPC] are calculated for each matrix with the help of MATLAB Identification of isomorphism in the mechanism kinematic chains. The novelty of this study is that the study may be used as guidelines for new researchers / design engineers and scientist in designing the best kinematic chain to do the pre-decided design in the initial stage of their research work. The study is explained with the help of examples of planar kinematic chain having simple joints.

Keywords: Mechanisms, Kinematic Chains; Isomorphism; Degree of Freedom. Identification Number.

I. INTRODUCTION

A number of researchers have discussed structural synthesis in the earlier days. Chang, et.al. [1] proposed method is based on the eigen vectors and eigen values to identify isomorphism of mechanism kinematic chain. Mruthyunjaya [2] made an effort to develop a fully computerized approach for structural synthesis of kinematic chains. Agrawal and Rao [3] investigated a systematic method of analysis of the mobility properties of the kinematic chains by its loop freedom matrix and its permanent function which are used to identify it. Sethi and Agrawal [4] proposed a classification scheme on the basis of structural properties. Madan and Jain [5] considered the kinematic chains-isomorphism, inversions and degree of similarity using the concept of connectivity. Rao [6] threw the light on the enumeration of distinct planar kinematic chains. Misti [7] presented the position analysis in polynomial form of planar mechanisms with Assur groups of class 3 including revolute and prismatic joints. Uicker and Raicu [8] presented a method for the identification and recognition of equivalence of kinematic chains. Later on, this method failed. Mruthyunjaya and Balasubramanian [9] proved that the method proposed by Uicker and Raicu [8] is not reliable. Shende and Rao [10] work on the problem of detection of isomorphism. Chu Jin-Kui and Cao Wei-Qing [11] proposed a method for identification of isomorphism among kinematic chains and inversions using Link's adjacent-chain-table. Yadav, et.al. [12] Proposed a computer aided detection method of isomorphism among kinematic chains and mechanisms using the concept of modified distance. Yadav, et.al.[13] presented a paper mechanism of a kinematic chain and the degree of structural similarity based on the concept of link path code'. Yadav, et.al.[14] presented a paper 'computer aided detection of isomorphism among binary chains using the link-link multiplicity distance concept. Rao [15] sued the application of fuzzy logic for the study of isomorphism, inversions, symmetry, parallelism and mobility in kinematic chains with some necessary and sufficient conditions. Kong, et.al. [16] Proposed a new method based on artificial neural network (ANN) to identify the isomorphism of the mechanism kinematic chain. Rao and Deshmukh [17] proposed method does not require any separate test for isomorphism in the generation of kinematic chains. He and Jhang [18] proposed a new method for detection of graph isomorphism based on the quadratic form. Tang and Liu [19] established a method 'the degree code' as a new mechanism identifier. Later on this method also failed. Zhao, et.al [20] put forward and more complete theory of degrees of freedom (DOF) for mechanisms. Hasan et al. [21] but the concept that these methods are based on seems to be unjustified as either link-link adjacency or joint-joint adjacency hardly differ in nature and are likely to fail at some stage or the other. Hasan [22],[23] proposed a new method in which kinematic chains are represented in the form of the Joint-Joint [JJ] matrix. Dargar et al. [24],[25] proposed Link adjacency value method to identify the isomorphism by calculating the first and second link adjacency values. Rizvi et al. [26] presented a new method for distinct inversions and isomorphism based on a link identity matrix and link signature. Leiying He et al. [27] proposed a method with the application of variable high-order adjacency link values for identification of isomorphism among kinematic chains. The authors redefined the high order adjacency link values. Later these redefined values were used in the characteristics of the kinematic chains in depth. These values were determined again and again through reassignment manner according to the presentation of one kinematic chain element. In the last, the isomorphism were found matching both the higher order adjacency link strings from both the kinematic chains. The authors tested their proposed method using 8-bars, 15-bars and 28-bars kinematic chains. Sun L. et al. [28] used graph theory to design the gear trains applicable for transplanting mechanism. The authors screened the topology graphs among the gear trains use a specific gear train mechanism for pot seedling transplantation. In the end, the authors verified the feasibility of proposed scheme. K.R. Rajnesh and P. Sunil [29] proposed a new algorithm for labeling the bars of kinematic chains along with binary code. The method was used for the verification of isomorphism among kinematic chains of 6,8,9,10,11,12 and 15 links having simple joints as well as 4,5 and 6 links epicyclic gear chains. S. H. Rizvi et al. [30] developed a new algorithm with adjacency matrices to calculate the distinct mechanisms of a closed kinematic chain. The worth of the developed algorithm was proved with the help of several examples. W. Sun, J. Kong and L. Sun [31] proposed a noval method for isomorphism calculation of planar kinematic chains along with multiple joints using joint–joint matrix. A joint–joint matrix was defined, then links and joints details were taken from the matrix. The scientist developed link code and joint code for the purpose. The author showed their proposed method with the help of examples. V. Dharanipragada and M. Chintada [32] work on the revolute as well as prismatic pairs. Here joint-joint matrices were used by labelling the revolute joint first and then prismatic pair. It was used the method like hamming number technique and split the matrices into three parts. The authors used a computer program for one degree of freedom, six bar chains having simple joints.W. Sun, J. Y. Kong and L. B. Sun [33] proposed a hamming number technique for isomorphism finding among kinematic chains with multiple joints. They used joint-joint matrix to define the chain. They discussed the joint Hamming number, chain Hamming number and joint Hamming string with the help of examples.

