Research Article

New analytical Solutions of (3+1)-dimensional Shallow Water Wave equation (SWW)

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Abstract: In this paper, we have obtained new analytical solutions of (3+1)-dimensional SWW equation with Kudryashov method. The study of Shallow water wave equation plays an imperative role in wave theory. For calculation software Maple is used. The solutions obtained by this method are new.

Keywords: (3+1)-dimensional SWW equation, Kudryashov method, Analytical solutions.

1. Introduction

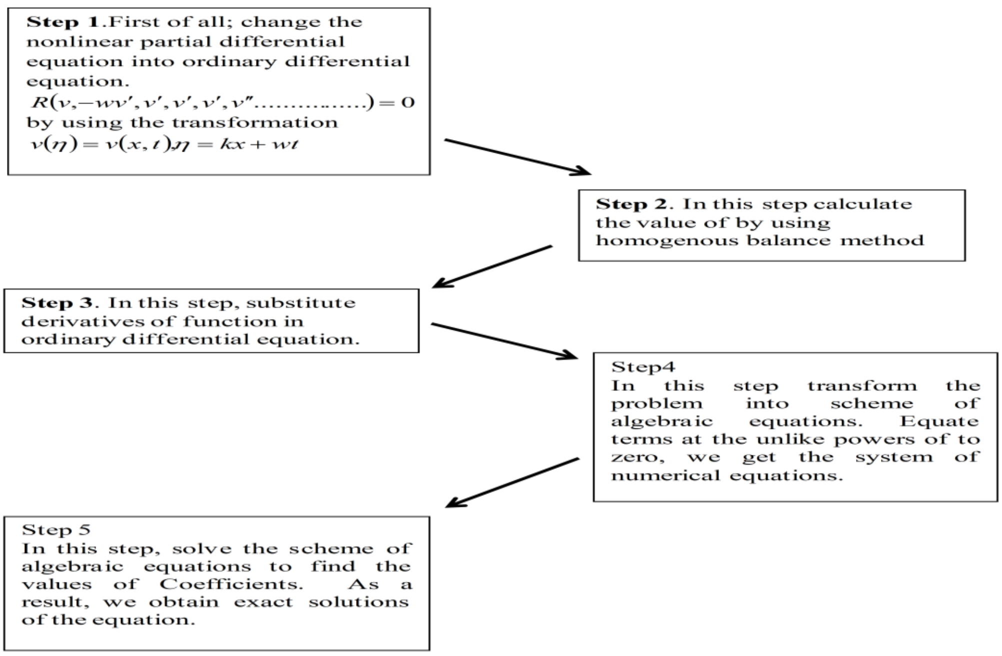
Waves in different parts of oceans are referred to by different names. Those differences are dependent on wave length. A shallow water wave is one that occurs at depths shallower than wave length of wave divided by 20. The (3+1)-dimensional SWW is

$$v_{xxxx} + 3v_{xx}v_y + 3v_x v_{xy} - v_{yt} - v_{xz} = 0 \qquad (1.1)$$

Many researchers have obtained exact and numerical solutions of SWW equation. Peter A. Clarkson *et al.* [8] studied symmetry reduction and accurate solutions of SWW equation. B. Aydm *et al.* [4] obtained hodograph transform solutions of one dimensional nonlinear SWW equation by using integral transform techniques.

A.A Imani *et al.* [2] use reform of variation iteration method for computing coupled Whitham-Broer-Kaup shallow water. Elizabeth L. Mansfield *et al*. [6] used classical and non classical reduction for (2+1)-dimensional SWW equation. N.A. Matskevich *et al*. [7] present a few approaches to solve problems of shallow water oscillation in parabolic basin. Wiiliam Carlisle [9] find analytical solutions correspond to time dependent motions in parabolic basins. Ahmet *et al*. [3] obtained analytical solution for (2+1) and (3+1)-dimensional SWW equation by using (G'/G) expansion method. Bijan *et al.* [5] used an extended elliptic function method to find solution of generalized shallow water wave equation. Anjali Verma *et al.* [1] used (G'/G) expansion method to discover traveling wave solutions of SWW equation.

2. Methodology



3. Application of the Method

Consider the application of the method for finding analytical solutions of (3+1)-dimensionl SWW equation

$$v_{xxxx}+3v_{xx}v_{y}+3v_{x}v_{xy}-v_{yt}-v_{xz}=0 \qquad (3.1)$$

In order to attain the solutions of equation (3.1), we make alteration

$$v(x,y,z,t)=v(\eta),\eta=x+y+z-ct \qquad (3.2)$$

By using this transformation, we have obtained ODE

$$v'''''+3v'v''+3v'v''+cv''-v''=0 \qquad (3.3)$$

Now integrate equation (3.3) with respect to η.

$$v'''+3(v')^{2}+(c-1)v'=0 \qquad (3.4)$$

Now by using homogenous balance method we find $N=2$.

In the third step substitute the derivatives of function $v(\eta)$ into equation (3.4). In this case these derivatives can be written as

$$v_{\eta\eta\eta}=a_1Q(Q-1)(6Q^2-6Q+1)+2a_2Q^2(Q-1)(12Q^2-15Q+4) \qquad (3.5)$$

$$v_{\eta\eta}=a_1Q(Q-1)[2Q-1]+2a_2Q^2(Q-1)[(3Q-2)] \qquad (3.6)$$

$$v_{\eta}=a_1Q(Q-1)+2a_2Q^2(Q-1) \qquad (3.7)$$

the expression $v(\eta)$ in the form

$$v=a_0+a_1Q+a_2Q^2 \qquad (3.8)$$

As result of the third step we have the following equation

$$12{a_2}^2Q^6+(12a_1a_2-24{a_2}^2+24a_2)Q^5+(6a_1-54a_2-24a_1a_2+3{a_1}^2+12{a_2}^2)Q^4+$$
$$(-12a_1+12a_1a_2+36a_2+2ca_2-6{a_1}^2)Q^3+(-6a_2-2ca_2+ca_1+3{a_1}^2+6a_1)Q^2-ca_1Q=0$$
(3.9)

4. Now equate terms of equation (3.9) equal to zero

$$12{a_2}^2=0$$
$$(12a_1a_2-24{a_2}^2+24a_2)=0$$
$$(6a_1-54a_2-24a_1a_2+3{a_1}^2+12{a_2}^2)=0$$
$$(-12a_1+12a_1a_2+36a_2+2ca_2-6{a_1}^2)=0$$
$$(-6a_2-2ca_2+ca_1+3{a_1}^2+6a_1)=0$$
$$-ca_1=0$$

Solving the system of equations by Maple Software

Case 1 $a_1=0=a_2, c=c$

Case 2 $a_1=-2, a_2=0=c$

6. Analytical solutions of the SWW equation take the form

$$v(\xi)=a_0+a_1\left(\frac{1}{1+e^{\eta}}\right)+a_2\left(\frac{1}{1+e^{\eta}}\right) \qquad (3.9)$$

Where $\eta=x+y+z-ct$.

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