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# A Graph Theoretical Approach for Frequency Reuse in a Mobile Computing Environment

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**Abstract:** Effective usage of frequency resource in a mobile omputing nvironment is a challenging problem.The key idea is to control mutual interference among neighboring cells in a systematic way and at the same time minimizing the usage of frequency utilization. In this paper we present a graph theoretic approach to address this problem. The mutually interfering cells are represented as the vertices of a graph and a mathematical assignment of frequencies is done in order to avoid interference up to four levels simultaneously focusing on the minimal usage of frequency resources. Then the minimum value of the maximum assigned frequency among all such assignments, called the span of the graph is obtained, which serves as the highest frequency to be used in order to avoid interference up to four levels. In this paper, we define a labeling for any connected graph with at least two vertices having a Hamiltonian path and mathematically obtain the span, so as avoid interference at four levels.

**Keywords:** Frequency areuse, amobile acellular asystems, aMulti alevel adistance alabeling, apath acoloring.

## 1. Introduction

A simple, undirected, finite and connected graphs are considered in this present paper. The standard graph theory terminologies are from [1], [2]. Due to the rapid growth of wireless networks, the frequency reuse problem has gained importance since recent years. In a mobile cellular environment, the frequency resource should be reused to minimize its usage , besides keeping the interference at tolerable limits [3], [4]. An efficient system capacity could be obtained through reuse of frequency in the mobile cellular environment. In this context, we model the frequency assignment problem as a graph labeling or graph coloring problem. We introduce a novel concept of path coloring of graphs to avoid interference upto four levels. The mutually interfering cells are treated as vertices of a graph and edges are drawn between them to show possible interference between them. A mapping from the vertex set of the graph to the set of positive integers is defined in such a way that there exists at least one path between every pair of vertices in the graph, where interference upto four levels can be avoided. Over all such labelings the minimum of the highest label used, called the span of the graph is obtained, which represents the highest frequency required for a interference free transmission. In section II of the article we define some basic graph theory terminologies, discuss work done in the area earlier and the motivation for our work. In section III we provide our main results and how it can be applied to the frequency reuse problem. Section IV has some concluding remarks about future work.

## 2. RELATED WORK

A Graph, $G(V,E)$, is a mathematical entity consisting of two sets, called as the vertex set, denoted by $V$ and edge set denoted by $E$. The set $V$ consists of vertices or points and the set $E$, consists of lines joining the vertices called edges, indicating some relation between the vertices. Graphs are versatile to use in a discussion which involves a set of discrete objects and relations between them. Two vertices $u$ and $v$ are said to be adjacent if they are joined by an edge. A $uv-$ Path in a graph $G$ is an alternating sequence of vertices and edges, beginning at a vertex $u$ and ending at another vertex $v$ in which no vertex is repeated. There may be several paths between a pair of vertices. The length of the shortest path between two vertices $u$ and $v$ in $G$ is called as the Distance between them. The largest distance between any two vertices of a graph is called as the Diameter of the graph. The number of edges incident to a vertex $v$ of a graph $G$ is called the Degree of the vertex $v$. The Maximum degree of $G$ is the degree of the vertex with highest degree. A graph is said to be Connected if there is at least one path between every pair of vertices. A graph $H$ is called a Subgraph of a graph $G$