II. THE JOINT-JOINT [JJ] MATRIX

This matrix is based upon the connectivity of the joints through the links and defined, as a square symmetric matrix of size n x n, where n is the number of Kinematic pairs (joints) in a kinematic chain.

$[JJ] = \{L_{ij}\}_{nxn}$ ------(1)

Where, $\mathbf{L_{ij}}$ {= Degree or Type of Link between ith and jth joints, those are directly connected, off course, all the diagonal elements Lii= Ljj=0}

Note: Degree or type of link means: 2 for binary link, 3 for ternary link, 4 for quaternary link and so on.

III. METHODOLOGY

Two similar square symmetric matrices have the same characteristic polynomials. Plain kinematic chains of combination of binary, ternary, and other higher order links. These links are joined together by simple kinematic pairs or simple pin joints. An identification number is assigned to links. Therefore, a binary link has a value of two, ternary three, quarter nary four and so on. Link values are used to assigned values to [JJ] matrix and these are utilized to identify the layout of the kinematic chains. For detecting isomorphism in kinematic chains, [JJ] matrices are written and identification number (composite structural invariants) [SCPC] and [MCPC] for [JJ] matrix are calculated. If the structural invariants [SCPC] and '[MCPC] of the two or more kinematic chains are same then the kinematic chains are considered as isomorphic otherwise non-isomorphic.

IV. ILLUSTRATIVE EXAMPLE

We have two kinematic chains with 12 links, 16 joints, one degree of freedom shown in Figure 1 and Figure 2. Now, we have to check whether these two KCs are isomorphic. [A] and [B] represent the [JJ] matrices for these KCs respectively.

The values of Identification Numbers (structural invariants are :

For KC shown in Figure 1: **[**SCPC] = 2.9971e+010 , [MCPC] =1.0219e+010 and

For KC shown in Figure 2: **[**SCPC] = 3.2201e+010, [MCPC] =1.1287e+010.

We note that KCs shown in Figure 1 and Figure 2 are non-isomorphic because the values of their identification numbers [SCPC] and [MCPC] are different for both the KCs. We Note that the author results are validated by using another method Chang [1] and Kong [16], already available in the literature.

