Research Article

Optimization Of Queueing Model

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Abstract: -in thepaper, we are considering the single server queueing system have interdependent arrival of the service processes having bulk service.In this article, we consider that the customers are served k at any instance except when less then k are in the system &ready to provide service at which time customers are served.

Keyword: -Interdependent queueing models, arrival process, service process, waiting line system, mean dependence.

1. OPTIMIZATION $M/M^{[K]}/1$ QUEUEING MODEL WITH VARYING BATCH SIZE :-

In this type of systems, the interdependence could be induced by considering the dependent structure with parameters λ , μ and $\in$as marginal arrival rate , service rate and mean dependence rate respectively .

Let $P_n(t)$ be the probability when there are n customers in system at time t . The difference – differential equations of above modelmay have written as,

$$P'_n(t) = -(\lambda + \mu - 2 \in)P_n(t) + (\lambda - \in)P_{n-1}(t) + P_{n-k}(t); \quad n \geq 1$$

$$P'_0(t) = -(\lambda - \in)P_0(t) + (\mu - \in)\sum_{i=1}^{k} P_i(t)$$

……………… (1)

Let us considerthat, the system achieved the steady state, therefore the transition equations of considered model ar,

$$-(\lambda + \mu - 2 \in)P_n + (\lambda - \in)P_{n-1} + (\mu - \in)P_{n-k} = 0 \quad ; \quad n \geq 1$$

$$-(\lambda - \in)P_0 + (\mu - \in)\sum_{i=1}^{k} P_i = 0$$

…………………. (2)

Applying heuristic arguments of "Gross and Harris" (1974). One can obtain the solution of mentioned equationsas,

$P_n = Cr^n n \geq 0$, $0 < r < 1$ ……………………(3)

Where r, is the root of equations which lie in (0,1) of the characteristic equation .

$[(\mu - \in)D^{k+1} - (\lambda + \mu - 2 \in)D + (\lambda - \in)]P_n = 0$ …………………… (4)

Here Drepresents theoperator.

2. MEASURES OF EFFECTIVENESS: -

The probability that the system is empty is,

$P_0 = (1 - r)$ ……………………. (5)

Where r is as given in equation (3).

For different values of $\in$&k, for the given values of λ and μ , we are able to compute P_0 values & are given in table (5.1). The values of $P_0$for the fixed k , $\in$and for varying λ , μmentioned in the table (5.2).

From tables (5.1), (5.2) and equation number---(5), we observe that for fixed of λ , μ and $\in$, the value of P_0 increases with respect to increase in$k$. As the dependence parameter$\in$increases the value of P_0increases for fixed values of λ , μ and k .The value of P_0decreases for fixed values of μ, k and $\in$ as λincreases. As μ increases the valueof P_0 increases for fixed values of the μ , k and dependence parameter$\in$. If the mean dependence rate, is zero then the value of P_0 is also same as in the $M/M^{[K]}/1 - model$.

The average no. of customers in the system can obtained as

$$L = \frac{r}{1 - r}$$

……………… (6)

and mean number, of customers in the queue are

$$L_q = \frac{r^2}{1 - r}$$

……………… (7)

where r is as given in equation (3).

The value of L and L_q has been computed and given in tables-5.3 and table-5.5 for provided values of λ, μ and for different values $\in$ and k respectively. The values of L and L_q for fixed values of $\in$ and k & for varying μ and λ also given in tables-5.4 and table-5.6.

By equations 6 and 7, also for the corresponding tables we observe, that as $\in$ increase, the values of L and L_q are decreasing and also as k increases the values of L and L_q are decreasing for fixed values of other parameters. As the arrival rate increases, the values of L and L_q are increasing for fixed values of μ, k and $\in$. As μ increases the values of L and L_q are decreasing for fixed values of λ, k and $\in$. When the dependence parameter $\in = 0$ then the average queue length is same as that of $M/M^{[K]}/1$ model. When $k = 1$ this is same as $M/M/1$ interdependence model.

The variability of this model can be obtained as

$$V = \frac{r}{(1-r)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where r is as given in equation (3).

The coefficient of variation of the model is

$$C.V = \frac{\sqrt{V}}{L} \times 100 \quad \ldots\ldots\ldots\ldots\ldots (9)$$

Where L & V are provided as in equations (6) and (7).

The values of 'variability of system' and 'coefficient of variation' for various values of $k, \in$ for fixed values of λ, μ are computed which are given in tables-5.7& 5.9 . The values of 'variability of the system' and 'coefficient of variation' for fixed values of $k, \in$ and for various values of λ, μ are provided in tables (5.8) and (5.10).

From equation-9 a& from the corresponding table we can observe that as μ increases the "variability of the system size" decreases and "coefficient of variation" increases. As λ increases and for fixed values of $\mu, \in$ and k, the 'variability of the system size' increases & the 'coefficient of variation decreases. We may observe that as $\in$ increases the 'variability of the system size, decreases and 'coefficient of variation' increases for fixed values of λ, μ and k. As k increases, the 'variability of the system' decreases and the 'coefficient of variation increases'.

For this model $\in = 0$ and $k = 1$ reduces to $M/M/1$ classical model. The mean-queue length & 'variability of the system size' of this model are less than that of the classical. When $k = 1$, this model becomes $M/M/1$ independent model for $\in = 0$, this model is same as $M/M^{[K]}/1$ model.

