Research Article

Exact solutions of (1+1)-Dimensional Kaup-Kupershmidt equation

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Abstract: In this paper, we have obtained new analytical solutions of Kaup-Kupershmidt equation by using one method. We conclude that One method present a wider applicability for managing nonlinear partial differential equation. The solutions obtained in this paper are new.

Keyword: One Method, Exact solutions, Kaup-Kupershmidt equation.

1. Introduction

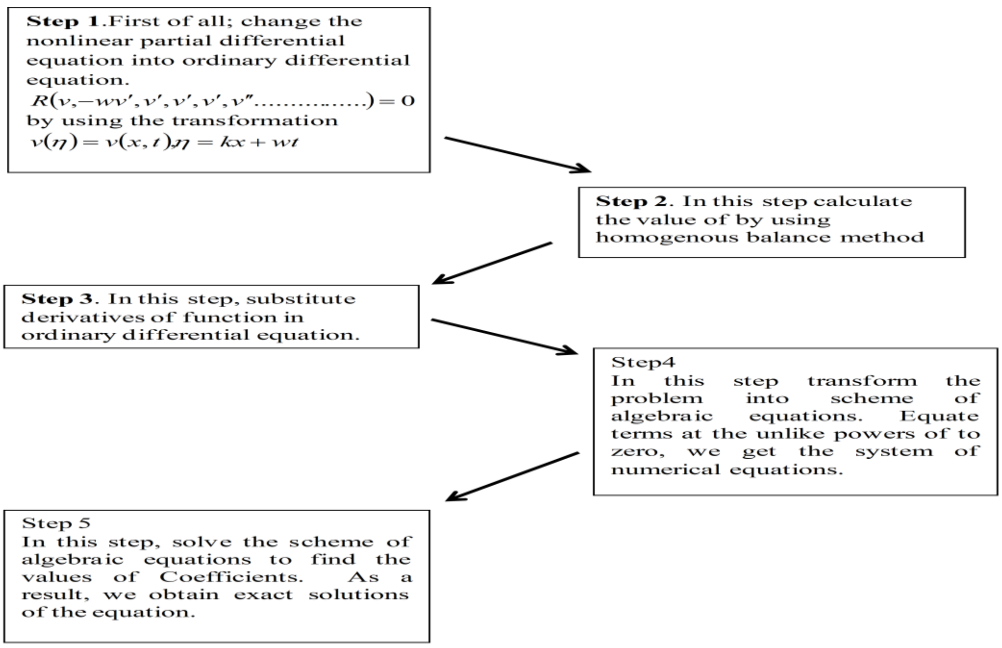
Nonlinear partial differential equations take part an essential role in Mathematical Physics. In this paper we will discuss about exact solutions of Kaup-Kupershmidt equation.

$$u_{xxxxx} + u_t + 45u_x u^2 - \frac{75}{2} u_{xx} u_x - 15uu_{xxx} = 0 \qquad (1.1)$$

This equation was firstly introduce by Kaup in 1980. S. Sahoo et al. [5] derived exact solutions of time fractional Kaup-Kupershmidt equation by using improved $\left(\frac{G'}{G}\right)$-expansion method and extended $\left(\frac{G'}{G}\right)$-expansion method. Ming *et al.* [3] obtained exact solutions of Broer-Kaup-Kupershmidt equation by using bifurcation method. A. H. Bhrawy et al. [1] used exp-function method to find new analytic solutions of (1+1)-dimensional and (2+1)-dimensional Kaup-Kupershmidt equation.

Alvaro [2] applied the Cole-Hopf transformation to find soliton solutions and Hirota method to find 1 and 2 soliton solutions of generalized Kaup-Kupershmidt equation. Syed et al. [6] obtained soliton solutions of Kaup-Kupershmidt equation with intial conditions. M.F. El-Sabbagh et al. [4] used improved exp method to find analytic solutions of Kaup-Kupershmidt equation.

2.Methodology



3. Application of the Method

Consider the appliance of the method for verdict analytical solutions of Kaup-Kupetshmidt equation

$$v_{xxxxx} + v_t + 45 v_x v^2 - \frac{75}{2} v_{xx} v_x - 15 v v_{xxx} = 0 \qquad (3.1)$$

In find travelling wave solutions of equation (3.1), we formulate alteration

$$v(x,t) = v(\xi), \xi = rx + wt \qquad (3.2)$$

By using this transformation, we have obtained ODE

$$r^5 v'''''+ wv' + 45 r v' v^2 - \frac{75}{2} r^3 v' v'' - 15 r^3 v v''' = 0 \qquad (3.3)$$

Now integrate equation (3.3) with respect to ξ.

$$r^5 v'''' + wv + 15 r v^3 - \frac{45}{4} r^3 (v')^2 - 15 r^3 v v'' = g \qquad (3.4)$$

Now by using homogenous balance method, we obtain $N = 2$.