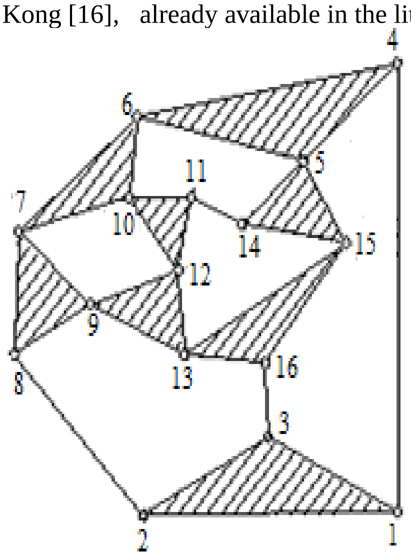


Figure 1: 12 Links, 1 F KC1

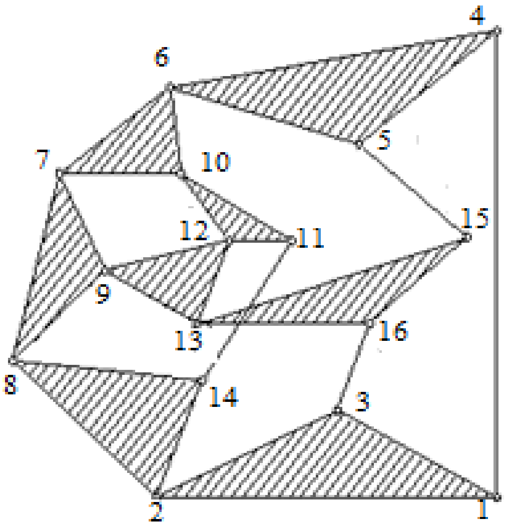


Figure 2: 12 Links, 1 F KC2

$$A=\begin{bmatrix}
0 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 0 & 3 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\\
2 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0\\
0 & 0 & 0 & 3 & 3 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 2 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 2 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 3 & 0 & 3 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 3\\
0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 3 & 0\\
0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 3\\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0
\end{bmatrix}$$

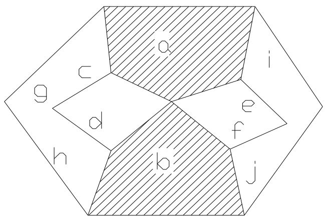
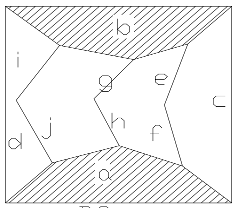
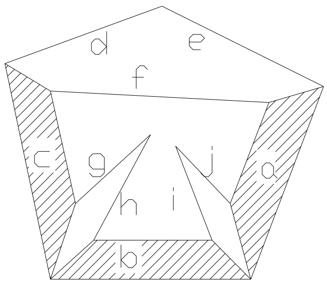
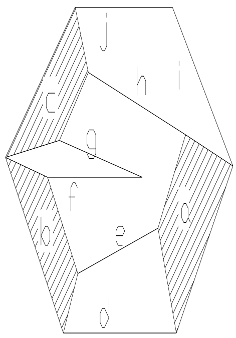
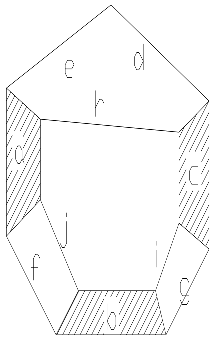
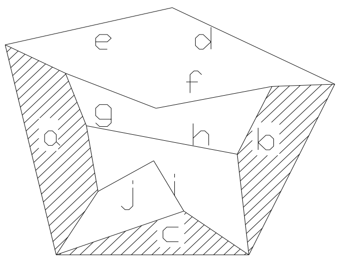
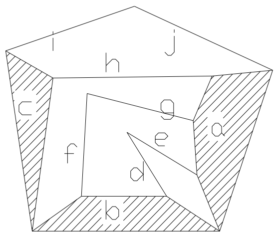
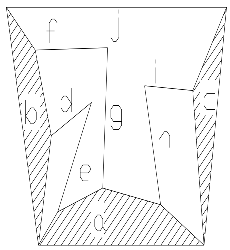
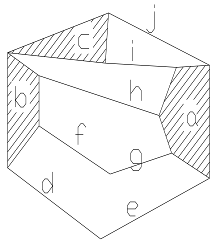
$$B=\begin{bmatrix}
0 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0\\
3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\\
2 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0\\
0 & 0 & 0 & 3 & 3 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 2 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 3 & 0 & 3 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 3\\
0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3\\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0
\end{bmatrix}$$

V. RESULTS AND CONCLUSIONS

The structural invariants [SCPC] and [MCPC] are used as the identification number of the kinematic chains having simple joints. The detailed identification numbers (values of SCPC and MCPC) of all 1-dof kinematic chains up to 10-Links are with the author. These identification numbers are also capable to detect isomorphism among the kinematic chains with multiple joints too. The kinematic chains of 10 Links, 13 Joints, 1-dof Kinematic Chains Group 'IV D, E and F' are redrawn from Jensen [34] and given in Table 1. The identification Numbers of Kinematic Chains Group-IV D, E and F are given in Table 2. In the present study, a simple, efficient, and reliable Joint-Joint [JJ] method proposed by Hasan[22] is used for isomorphism identification.