if its vertices and edges are subsets of the vertex and edge sets of the graph $G$ respectively. A sub graph of $G$ that contains all the vertices of $G$ is called a Spanning sub graph of $G$. A Cycle is a closed path which begins and ends at the same vertex,denoted by $C_n$, where $n$ is the number of vertices. A graph in which every pair of vertices are adjacent is called a Complete graph or Clique, denoted by $K_n$, where $n$ is the number of vertices in the graph. A graph whose vertices can be partitioned into two subsets such that, every vertex of one set is adjacent to every vertex of the other set only is called a Complete Bipartite graph, denoted by $K_{m,n}$, where $m$ and $n$ are the number of vertices in the two sets. If $m = 1$, the graph is called a star graph denoted by $K_{1,n}$. An Interval graph is a graph in which vertices represent some interval on the real line and an edge between vertices exist if the corresponding intervals intersect. A Hamiltonian path is a path that traverses all the vertices of the graph $G$ exactly once. Two graphs $G$ and $H$ having an equal number of vertices, equal number of edges, that preserve adjacency are called Isomorphic graphs, denoted by $G \cong H$. A connected graph in which there is only one path between every pair of vertices is called a tree, denoted by $T_n$. A set of vertices in a graph is said to be an Independent set if no two vertices in the set are adjacent. A split graph is a graph whose vertex set can be partitioned into two subsets. One of which is an independent set and the other is a clique. A 2-edge connected split graph is obtained by removing minimum two edges of a split graph which then forms a clique and an independent set. The Cartesian Product of two graphs $G$ and $H$, denoted by $G \times H$ is the graph with $V(G) \times V(H)$ as the vertex set and $E(G \times H)$, as the edge set such that $\{((u,v),(u',v')) | (u = u' \text{ and } vv' \in E(H)) or (uu' \in E(G) \text{ and } v = v')\}$ where $u,v \in V(G)$ and $u',v' \in V(H)$. The Join of two graphs is the graph obtained by connecting all the vertices of one graph to all the vertices of the other graph.

In 1980, Hale et al. [5], [6], turned the frequency assignment problem, where channels had to be assigned to radio stations, in order to avoid interference as a graph labeling problem which is defined as follows: An $L(p_1, p_2, p_3, ..., p_m)$labeling of a graph $G$, is labelling of vertices with non negative integers such that the vertices at distance $i$ are assigned with the labels whose difference is at least $p_i$ .

Later, Roberts proposed a concept ”close” or ”very close” in FM radio stations during 1991 in which ”very close” represents the adjacent vertices and ”close” represents the vertices at distance two [7].

In case of very close stations, frequencies assigned to them must differ by at least 2 and when they are close to each other by at least 1. This is called as the distance two labeling which is extensively studied as $L(2, 1)$-labelling in [8]–[19].

Practically, interference can occur at levels more than two also. Jean Clipperton et al., studied $L(3, 2, 1)$-labeling problems and defined $L(3, 2, 1)$-labelling as an assignment of non negative integers to each vertex of $G$ such that the vertices at distance 1, 2, 3 are labelled with integers that differ by at least 3, 2, 1 respectively [20]. Later, Soumen Atta and Priya Ranjan Sinha Mahapatra defined the $L(4, 3, 2, 1)$-labeling as an assignment of non negative integers to each vertex of $G$ such that the vertices at distance 1, 2, 3, 4 are labelled with a difference of at least 4, 3, 2, 1 respectively. The smallest positive integer $k$, where $k$ is the maximum label in a $L(4, 3, 2, 1)$-labeling of $G$ is called $L(4, 3, 2, 1)$-labeling number of graph $G$, denoted by $k(G)$. They obtained $k(G)$ for paths, cycles, complete graphs and complete bipartite graphs [21]. In [22], R. Sweetly and J. Paulraj Joseph also defined $L(4, 3, 2, 1)$-labeling of $G$ and obtained an upper bound for $k(G)$ in terms of maximum degree of $G$. A $L(4, 3, 2, 1)$labeling of $K_5$ is shown in Figure 1(a). In $L(4,3,2,1)$-labeling , the condition must be satisfied between every pair of vertices. Here in Figure 1(a), $L(4,3,2,1)$-labeling condition is satisfied between all pairs of vertices. Hence, $L(4,3,2,1)$- labeling number, $k(G) = 17$.

Sk Amanathulla, Madhumangal Pal discussed the $L(3, 2, 1)$ and $L(4, 3, 2, 1)$-labeling problem on interval graphs [23]. Variations of the problem for higher levels of interference has been studied in [24]–[27], [29].

According to Ruxandra Marinescu-Ghemeci [28], arbitrary paths provide a safe communication in networks. In order to solve interference or security problems, it is necessary to have at least one path between every pair of vertices such that the labeling restricted to that path satisfies interference condition. Hence rather than seeking interference-free condition between every pair of vertices, they look for at least one such path between every pair of vertices and call this as a path coloring. Restricting the levels of interference to two, in [28], they studied $L(2, 1)$-path coloring.

