Exact solutions of (1+1)-Dimensional Kaup-Kupershmidt equation

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Abstract: In this paper, we have obtained new analytical solutions of Kaup-Kupershmidt equation by using one method. We conclude that One method present a wider applicability for managing nonlinear partial differential equation. The solutions obtained in this paper are new.

Keyword: One Method, Exact solutions, Kaup-Kupershmidt equation.

1. Introduction
Nonlinear partial differential equations take part an essential role in Mathematical Physics. In this paper we will discuss about exact solutions of Kaup-Kupershmidt equation.

\[ u_{xxxx} + u_x + 45u_x u^2 - \frac{75}{2} u_{xx} u_x - 15u u_{xxx} = 0 \]  

(1.1)

This equation was firstly introduce by Kaup in 1980. S. Sahoo et al. [5] derived exact solutions of time fractional Kaup-Kupershmidt equation by using improved \( \left( \frac{G'}{G} \right) \)-expansion method and extended \( \left( \frac{G'}{G} \right) \)-expansion method. Ming et al. [3] obtained exact solutions of Broer-Kaup-Kupershmidt equation by using bifurcation method. A. H. Bhrawy et al. [1] used exp-function method to find new analytic solutions of (1+1)-dimensional and (2+1)-dimensional Kaup-Kupershmidt equation.


2. Methodology

Step 1. First of all, change the nonlinear partial differential equation into ordinary differential equation.

\[ R(v, v', v'', v''', v''''; \ldots) = 0 \]

by using the transformation \( v(\eta) = v(x, t) \eta = kx + wt \)

Step 2. In this step calculate the value of by using homogenous balance method

Step 3. In this step, substitute derivatives of function in ordinary differential equation.

Step 4. In this step transform the problem into scheme of algebraic equations. Equate terms at the unlike powers of to zero, we get the system of numerical equations.

Step 5. In this step, solve the scheme of algebraic equations to find the values of Coefficients. As a result, we obtain exact solutions of the equation.
3. Application of the Method

Consider the appliance of the method for verdict analytical solutions of Kaup-Kupetshmidt equation
\[ v_{xxxx} + v_x + 45v_xv^2 - \frac{75}{2}v_{xx}v_x - 15vv_{xxx} = 0 \]  
(3.1)

In find travelling wave solutions of equation (3.1), we formulate alteration
\[ v(x,t) = v(\xi), \xi = rx + wt \]  
(3.2)

By using this transformation, we have obtained ODE
\[ r^3v'''' + r\v'' + 45rv'v^2 - \frac{75}{2}r^3v''v - 15r^3vv'' = 0 \]  
(3.3)

Now integrate equation (3.3) with respect to \( \xi \).
\[ r^5v''' + rv' + 15r^3v = \frac{45}{4}r^3(\v')^2 - 15r^3vv'' = g \]  
(3.4)

Now by using homogenous balance method, we obtain \( N = 2 \).

In this step put the derivatives of function \( \nu(\xi) \) into equation (3.4). In our case these derivatives can be written as
\[ \begin{align*}
v_{xxxx} &= a_1Q(Q - 1)[24Q^3 - 36Q^2 + 14Q - 1] + 2a_2Q^2(Q - 1)[60Q^3 - 108Q^2 + 57Q - 8] \\
v_{xx} &= a_1Q(Q - 1)[2Q - 1] + 2a_2Q^2(Q - 1)[3Q - 2] \\
v_x &= a_1Q(Q - 1) + 2a_2Q^2(Q - 1)
\end{align*} \]
(3.5-3.7)

the expression \( \nu(\xi) \) in the form
\[ v = a_0 + a_1Q + a_2Q^2 \]  
(3.8)

As effect of the previous step we have the following equation
\[ 
(15ra_2^3 - 135r^3a_2^2 + 120r^5a_2)Q^6 + (27r^5a_1 - 165r^3a_1a_2 - 336r^5a_2 + 45ra_1a_2^2 + 240r^3a_2^2)Q^5 \\
+ (45ra_0a_2^2 + 330r^5a_2 - 41.250000000r^3a_1^2 + 285r^3a_1a_2 - 90r^3a_1a_2 - 63r^5a_1 - 105r^3a_2^2 + \\
45ra_1^2a_2)Q^4 + (-30r^3a_0a_1 + 150r^3a_0a_2 - 139r^5a_2 + 15ra_1^3 + 50r^5a_1 + 67.5000000r^3a_1^2 \\
+ 90ra_0a_1a_2 - 120r^3a_1a_2)Q^3 + (-26.250000000r^3a_1^2 - 15r^5a_1 + 16r^5a_2 + 45r^3a_0a_1 + wa_2 + \\
45ra_0a_1^2 + 45ra_0^2a_2 - 60r^3a_0a_2)Q^2 + (45ra_0^2a_1 - 15r^3a_0a_1 + r^5a_1 + wa_1)Q - g + wa_0 + \\
15ra_0^3 = 0
\]
(3.9)

4. Now equate terms of equation (3.9) equal to zero
\[ 
(15ra_2^3 - 135r^3a_2^2 + 120r^5a_2) = 0 \\
(27r^5a_1 - 165r^3a_1a_2 - 336r^5a_2 + 45ra_1a_2^2 + 240r^3a_2^2) = 0 \\
(45ra_0a_2^2 + 330r^5a_2 - 41.250000000r^3a_1^2 + 285r^3a_1a_2 - 90r^3a_1a_2 - 63r^5a_1 - 105r^3a_2^2 + \\
45ra_1^2a_2) = 0 \\
(-30r^3a_0a_1 + 150r^3a_0a_2 - 130r^5a_2 + 15ra_1^3 + 50r^5a_1 + 67.5000000r^3a_1^2 \\
+ 90ra_0a_1a_2 - 120r^3a_1a_2) = 0 \\
(-26.250000000r^3a_1^2 - 15r^5a_1 + 16r^5a_2 + 45r^3a_0a_1 + wa_2 + 45ra_0a_1^2 + 45ra_0^2a_2 - 60r^3a_0a_2) = 0 \\
+(45ra_0^2a_1 - 15r^3a_0a_1 + r^5a_1 + wa_1)Q - g + wa_0 + 15ra_0^3 = 0
\]
Solving the scheme of equations by Maple Software

Case 1 \( a_0 = a_0, a_1 = a_1, a_2 = a_2, r = w = g = 0 \)

Case 2 \( a_0 = a_0, a_1 = 0, a_2 = 0, r = r, w = w, g = wa_0 + 15ra_0^3 \)

6. Analytical solutions of the Kaup-Kupershmidt equation take the form

\[ u(\xi) = a_0 + a_1\left(\frac{1}{1 + e^{\xi}}\right) + a_2\left(\frac{1}{1 + e^{\xi}}\right) \]  

(3.9)

Where \( \xi = rx + wt \).

References


5. S.Sahoo, S. Saha Ray and M. A. Abdou, “New exact solutions of time fractional Kaup-Kupershmidt equation by using improved \( \left( \frac{G'}{G} \right) \)-expansion and extended \( \left( \frac{G'}{G} \right) \)-expansion method.” Alexandria Engineering Journal. Vol. 59 (2020), pp. 3105-3110.