Using this methodology, the isomorphism of mechanisms kinematic chains can easily be identified and mechanisms determined. It incorporates all features of the kinematic chains and as such, violation o f the isomorphism test is rather difficult. The method has already been applied and found to be successful in distinguishing all known 16 kinematic chain of 8-links, 230 kinematic chain of 10-links having 1-F. The advantage is that they are very easy to compute using MATLAB software. It is not essential to determine both the structural invariants to compare two chains, only in case the [SCPC] is same then it is needed to determine [MCPC]for both kinematic chains. The [JJ] matrices can be written with very little effort, even by mere inspection of the chain. The proposed test is quite general in nature and can be used to detect isomorphism of not only planar kinematic chains of one degree of freedom, but also kinematic chains of multi degree of freedom.

Table 1: 10 Link, 13 Joints, 1F Kinematic Chains Group IV D, E and F

10 Link, 13 Joints, 1F Kinematic Chains Group IV D			
 D1	 D2		
10 Link, 13 Joints, 1F Kinematic Chains Group IV E			
 E1	 E2	 E3	
10 Link, 13 Joints, 1F Kinematic Chains Group IV F			
 F1	 F2	 F3	 F4

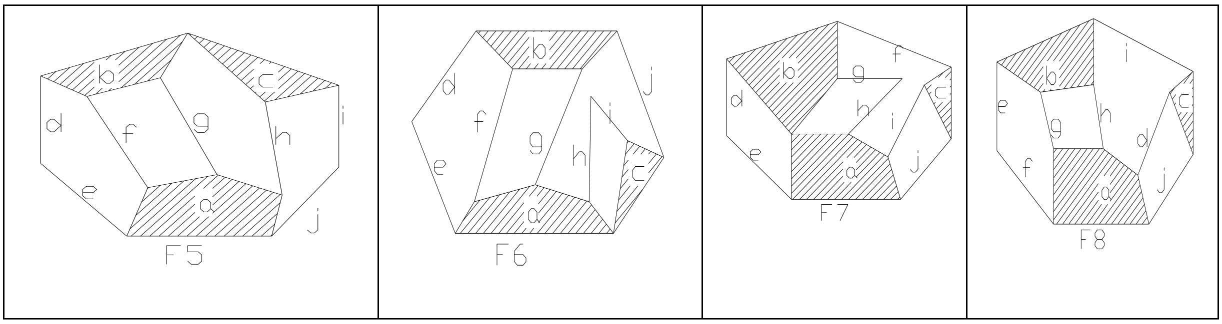


Table 2: Identification Number or Structural Invariants of KCs Group IV D, E and F

S.N.	KC	$n_5\,n_4\,n_3\,n_2$	SCPC	MCPC
Structural Invariants of KCs GROUP IV D				
1	D1	2008	1.0071e+008	4.5427e+007
2	D2	2008	2.3121e+008	1.0077e+008
Structural Invariants of KCs GROUP IV E				
3	E1	0307	1.3726e+007	5.9228e+006
4	E2	0307	1.6797e+007	6.3078e+006
5	E3	0307	1.1802e+007	4.9592e+006
Structural Invariants of KCs GROUP IV F				
6	F1	1117	3.8452e+007	1.2259e+007
7	F2	1117	7.2174e+007	2.3745e+007
8	F3	1117	3.6131e+007	1.5192e+007
9	F4	1117	1.1376e+008	4.4208e+007
10	F5	1117	7.9142e+007	3.2904e+007
11	F6	1117	3.8505e+007	1.3984e+007
12	F7	1117	8.8718e+007	3.7513e+007
13	F8	1117	1.1202e+008	4.5708e+007

