Travelling Wave solutions of hyperbolic Telegraph equation by Tanh method

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Abstract: Tanh method is utilized to find travelling solutions of second order nonlinear Telegraph equation. As a result, we attain dissimilar travelling wave solutions. Our aim is to show that this method is most efficient and convenient approach for verdict travelling wave solutions of nonlinear differential equations. For calculation the software MAPLE is used.

KeyWords: Tanh- Method, Travelling wave solutions, (1+1)-dimensional telegraph equation.

1. Introduction

Telegraph equation is frequently used in the learning of proliferation of electric signals in a wire conduction stripe and also in the wave phenomenon. The second order nonlinear telegraph equation is:

$$w_{tt} - w_{xx} + w_t + \alpha w + \beta w^3 = 0 \quad (1.1)$$

Equation of this kind arise in study of heat transfer, chemical kinetics, population dispersal and transmission lines. Various mathematical methods have been planned for obtaining exact and numerical solutions. Mehrdad Lakestani *et al.* [6] used interpolating scaling functions to find solutions of linear hyperbolic equation. J. Rashidinia *et al*. [3] used a collocation approach to find solutions of telegraph equation. Murat *et al.* [4] used DGJ method to find solutions of hyperbolic telegraph equation. A modified adomian decomposition method was used by Hind *et al.* [2] to find approximate solutions of nonlinear telegraph equation.

T. S. Jang [7] showed how the novel nonlinear telegraph equation can be changed into corresponding scheme of two integral equations of second type. Mustafa *et al.* [5] Proposed reproducing kernel method to solve telegraph equation with primary conditions used on reproduce kernel theory. A reduced differential transform method was used by Vineet K. Srivastava *et al.* [8] to solve telegraph equation. So there are many methods to find exact and numerical solutions of nonlinear equations. The method was firstly introduced by Malfliet [9] and Fan and Hon [1].

2. Methodology

In this section, a concise depiction of method as follows:
Let us suppose a nonlinear equations in one variable

$$Q(w, w_x, w_t, w_{xx}, w_{xxx}..............) = 0 \quad (2.1)$$

Now make the transformation:

$$w(x,t) = w(\xi), \quad \xi = x - ct$$

Using this transformation we obtain ODE

$$Q(w, w', -cw', w'', w'''.............) = 0 \quad (2.2)$$

Now set up a new independent variable in the form

$$K = \tanh(\xi), \quad \xi = x - ct \quad (2.3)$$

This leads to the change of derivatives

$$\frac{d}{d\xi}=\left(1-K^2\right)\frac{d}{dK}$$

$$\frac{d^2}{d\xi^2}=\left(1-K^2\right)\left(-2K\frac{d}{dK}+\left(1-K^2\right)\frac{d^2}{dK^2}\right) \tag{2.4}$$

$$\frac{d^3}{d\xi^3}=\left(1-K^2\right)\left(\left(6K^2-2\right)\frac{d}{dK}-6K\left(1-K^2\right)\frac{d^2}{dK^2}+\left(1-K^2\right)^2\frac{d^3}{dK^3}\right)$$

Now introduce ansatz

$$u(\xi)=\sum_{r=-u}^{v}a_rK^r \tag{2.5}$$

Here u and v are nonnegative integers. Now put (2.4) and (2.5) into ODE (2.2) .Now to find u and v use homogenous balance method and equate coefficients of powers of k to zero. As a result we find algebraic equations involving a_r . Solve these equations, and then we get values of parameter. Using (2.5) we obtain exact solutions.

3. Application of Tanh method

As described in previous section, we use transformation

$$w(x,t)=w(\xi),\quad \xi=(x-ct) \tag{3.1}$$

1. As a outcome of the first step, we get the nonlinear ordinary differential equation in the Type

$w(x,t)=w(\xi),\quad \xi=x-ct$. Then we get (3.2)

$$-c^2w''-w''-cw'+\alpha w+\beta w^3=0 \tag{3.3}$$

By balancing the nonlinear terms we get $u=v=1$
Now choose the solutions of (3.3) in the form

$$w=w(\xi)=a_{-1}K^{-1}+a_0+a_1K \tag{3.4}$$

Where $Y=\tanh(\xi)$. Now put (3.4) into (3.3) and equate like powers of Y equal to zero, then we get following algebraic equations:

$$\begin{aligned}
&\beta a_1^3-2a_1-2c^2a_1=0\\
&3\beta a_0a_1^2+ca_1=0\\
&3\beta a_0^2a_1+3\beta a_{-1}a_1^2+2a_1+2c^2a_1+\alpha a_1=0\\
&\beta a_0^3-ca_1-ca_{-1}+6\beta a_{-1}a_0a_1+\alpha a_0=0\\
&2c^2a_{-1}+3\beta a_{-1}^2a_1+3\beta a_{-1}a_0^2+\alpha a_{-1}+2a_{-1}=0\\
&3\beta a_{-1}^2a_0+ca_{-1}=0\\
&-2a_{-1}-2c^2a_{-1}+\beta a_{-1}^3=0
\end{aligned} \tag{3.5}$$

