New analytical Solutions of (3+1)-dimensional Shallow Water Wave equation (SWW)

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Abstract: In this paper, we have obtained new analytical solutions of (3+1)-dimensional SWW equation with Kudryashov method. The study of Shallow water wave equation plays an imperative role in wave theory. For calculation software Maple is used. The solutions obtained by this method are new.

Keywords: (3+1)-dimensional SWW equation, Kudryashov method, Analytical solutions.

1. Introduction

Waves in different parts of oceans are referred to by different names. Those differences are dependent on wave length. A shallow water wave is one that occurs at depths shallower than wave length of wave divided by 20. The (3+1)-dimensional SWW is

\[ v_{xxxx} + 3v_{xx}v_{yy} + 3v_{x}v_{xy} - v_{yy} - v_{zz} = 0 \]  

(1.1)


2. Methodology

**Step 1.** First of all, change the nonlinear partial differential equation into ordinary differential equation.

\[ R(v,v',v'',v''',v''''......) = 0 \]

by using the transformation

\[ v(\xi) = v(x, t, \eta) = \xi + \omega t \]

**Step 2.** In this step calculate the value of \( \xi \) by using homogenous balance method.

**Step 3.** In this step, substitute derivatives of function in ordinary differential equation.

**Step 4.** In this step transform the problem into scheme of algebraic equations. Equate terms at the unlike powers of to zero, we get the system of numerical equations.

**Step 5.** In this step, solve the scheme of algebraic equations to find the values of Coefficients. As a result, we obtain exact solutions of the equation.
3. Application of the Method

Consider the application of the method for finding analytical solutions of (3+1)-dimensional SWW equation
\[ v_{xxxx} + 3v_{xv}v_x + 3v_{xv}v_y - v_{yy} - v_{z} = 0 \]  
(3.1)
In order to attain the solutions of equation (3.1), we make alteration
\[ v(x, y, z, t) = v(\eta), \eta = x + y + z - ct \]  
(3.2)
By using this transformation, we have obtained ODE
\[ v^{xxxx} + 3v^{xv}v_x + 3v^{xv}v_y + cv^y - v^x = 0 \]  
(3.3)
Now integrate equation (3.3) with respect to \( \eta \).
\[ v'' + 3(v')^2 + (c - 1)v' = 0 \]  
(3.4)
Now by using homogenous balance method we find \( N = 2 \).
In the third step substitute the derivatives of function \( v(\eta) \) into equation (3.4). In this case these derivatives can be written as
\[ v_{\eta\eta\eta} = a_1Q(Q-1)(6Q^2 - 6Q + 1) + 2a_2Q^2(Q-1)(12Q^2 - 15Q + 4) \]  
(3.5)
\[ v_{\eta\eta} = a_1Q(Q-1)[2Q-1] + 2a_2Q^2(Q-1)[3Q - 2] \]  
(3.6)
\[ v_\eta = a_1Q(Q-1) + 2a_2Q^2(Q-1) \]  
(3.7)
the expression \( v(\eta) \) in the form
\[ v = a_0 + a_1Q + a_2Q^2 \]  
(3.8)
As result of the third step we have the following equation
\[ 12a_2^2Q^6 + [12a_1a_2 - 24a_2^2 + 24a_2]Q^5 + (6a_1 - 54a_2 - 24a_1a_2 + 3a_1^2 + 12a_2^2)Q^4 + \left( -12a_1 + 12a_1a_2 + 36a_2 + 2ca_2 - 6a_1^2 \right)Q^3 + (-6a_2 - 2ca_2 + ca_1 + 3a_1^2 + 6a_1)Q^2 - ca_1Q = 0 \]  
(3.9)
4. Now equate terms of equation (3.9) equal to zero
\[ 12a_2^2 = 0 \]
\[ (12a_1a_2 - 24a_2^2 + 24a_2) = 0 \]
\[ (6a_1 - 54a_2 - 24a_1a_2 + 3a_1^2 + 12a_2^2) = 0 \]
\[ (-12a_1 + 12a_1a_2 + 36a_2 + 2ca_2 - 6a_1^2) = 0 \]
\[ (-6a_2 - 2ca_2 + ca_1 + 3a_1^2 + 6a_1) = 0 \]
\[-ca_1 = 0 \]
Solving the system of equations by Maple Software
Case 1 \( a_1 = 0 = a_2, c = c \)
Case 2 \( a_1 = -2, a_2 = 0 = c \)
6. Analytical solutions of the SWW equation take the form
\[ v(\xi) = a_0 + a_1\left( \frac{1}{1 + e^\eta} \right) + a_2\left( \frac{1}{1 + e^\eta} \right) \]  
(3.9)
Where \( \eta = x + y + z - ct \).

References