REFERENCES

1. Zongyu Chang, Ce Zhan, Yuhu Yang and Yuxin Wang, “A new Method to Mechanism Kinematic Chain Isomorphism Identification,” Mech. Mach. Theory, Vol. 37, pp. 411-417, 2002. https://doi.org/10.1016/S0094-114X(01)00084-2
2. Mruthyunjaya T. S., “A Computerized Methodology for Structural Synthesis of Kinematic Chains, Part – 1, Formulation,” Mech. Mach. Theory, Vol. 19, No. 6, pp. 487 – 495, 1984. https://doi.org/10.1016/0094-114X(84)90055-7
3. Agrawal V.P. and Rao J.S., “The Mobility Properties of Kinematic Chains,” Mech. Mach. Theory, Vol. 22, pp. 497-504, 1987. https://doi.org/10.1016/0094-114X(87)90067-X
4. Sethi V. K. and Agrawal V. P., “Hierarchical Classification of Kinematic Chains – A Multigraph Approach,” Mech. Mach. Theory, Vol. 28, pp. 601 – 614, 1993. https://doi.org/10.1016/0094-114X(93)90039-X
5. Madan D. R. and Jain R. C., “Kinematic Chains-Isomorphism, Inversions and Degree of Similarity Using Concept of Connectivity,” Journal of Institution of Engineers (India), Vol. 82, pp. 164-169, 2002.
6. Rao a.b.s., Srinath A. and Rao A.C., “Synthesis of Planar Kinematic Chains,” Journal of Institution of Engineers (India), vol.86, pp 195-201, 2006.
7. Mitsi S., Bouzakis K. D., Mansour G. and Popescu I., “Position Analysis in Polynomial Form of Planer Mechanisms with Assure Groups of Class – 3 including revolute and Prismatic Joints,” Mech. Mach. Theory, Vol. 38, pp. 1325-1344, 2003. DOI: 10.1016/S0094-114(03)00090-9
8. Uicker J. J. and Raicu A., “A method for the Identification and Recognition of Equivalence of Kinematic Chains,” Mech. Mach. Theory, Vol. 10, pp. 375 – 383, 1975. https://doi.org/10.1016/0094-114X(75)90037-3
9. Mruthyunjaya T.S. and Balasubramanium H.R., “In Quest of Reliable and Efficient Computational Test for Detection of Isomorphism in Kinematic Chains,” Mech. Mach theory, Vol. 22, No 4, pp 131-139, 1987. https://doi.org/10.1016/0094-114X(87)90036-X
10. Schende S. and Rao A. C.,”Isomorphism in Kinematic Chains,” Mech. Mach. Theory, Vol. 29, No. 7 pp. 1065 – 1070, 1994. https://doi.org/10.1016/0094-114X(94)90073-6
11. Chu Jin-Kui and Cao Wei-Qing, “Identification of Isomorphism among Kinematic Chains and Inversions Using Link’s Adjacent-Chain-Table,” Mech. Mach. Theory, Vol. 29, pp. 53 – 58 1994. https://doi.org/10.1016/0094-114X(94)90019-1