Solve the following structure with the support of Maple and get following cases:

1.

$$a_0=a_0,\ a_1=0,\ a_{-1}=0,\ c=c,\ \alpha=-\beta a_0^2,\ \beta=\beta$$

2.

$$a_0=0,\ a_1=0,\ a_{-1}=a_{-1},\ c=0,\ \alpha=-2,\ \beta=\frac{2}{a_{-1}^2}$$

3.

$$a_{-1}=a_0,\, a_0=a_0,\, a_1=0,\, c=RootOf\left(6+6\_Z^2+\_Z\right),\, \alpha = \frac{4}{3}\, RootOf\left(6+6\_Z^2+\_Z\right),\, \beta = -\frac{1}{3}\, \frac{RootOf\left(6+6\_Z^2+\_Z\right)}{a_0^2}$$

4.

$$a_0=a_0,\, a_1=0,\, a_{-1}=-a_0,\, c=RootOf\left(6\_Z^2-\_Z+6\right),\, \alpha = -\frac{4}{3}\, RootOf\left(6\_Z^2-\_Z+6\right),\, \beta = \frac{1}{3}\, \frac{RootOf\left(6\_Z^2-\_Z+6\right)}{a_0^2}$$

5.

$$a_0=0,\, a_1=a_1,\, a_{-1}=0,\, c=0,\, \alpha=-2,\, \beta=\frac{2}{a_1^2}$$

6.

$$a_0=a_0,\, a_1=a_0,\, a_{-1}=0,\, c=RootOf\left(6+6\_Z^2+\_Z\right),\, \alpha = \frac{4}{3}\, RootOf\left(6+6\_Z^2+\_Z\right),\, \beta = -\frac{1}{3}\, \frac{RootOf\left(6+6\_Z^2+\_Z\right)}{a_0^2}$$

7.

$$a_0=a_0,\, a_1=-a_0,\, a_{-1}=0,\, c=RootOf\left(6\_Z^2-\_Z+6\right),\, \alpha = -\frac{4}{3}\, RootOf\left(6\_Z^2-\_Z+6\right),\, \beta = \frac{1}{3}\, \frac{RootOf\left(6\_Z^2-\_Z+6\right)}{a_0^2}$$

8.

$$a_0=0,\, a_{-1}=a_1,\, a_1=a_1,\, c=0,\, \alpha=-8,\, \beta=\frac{2}{a_1^2}$$

9.

$$a_0=a_0,\, a_1=-\frac{1}{2}\, a_0,\, a_{-1}=-\frac{1}{2}\, a_0,\, c=RootOf\left(12\_Z^2-\_Z+12\right),\, \alpha = -\frac{8}{3}\, RootOf\left(12\_Z^2-\_Z+12\right),\, \beta = \frac{2}{3}\, \frac{RootOf\left(12\_Z^2-\_Z+12\right)}{a_0^2}$$

10.

$$a_0=a_0,\, a_1=\frac{1}{2}\, a_0,\, a_{-1}=\frac{1}{2}\, a_0,\, c=RootOf\left(12\_Z^2+\_Z+12\right),\, \alpha = \frac{8}{3}\, RootOf\left(12\_Z^2+\_Z+12\right),\, \beta = -\frac{2}{3}\, \frac{RootOf\left(12\_Z^2+\_Z+12\right)}{a_0^2}$$

11.

$a_0 = 0, a_1 = a_1, a_{-1} = -a_1, c = 0, \alpha = 4, \beta = \frac{2}{a_1^2}$

Substituting these results into (3.4), then we get exact solutions. For example in case 10, we have following exact solution

Case 10

$$u_1 = a_0 + \frac{1}{2}a_0 \tanh\left(x - Rootof\left(12\_Z^2 + \_Z + 12\right)t\right) + \frac{1}{2}a_0 \coth\left(x - Rootof\left(12\_Z^2 + \_Z + 12\right)t\right)$$

We can choose any value of a_0. Similarly, we can find other exact solutions.

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