In this step put the derivatives of function $u(\xi)$ into equation (3.4). In our case these derivatives can be written as

$$v_{\xi\xi\xi\xi} = a_1 Q(Q-1)[(24Q^3 - 36Q^2 + 14Q - 1)] + 2a_2 Q^2 (Q-1)[(60Q^3 - 108Q^2 + 57Q - 8)]$$

(3.5)

$$v_{\xi\xi} = a_1 Q(Q-1)[2Q-1] + 2a_2 Q^2 (Q-1)[(3Q-2)] \qquad (3.6)$$

$$v_{\xi} = a_1 Q(Q-1) + 2a_2 Q^2 (Q-1) \qquad (3.7)$$

the expression $v(\xi)$ in the form

$$v = a_0 + a_1 Q + a_2 Q^2 \qquad (3.8)$$

As effect of the previous step we have the following equation

$$(15ra_2^3 - 135r^3 a_2^2 + 120r^5 a_2)Q^6 + (27r^5 a_1 - 165r^3 a_1 a_2 - 336r^5 a_2 + 45ra_1 a_2^2 + 240r^3 a_2^2)Q^5$$
$$+ (45ra_0 a_2^2 + 330r^5 a_2 - 41.25000000r^3 a_1^2 + 285r^3 a_1 a_2 - 90r^3 a_0 a_2 - 63r^5 a_1 - 105r^3 a_2^2 +$$
$$45ra_1^2 a_2)Q^4 + (-30r^3 a_0 a_1 + 150r^3 a_0 a_2 - 130r^5 a_2 + 15ra_1^3 + 50r^5 a_1 + 67.5000000r^3 a_1^2$$
$$+ 90ra_0 a_1 a_2 - 120r^3 a_1 a_2)Q^3 + (-26.25000000r^3 a_1^2 - 15r^5 a_1 + 16r^5 a_2 + 45r^3 a_0 a_1 + wa_2 +$$
$$45ra_0 a_1^2 + 45ra_0^2 a_2 - 60r^3 a_0 a_2)Q^2 + (45ra_0^2 a_1 - 15r^3 a_0 a_1 + r^5 a_1 + wa_1)Q - g + wa_0 +$$
$$15ra_0^3 = 0$$

(3.9)

4. Now equate terms of equation (3.9) equal to zero

$$(15ra_2^3 - 135r^3 a_2^2 + 120r^5 a_2) = 0$$
$$(27r^5 a_1 - 165r^3 a_1 a_2 - 336r^5 a_2 + 45ra_1 a_2^2 + 240r^3 a_2^2) = 0$$
$$(45ra_0 a_2^2 + 330r^5 a_2 - 41.25000000r^3 a_1^2 + 285r^3 a_1 a_2 - 90r^3 a_0 a_2 - 63r^5 a_1 - 105r^3 a_2^2 +$$
$$45ra_1^2 a_2) = 0$$
$$(-30r^3 a_0 a_1 + 150r^3 a_0 a_2 - 130r^5 a_2 + 15ra_1^3 + 50r^5 a_1 + 67.5000000r^3 a_1^2$$
$$+ 90ra_0 a_1 a_2 - 120r^3 a_1 a_2) = 0$$
$$(-26.25000000r^3 a_1^2 - 15r^5 a_1 + 16r^5 a_2 + 45r^3 a_0 a_1 + wa_2 + 45ra_0 a_1^2 + 45ra_0^2 a_2 - 60r^3 a_0 a_2) = 0$$
$$+ (45ra_0^2 a_1 - 15r^3 a_0 a_1 + r^5 a_1 + wa_1)Q - g + wa_0 + 15ra_0^3 = 0$$

Solving the scheme of equations by Maple Software

Case 1 $a_0 = a_0, a_1 = a_1, a_2 = a_2, r = w = g = 0$

Case 2 $a_0 = a_0, a_1 = 0, a_2 = 0, r = r, w = w, g = wa_0 + 15ra_0^{\ 3}$

6. Analytical solutions of the Kaup-Kupetshmidt equation take the form

$$u(\xi) = a_0 + a_1\left(\frac{1}{1+e^{\xi}}\right) + a_2\left(\frac{1}{1+e^{\xi}}\right) \tag{3.9}$$

Where $\xi = rx + wt$.

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