12. Yadav J. N. , Pratap C. R.and Agrawal V. P., "Mechanisms of a kinematic chain and degree of structural similarity based on the concept of link – path code," Mech. Mach. Theory, Vol. 31, pp. 865 – 871, 1996. https://doi.org/10.1016/0094-114X(96)00001-8
13. Yadav J. N. , Pratap C. R.and Agrawal V. P., "Computer Aided detection of isomorphism among binary chains using link – link multiplicity distance concept," Mech. Mach. Theory, Vol. 31 , pp. 873 – 877, 1996. https://doi.org/10.1016/0094-114X(96)00002-X
14. Yadav J. N. , Pratap C. R.and Agrawal V. P.,, "Computer Aided Detection of Isomorphism Among Kinematic Chains and Mechanisms," Institution of Engineers (India), Vol. 82, pp. 51-55, 2002
15. Rao A. C., "Application of Fuzzy Logic for the Study of Isomorphism, Inversions, Symmetry, Parallelism and Mobility in Kinematic Chains," Mech. Mach. Theory, Vol. 35, pp. 1103-1116, 2000. https://doi.org/10.1016/S0094-114X(99)00060-9
16. Kong F. G., Li Q. and Zhang W., J., "An Artificial Neural Network Approach to Mechanism Kinematic Chains Isomorphism Identification," Mech. Mach. Theory, Vol. 34, pp. 271-283, 1999. https://doi.org/10.1016/S0094-114X(98)00035-4
17. Rao A. C., Pratap and B. Deshmukh, "Computer Aided Structural Synthesis of Planer Kinematic Chains Obviating the Test of Isomorphism," Mech. Mach. Theory, Vol. 36, pp. 489-506, 2001. https://doi.org/10.1016/S0094-114X(00)00030-6
18. He.P.R., Zhang W.J., Li Q. and Wu F.X., "A New Method For Detection of Graph Isomorphism Based On The Quadratic Form," ASME Journal of mechanical design, vol.125,pp 640-646,2003
19. Tang C. S. and Liu Tyang, "The degree code – A New Mechanism Identifier," Trends and Developments in Mechanisms, Machines and Robotics, Kissimmee, Florida, U.S.A., 1, pp. 147-151, 1988. https://doi.org/10.1115/1.2919236
20. Zhao J.S., Zhou K.and feng Z.J., "A theory of freedom for mechanisms," Mech. Mach. Theory, Vol. 39, pp. 621-643, 2004. https://doi.org/10.1016/j.mechmachtheory.2003.12.005
21. Hasan A., Dargar A. and Khan R. A., "Isomorphism and inversions of kinematic chains up to 10-links," Proceedings of the 13th National Conference on Mechanisms and Machines, 2007.
22. Hasan A., "Some Studies on Characterization and Identification of Kinematic Chains and Mechanisms." Ph D Thesis,Jamia Millia Islamia, New Delhi, India, 2007.
23. Hasan A., " Isomorphism and Inversions of Kinematic Chains up to 10 Links, "Journal of 'Institution of Engineers (India), Vol. 90, pp.10-14, 2009.
24. Dargar A. , Khan R..A.,Hasan A., " Identification of Isomorphism among Kinematic Chains and Inversions Using Link Adjacency Values,"International J. of Mech. and Materials Engineering (IJMME), pp.309-315, No.3, Vol. 4(2009)
25. Dargar A. , Khan R..A.,Hasan A., " Application of Link Adjacency Values to Detect Isomorphism among Kinematic Chains,"Int. J. Mech. Mater. Design, 6,157-162,2010.
26. Rizvi S.S.H., Hasan A., Khan R.A.," A New for distinct inversions and isomorphism detection in kinematic chains," Int. J. Mechanisms and Robotic Systems, Inderscience Enterprises Ltd ,Vol. 3, No. 1, pp. 48-59, 2016.
27. Leiying He, Faxin Liu, Liang Sun & Chuanyu Wu.: Isomorphic identification for kinematic chains using variable high-order adjacency link values. J Mech Sci Technol 33, 4899–4907 (2019). https://doi.org/10.1007/s12206-019-0930-9.
28. L. Sun, X. Chen, C. Wu, G. Zhang and Y. Xu.: Synthesis and design of rice pot seedling transplanting mechanism based on labeled graph theory. Comput. Electron. Agr. 143, 249–261(2017). https://doi.org/10.1016/j.compag.2017.10.021.
29. K. R. Rajnefsh and Sunil P.: A new algorithm of links labelling for the isomorphism detection of various kinematic chains using binary code. Mechanism and Machine Theory 131,1–32 (2019). https://doi.org/10.1016/j.mechmachtheory.2018.09.010.
30. S. H. Rizvi, A. Hasan and Khan R. A.: An efficient algorithm for distinct inversions and isomorphism detection in kinematic chains. Perspectives in Science 8, 251–253(2016). https://doi.org/10.1016/j.pisc.2016.03.022Get rights and content
31. W. Sun, J. Kong and Sun L.: A joint-joint matrix representation of planar kinematic chains with multiple joints and isomorphism identification. Adv. Mech. Eng. 10 (6), 1–10(2018). https://doi.org/10.1177/1687814018778404.
32. V. Dharanipragada and Chintada M.: Split hamming string as an isomorphism test for one degree-of-freedom planar simple-jointed kinematic chains containing sliders. ASME J. Mech. Des. 138 (8), 082301–1–082301–8(2016). https://doi.org/10.1115/1.4033611.
33. W. Sun, J. Y. Kong and Sun L. B.: The improved hamming number method to detect isomorphism for kinematic chain with multiple joints. J. Adv. Mech. Des. Syst. Manuf. 11 (5), 17–00479–1–10(2017).

34. Preben W. Jensen, Classical and Modern Mechanism for Engineers and Inventors, Marcel Decker, Inc, New York ,1991.