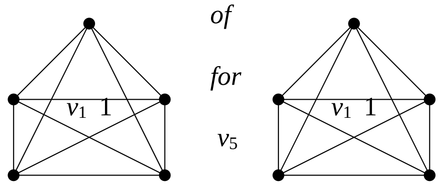
If there exists a $L(2, 1)$-path between every pair of vertices, then they called $G$ as 2-radio connected. If $c : V(G) \rightarrow N^*$ is a $L(2, 1)$-path coloring of $G$, the highest label used was called as *values* of $c$, denoted by $val(c)$. The minimum of $val(c)$ over all such labelings of $c$ was called 2-radio connection number, denoted by $\lambda_c(G)$. They obtained the upper bound and lower bound for $\lambda_c(G)$ where $G$ is connected graph with at least 5 vertices, exact values for graphs having Hamiltonian path, complete graphs, cycles, complete bipartite graphs, 2edge connected split graphs, graph obtained by cartesian product, join of two graphs.

Motivated by this we seek at least one path between every pair of vertices along which interference is avoided at 3 levels and hence defined $L(3, 2, 1)$-path coloring of graph $G$ in [30]. Analogous to 2-connection number of $G$, we defined 3-connection number, $k_c(G)$ in [30] and obtained results for $k_c(G)$ where $G$ ' $C_n$ or $K_n$, $K_{m,n}$, 2-edge connected Split graph, Cartesian product of two graphs, Join of two graphs.

In this paper, we extend this concept to 4 levels by defining $L(4, 3, 2, 1)$-path coloring of graphs and we find $k_c(G)$ for any graph with at least 2 vertices and containing a hamiltonian path.

Definition II.1. *A labeling $g : V(G) \rightarrow Z^+$ defined such that there exists at least one path $P$ between every pair of vertices in which the labeling imputed in this path must be a $L(4, 3, 2, 1)$-labeling is called $L(4, 3, 2, 1)$-path coloring of G. The maximum label assigned to any vertex of G under g, is called the span of g. The minimum value of span of g taken over all such labelings g is the 4−connection number or $L(4, 3, 2, 1)$-connection number of G, denoted by $k_c(G)$.*

Example II.2. *A $L(4, 3, 2, 1)$-path coloring of $K_5$ is shown in Figure 1(b). In the Figure 1(a), span of $L(4, 3, 2, 1)$labelling of $K_5$ is 17. In the Figure 1(b), $v_1$-$v_2$-$v_3$-$v_4$-$v_5$ is a $L(4, 3, 2, 1)$-path coloring of $K_5$ with span 11. Hence, in general, a path coloring reduces the span of a given graph. A heuristic approach in finding the span of a graph goes out of hand as the number of vertices increases. In the next section we develop a labeling algorithm and also give mathematical proof for the minimality of span obtained.*




(a) $L(4, 3, 2, 1)$-labelling of $K_5$ (b) $L(4, 3, 2, 1)$-path coloring of $K_5$

Fig. 1

Remark II.3. *We first make the following simple observation: As tree is a graph in which there exists only one path between every pair of vertices, the definitions of $L(4,3,2,1)$ and $L(4,3,2,1)$−path coloring coincide for trees. Since paths and star graphs, are special cases of trees, for any path $P_n$ with $n$ vertices, $k_c(P_n) = k(P_n)$ and for the star graph also $kc(K1,n) = k(K1,n)$.*
*In [22] determined*

$$k(P_n) = \begin{cases} 5 & \text{if } n = 2, \\ 8 & \text{if } n = 3, \\ 9 & \text{if } n = 4, \\ 11 & \text{if } 5 \le n \le 7, \\ 12 & \text{if } 8 \le n \le 12, \\ 13 & \text{if } n \ge 13. \end{cases}$$

*and $k(K_{1,n}) = 3n + 2$. Hence the above results are also the span for $L(4,3,2,1)$path coloring of paths and star graph respectively.*

**3. MAIN RESULTS**

Theorem III.1. *For any connected graph $G$ with $n$ vertices,*

1) $k_c(G) = 5$ *if* $n = 2$
2) $k_c(G) = 8$ *if* $n = 3$
3) $k_c(G) = 9$ *if* $n = 4$ *and* $G$ 6' $K_{1,3}$

*Proof:*

1) The only connected graph with two vertices is $P_2$. Define $g : V(G) \rightarrow Z^+$, such that $g(v_1) = 1$, $g(v_2) = 5$. Hence span for $P_2$ is 5 .
2) The only connected graphs with three vertices are $P_3$ or $C_3$. Define $g : V(G) \rightarrow Z^+$, such that $g(v_1) = 8$, $g(v_2) = 1$, $g(v_3) = 5$. Hence span for $P_3$ or $C_3$ is 8. $\therefore$ $k_c(G) = k_c(P_3) = k_c(C_3) = 8$.

3) If $G$ is any connected graph on 4 vertices and $G$ 6' $K_{1,3}$ then $G$ contains $P_4$ as a spanning connected sub graph which contains all the vertices of $G$, therefore, $k_c(G) \leq k_c(P_4) = 9$. To prove that $k_c(G) \nless 9$. Suppose $k_c(G) = 8$. Then there exists two vertices $v_i$ and $v_j$ such that $g(v_i) = 1$ and $g(v_j) = 8$. Let $P$ be the $L(4, 3, 2, 1)$-path between $v_i$ and $v_j$. Then $P$ contains at most two vertices between $v_i$ and $v_j$. Let $x$ and $y$ be the other two vertices of $G$.

Case 1: $P$ is of the form $v_i - x - y - v_j$.

Then $g(x) = 5$ and $g(y) \geq 12$, a contradiction.

Case 2: $P$ is of the form $v_i - x - v_j$.

Then $g(x) \geq 5$ and $g(x) \leq 4$, a contradiction.

Case 3: There are no vertices between $v_i$ and $v_j$ on $P$. Then the edge $v_iv_j$ itself is a $L(4, 3, 2, 1)$-path between $v_i$ and $v_j$. The edge $v_iv_j$ can be extended to at most two other vertices say $x$ and $y$ of $G$ as follows: Subcase 3.1: Suppose the edge $v_iv_j$ can be extended to both $x$ and $y$ in the form $v_i - v_j - x - y$.

Then $g(x) = 4$ and $g(y) \geq 11$, a contradiction. Subcase 3.2: Suppose the edge $v_iv_j$ can be extended in the form $y - x - v_i - v_j$. Then $g(x) = 5$. If $g(x) = 5$ then $g(y) \geq 10$, a contradiction.

Subcase 3.3: Suppose the edge $v_iv_j$ can be extended on both sides to exactly one vertex say $x{-}v_i{-}v_j{-}y$. Then $g(x) = 5$ and $g(y) \geq 12$, a contradiction. Subcase 3.4: If edge $v_iv_j$ can be extended to exactly one vertex say $x$ in the form $v_i - v_j - x$ then the other vertex $y$ should be adjacent to $v_j$. Hence $G$ ' $K_{1,3}$, a contradiction.

A similar proof holds if $x$ is adjacent to $v_i$. In all the cases, there is no $L(4, 3, 2, 1)$-path coloring of $G$ with eight or fewer colors. Hence the result.

Remark III.2. *If* $G$ 6' $K_{1,3}$, *then,* $k_c(K_{1,3}) = k(K_{1,3}) = 11$ *(By R. Sweetly et al.,* $k(K_{1,n}) = 3n + 2$ *[22]). By the above theorem III.1 point 3, we observe that if G is a connected graph on n vertices containing a Hamiltonian path, then* $k_c(G) = k_c(P_n) = k(P_n)$. *Also, Span of the graph not containing a Hamiltonian path will be more than that of the graph containing Hamiltonian path with same number of vertices.*

Theorem III.3. *Let G be a connected graph with* $5 \leq n \leq 7$ *vertices and containing Hamiltonian path. Then* $k_c(G) = 11$.

*Proof:* Assume that $G$ contains a Hamiltonian path. Since a Hamiltonian path is a spanning connected sub graph of $G$, $k_c(G) \leq k_c(P_n) \leq 11$ for $5 \leq n \leq 7$.

To show that $k_c(G) \nless 11$.

Suppose $k_c(G) = 10$, there exists a $L(4, 3, 2, 1)$-path $P$ between $v_i$ and $v_j$ such that $g(v_i) = 1$ and $g(v_j) = 10$. Now, $P$ contains at most 5 vertices between $v_i$ and $v_j$.

Case 1: $P$ contains 5 vertices between $v_i$ and $v_j$.

Let the 5 vertices be in the order $v_i - x - y - z - u - v - v_j$. Then $g(x) \geq 5$.

Let $g(x) \in \{5, 6, 7\}$. Since $g(v_j) = 10$, $g(y) = 9$ and $g(z) \geq 13$, a contradiction.

If $g(x) \geq 8$ then $g(y) \geq 4$ and $g(z) \geq 12$, a contradiction.

Case 2: $P$ contains 4 vertices between $v_i$ and $v_j$.

Let the 4 vertices be in the order $v_i - x - y - z - u - v_j$. Since $g(v_i) = 1$, $g(x) \geq 5$ and since $g(v_j) = 10$, $g(x) \leq 9$ which implies $g(x) \in \{5,6,7,8,9\}$.

Subcase 2.1: If $g(x) \in \{5,6,7\}$, then $g(y) \geq 12$, a contradiction.

Subcase 2.2: If $g(x) = 8$ or $g(x) = 9$ then $g(z) \geq 13$, a contradiction.

Case 3: $P$ contains three vertices between $v_i$ and $v_j$. Let $x$, $y$ and $z$ be the three vertices in the order $v_i - x - y - z - v_j$. Since $g(v_i) = 1$ and $g(v_j) = 10$, $g(x) \in \{5,6,7,8\}$.

Subcase 3.1: If $g(x) \in \{5,6,7\}$, then $g(y) \geq 13$, a contradiction.

Subcase 3.2: If $g(x) = 8$, then $g(y) = 4$ and $g(z) \geq 14$, a contradiction.

In any case, it is a contradiction.

Case 4: $P$ contains two vertices between $v_i$ and $v_j$.

Let $x$ and $y$ be the two vertices in the order $v_i - x - y - v_j$. As $g(v_i) = 1$, $g(x) \in \{5,6,7\}$. In any case, $g(y) = 14$, a contradiction.

Case 5: $P$ contains one vertex between $v_i$ and $v_j$. Let $x$ be the vertex between $v_i$ and $v_j$. Let the path $v_i{-}x{-}v_j$ be $P^0$.

Then, as $g(v_i) = 1$, $g(x) \geq 5$. As $g(v_j) = 10$, $g(x) \leq 6$ which implies that $g(x) = 5$ or 6.

Now, $P^0$ can be extended to the other vertices say $y$ and $z$ as follows:

Subcase 5.1: $P^0$ can be extended to the form $z{-}y{-}v_i{-}x{-}v_j$. If $g(x) = 5$ or $g(x) = 6$ then $g(z) \geq 12$, a contradiction.

Subcase 5.2: $P^0$ can be extended to the form $v_i{-}x{-}v_j{-}y{-}z$. If $g(x) = 5$ or $g(x) = 6$ then $g(y) \geq 14$ or $g(z) \geq 13$ respectively, a contradiction.

Subcase 5.3: $P^0$ can be extended to the form $y{-}v_i{-}x{-}v_j{-}z$. As in previous two cases, $g(z) \geq 14$ or $g(y) \geq 12$, a contradiction.

Case 6: $P$ contains no vertices between $v_i$ and $v_j$. Then an edge $v_i - v_j$ itself is a $L(4, 3, 2, 1)$-path between $v_i$ and $v_j$, say $P^{00}$.
This path $P^{00}$ can be extended to atleast 3 vertices say $x$, $y$ and $z$ of $G$ as follows:
Subcase 6.1: $P^{00}$ can be extended to all $x$, $y$ and $z$ in the form $v_i - v_j - x - y - z$
Since $g(v_i) = 1$, $g(x) \geq 4$. Since $g(v_j) = 10$, $g(x) \leq 6$ which implies that $g(x) \in \{4,5,6\}$.
If $g(x) \in \{4,5\}$ then $g(y) \geq 13$, a contradiction.
If $g(x) = 6$ then $g(y) = 3$ and $g(z) = 12$ , a contradiction. Subcase 6.2: $P^{00}$ can be extended to the form $z{-}y{-}x{-}v_i{-}v_j$ Here, for any values $g(x)$, $g(y) \geq 12$, a contradiction. Subcase 6.3: $P^{00}$ can be extended to the form $x{-}v_i{-}v_j{-}z{-}y$ Here $g(x) \in \{5,6,7\}$.
If $g(x) = 5$ then $g(y) \geq 14$, a contradiction If $g(x) \in \{6,7\}$ then $g(y) = 4$ and $g(z) = 13$, a contradiction. Subcase 6.4: $P^{00}$ can be extended to the form $y{-}x{-}v_i{-}v_j{-}z$ As in the previous two cases, $g(y) \geq 13$, a contradiction.
$\therefore$ In all the cases, there is no $L(4, 3, 2, 1)$-path coloring of $G$ with ten or fewer colors.
Hence, if $G$ contains a Hamiltonian path, $k_c(G) = 11$.

Theorem III.4. *Let G be a connected graph with n vertices. Then*

1) $k_c(G) = 12$ *if* $8 \leq n \leq 12$ *and G contains a Hamiltonian path.*
2) $k_c(G) \geq 13$, *otherwise*

*Proof:*

1) Assume that $G$ contains a Hamiltonian path. Since a Hamiltonian path is a spanning connected sub graph of $G$, $k_c(G) \leq k_c(P_n) \leq 12$ for $8 \leq n \leq 12$.
To show that $k_c(G) \not< 12$.
Suppose $k_c(G) = 11$, there exists a $L(4, 3, 2, 1)$path $P$ between $v_i$ and $v_j$ such that $g(v_i) = 1$ and $g(v_j) = 11$. Now, $P$ contains at most 10 vertices between $v_i$ and $v_j$.
Case 1: $P$ contains $m, 6 \leq m \leq 10$ vertices between $v_i$ and $v_j$.
Let $x$, $y$, $z$ be the at least three vertices between $v_i$ and $v_j$ in the order $v_i{-}x{-}y{-}z{-}\cdots{-}v_j$. Then $g(x) \geq 5$, $g(y) \geq 9$ and $g(z) \geq 13$, a contradiction. Case 2: $P$ contains 5 vertices between $v_i$ and $v_j$. Let $x$, $y$, $z$ be the at least three vertices between $v_i$ and $v_j$ in the order $v_i{-}x{-}y{-}z{-}\cdots{-}v_j$. Since $g(v_i) = 1$, $g(x) \geq 5$.
Subcase 2.1: Let $g(x) \in \{5, 6, 7\}$. Since $g(v_j) = 11$, $g(y) \in \{9, 10\}$ and $g(z) \geq 13$, $g(z) \geq 14$ respectively which is a contradiction.
Subcase 2.2: If $g(x) \geq 8$ then $g(y) \geq 4$ and $g(z) \geq 13$, a contradiction.
Case 3: $P$ contains 4 vertices between $v_i$ and $v_j$. Let $x$, $y$, $z$ and $u$ be the four vertices between $v_i$ and $v_j$ in the order $v_i - x - y - z - u - v_j$.
Since $g(v_i) = 1$, $g(x) \geq 5$. Since $g(v_j) = 11$, $g(x) \leq 10$ implies that $g(x) \in \{5,6,7,8,9,10\}$. Subcase 3.1: If $g(x) = 5$ then $g(y) = 9$ and $g(z) \geq 13$ which is a contradiction.
Subcase 3.2: Let $g(x) \in \{6, 7\}$. Since $g(v_j) = 11$, $g(y) \geq 13$ and $g(y) \geq 14$ respectively which is a contradiction.
Subcase 3.3: If $g(x) \in \{8,9,10\}$ then $g(y) \geq 4$ and $g(z) \geq 14$, a contradiction. Case 4: $P$ contains 3 vertices between $v_i$ and $v_j$. Let $x$, $y$ and $z$ be the vertices between $v_i$ and $v_j$ in the order $v_i - x - y - z - v_j$.
Then, as $g(v_i) = 1$, $g(x) \geq 5$. As $g(v_j) = 11$, $g(x) \leq 8$ which implies that $g(x) \in \{5,6,7,8\}$.
Also, Since $g(x) \in \{5,6,7\}$, $g(y) \geq 9$ and since $g(v_j) = 11$, $g(y) \leq 8$, a contradiction.
If $g(x) = 8$ then $g(y) = 4$ and $g(z) \geq 15$, a contradiction.
Case 5: $P$ contains 2 vertices between $v_i$ and $v_j$.
Let $x$ and $y$ be the vertices between $v_i$ and $v_j$. Then $g(x) \geq 5$ as $g(v_i) = 1$ and $g(x) \leq 8$ as $g(v_j) = 11$ implies that $g(x) \in \{5,6,7,8\}$.
Also, $g(y) \geq 9$ as $g(x) \in \{5,6,7\}$ and $g(y) \leq 7$ as $g(v_j) = 11$, a contradiction.
If $g(x) = 8$ then the path $v_i - x - y - v_j$ be $P$ which can be extended to the remaining four or more vertices of $G$ as follows:
Subcase 5.1: $P$ is extended on the right in the form $v_i - x - y - v_j - z - \ldots$
Since $g(x) = 8$, $g(y) = 4$. Since $g(y) = 4$, $g(z) = 15$,
a contraction.
Subcase 5.2: $P$ is extended on the left in the form $\cdots - z - v_i - x - y - v_j$
Then $g(x) = 8$ and $g(y) = 4$. Since $g(x) = 8$, $g(z) \geq 12$, a contradiction.
Case 6: $P$ contains one vertex between $v_i$ and $v_j$.
Let $x$ be the vertices between $v_i$ and $v_j$.
Then, as $g(v_i) = 1$, $g(x) \geq 5$. As $g(v_j) = 11$, $g(x) \leq 7$ which implies that $g(x) \in$

{5,6,7}.

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Let this path $v_i - x - v_j$ be $P$ which can be extended to the remaining five or more vertices of $G$ as follows:

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Subcase 6.1: $P$ can be extended to the form $v_i - x - v_j - y - z - ....$

Here if $g(x) = 5$ then $g(y) \geq 15$, a contradiction.

If $g(x) = 6$ or $g(x) = 7$ then $g(z) \geq 14$, a contradiction.

Subcase 6.2: $P$ can be extended to the form $\cdots - z - y - v_i - x - v_j$.

Here if $g(x) \in \{5,6\}$ then $g(y) = 8$ or 9 and $g(z) \geq 12$ or 13 respectively, a contradiction.

If $g(x) = 7$ then $g(y) = 13$, a contradiction.

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Subcase 6.3: $P$ can be extended to the form $y - v_i - x - v_j - z - u$.

Here if $g(x) = 5$ then $g(y) = 8$ and $g(z) \geq 15$, a contradiction.

If $g(x) \in \{6,7\}$ then $g(y) \geq 9$ and $g(z) = 3$ which implies that $g(u) \geq 14$, a contradiction. Case 7: There are no vertices between $v_i$ and $v_j$. Then an edge $v_i - v_j$ itself a $L(4, 3, 2, 1)$-path between $v_i$ and $v_j$, say $P$ . Since there are at least six vertices

00 left other than $v_i$ and $v_j$, the path $P$ may be extended to the remaining vertices as follows:

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Subcase 7.1: $P$can be extended to four vertices in the form $v_i - v_j - x - y - z - u$

Then, Since $g(v_i) = 1$, $g(x) \geq 4$.

Since $g(v_j) = 11$, $g(x) \leq 7$ which implies that $g(x) \in \{4,5,6,7\}$.

If $g(x) = 4$, then $g(y) = 8$ and $g(z) = 13$, a contradiction

If $g(x) = 5$ then $g(y) \geq 14$, a contradiction.

If $g(x) = 6$ or 7 then $g(y) = 3$ and $g(z) \geq 13$, a contradiction.

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Subcase 7.2: $P$can be extended to four vertices in the form $u - z - y - x - v_i - v_j$

Then, as $g(v_i) = 1$, $g(x) \geq 5$. As $g(v_j) = 11$, $g(x) \leq 8$ which implies that $g(x) \in \{5,6,7,8\}$.

If $g(x) = 5$ then $g(y) = 9$ and $g(z) \geq 13$, a contradiction.

If $g(x) = 6$ or 7 then $g(y) \geq 13$, a contradiction.

If $g(x) = 8$ then $g(y) = 4$ and $g(z) = 12$, a contradiction.

∴ In all the cases, there is no $L(4, 3, 2, 1)$-path coloring of $G$ with eleven or fewer colors. Hence, if $G$ contains a Hamiltonian path, $k_c(G) = 12$.

Below we give an algorithm for a $L(4,3,2,1)$-path coloring of a graph $G$ which has a Hamiltonian path.

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Algorithm 1: $L(4, 3, 2, 1)$-connection number, $k_c(G)$ where $G$ has a Hamiltonian path.

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Input: The adjacency matrix of a graph $G$ with $n$ vertices and $diam(G)$ .

Output: A $L(4,3,2,1)-$ path coloring of $G$ and $k_c(G)$ .

Begin

1) Choose a Hamiltonian path $P$ and label the vertices as $v_1, v_2, v_3, ... v_n$

2) Set $g(v_1) = 1$ if $i \equiv 1 (mod\ 7)$ else $g(v_2) = 5$ if $i \equiv 2 (mod\ 7)$ else $g(v_3) = 9$ if $i \equiv 3 (mod\ 7)$ else $g(v_4) = 13$ if $i \equiv 4 (mod\ 7)$ else $g(v_5) = 3$ if $i \equiv 5 (mod\ 7)$ else $g(v_6) = 7$ if $i \equiv 6 (mod\ 7)$ else $g(v_7) = 11$ if $i \equiv 0 (mod\ 7)$

End

To achieve the goals of wider coverage range and higher data packet throughput wider bandwidth has to be used. By treating the mobile stations as vertices of a graph and showing edges between possibly interfering stations the problem can be modeled as a labeling problem in graph theory as described in the paper. Stations which are at a geographical distance of 100kms say, can be treated as distance one vertices and 200 kms as distance two vertices and so on. Scaling up the integers used in the labeling to available frequencies one can apply the above labeling procedure to avoid a 4-level interference, thus reducing the width of the bandwidth required.

## CONCLUSION

In this work, we deal with the problem of assigning frequencies to the very close transmitters which keeps down the maximum frequency used in wireless communication networks. The idea of path coloring is used, rather than the normal coloring problem. The path coloring when applied to various graphs leads to a span much lesser than the normal coloring as shown in the Table 1.

Table 1: Comparison between $L(4,3,2,1)$-coloring and $L(4,3,2,1)$- path coloring of paths, cycles and complete graphs :

| $G$ ' | $L(4,3,2,1)$coloring | $L(4,3,2,1)$- path coloring |
|---|---|---|
| $P_2$ | 5 | 5 |
| $P_3$ | 8 | 8 |
| $P_4$ | 9 | 9 |
| $P_n$, $5 \leq n \leq 7$ | 11 | 11 |
| $P_n$, $8 \leq n \leq 12$ | 12 | 12 |
| $P_n$, $n \geq 13$ | 13 | 13 |
| $C_3$ | 9 | 8 |
| $C_4$ | 11 | 9 |
| $C_n$, $5 \leq n \leq 7$ | 13 | 11 |
| $C_n$, $n$=8,11,16,17,23,29 | 14 | 12 ($\forall n, 8 \leq n \leq 12$) |
| $C_n$, $n \geq 13$ | 13 | 13 |
| $K_3$ | 9 | 8 |
| $K_4$ | 13 | 9 |
| $K_5$ | 17 | 11 |
| $K_6$ | 21 | 11 |
| $K_7$ | 25 | 11 |
| $K_8$ | 29 | 12 |
| $K_9$ | 33 | 12 |
| $K_{10}$ | 37 | 12 |
| $K_{11}$ | 41 | 12 |
| $K_{12}$ | 45 | 12 |
| $K_n$, $n \geq 13$ | $4n - 3$ | 13 |

Also, to reduce interference to the minimum in current and future communication networks, we proposed an algorithm which reduces the span of a given graph $G$ where $G$ is a graph containing Hamiltonian path. In our future work, we will be dealing with the graphs which do not contain a Hamiltonian path. Here, we observe that our value of $k_c(G)$ is better compared to the existing $k_c(G)$ value found by others in several bench marking labeling problems.

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