TABLE 1.1

VALUES OF P_0

$\boldsymbol{\lambda = 3 \quad , \quad \mu = 5}$

$K/\in$	0.0	0.2	0.4	0.6	0.8
1.	0.4000	0.4167	0.4348	0.4545	0.4762
2.	0.5780	0.5871	0.5971	0.6081	0.6203
3.	0.6106	0.6182	0.6264	0.6357	0.6459
4.	0.6201	0.6270	0.6347	0.6433	0.6529
5.	0.6214	0.6300	0.6374	0.6458	0.6551

(TABLE 1.2)

"VALUES OF P_0"

$\boldsymbol{for\ K = 2}$ & $\in = 5$

μ/λ	1	2	3	4	5
1.	0.9024	0.7681	0.6548	0.5551	0.4649
2.	0.9161	0.7983	0.6975	0.6081	0.5269
3.	0.9265	0.8214	0.7305	0.6493	0.5752
4.	0.9345	0.8397	0.7568	0.6823	0.6141
5.	0.9410	0.8545	0.7783	0.7094	0.6461

TABLE 1.3
VALUES OF L
$\lambda = 3 \quad , \quad \mu = 5$

$K/\in$	0.0	0.2	0.4	0.6	0.8
1.	1.5000	1.3998	1.2999	1.2002	1.1000
2.	0.7301	0.7033	0.6748	0.6445	0.6121
3.	0.6377	0.6176	0.5964	0.5731	0.5482
4.	0.6126	0.5949	0.5755	0.5545	0.5316
5.	0.6093	0.5873	0.5689	0.5485	0.5295

TABLE 1.4
VALUES OF L
$K = 2 \quad , \quad \in = 0.4$

μ/λ	1	2	3	4	5
1.	0.1082	0.3019	0.5272	0.8015	0.1510
2.	0.0916	0.2527	0.4337	0.6445	0.8979
3.	0.0793	0.2174	0.3689	0.5401	0.7385
4.	0.0701	0.1909	0.3214	0.4656	0.6284
5.	0.0627	0.1703	0.2849	0.4096	0.5477

TABLE 1.5
VALUES OF L_q
$\lambda = 3 \quad , \quad \mu = 5$

$K/\in$	0.0	0.2	0.4	0.6	0.8
1.	0.9000	0.8165	0.7347	0.6547	0.5762
2.	0.3081	0.2904	0.2228	0.2526	0.2324
3.	0.2483	0.2219	0.2228	0.2088	0.1941
4.	0.2327	0.2219	0.2102	0.1978	0.1845
5.	0.2307	0.2173	0.2063	0.1943	0.1816

TABLE 1.6
VALUES OF L_q
$K = 2 \quad , \quad \in = 0.4$

μ/λ	1	2	3	4	5
1.		0.700	0.1820	0.3566	0.6159
2.	0.0077	0.510	0.1321	0.2526	0.4248
3.	0.0058	0.0388	0.0994	0.1894	0.3137
4.	0.0046	0.0306	0.0782	0.1479	0.2425
5.	0.0037	0.0248	0.632	0.1190	0.1938

REFERENCE

1. Baccelli, F. Mssey, W. A. (1989): "A Sample Path Analysis Of The M/M/1 Queue", J Appl. Prob. 26, 418-422.
2. Boxma, O.J., Kelly, F.P. And Konheim, A. G. (1984): "The Product Form For Sojourn Time Distributions Incyclic Exponential Queues",J. Assoc. Comput. Mach., 31, 128-133.
3. Brien O . (1987) : " Extreme Values For Stationary And Markov Sequences" Journal. Soci. Indian Appl. Math ., 2, 133.
4. Chae, K. C ., Lee, H. W. And Ahn C. W. (2001): "An Arrival Time Approach To M /G/1 Type Queues With Generalized Vacations" , Queueing Systems 38, 91- 100.
5. Chaudhary , G . (1998) : "On A Batch Arrival Poisson Queue With A Random Setup And Vacation Period", Comm .Ops. Res., 25 , 1013-1026.
6. Chhikra (1972) : Statistical Inference Related To The Inversegaussian Distribution , Ph.D. Dissertation, Oklahoma State University .
7. Conti, P.L. And Giovanni , L.D. (2002) : " Queueing Models And Statistical Analysis For Atm Based Networks" ,Sankhaya : The Indian Journal Of Statistics , 64, 50-75.

8. Cooper, R.B.(1970) : "Queues Served In Cyclic Order : Waiting Times", Bell System Technical Journal , 49 399- 413 .
9. Erlang , A.K. (1909) : "Probability And Phone Calls" ,Nyt. Tidsskr Mat. Ser. B., 20 , 33-39.
10. Fry , T.C. (1982) : "The Theory Of Probability As Applied To Problems Of Congestion In Probability And Its Engineering Uses" , D.Van Nostrand Co . Inc., Princeton New Jersy.
11. Griffitins , J.D ., Leomenko, G.M., Williams , J .E. (2006) : The Transient Solution To M/E/1 Queue , Oper. Res. Letters , Vol. 34 , Issue 4 Pp. 349-354.
12. Hiller, F.S. And Boiling , R.W. (1967) : Finite Queues In Series With Exponential Or Erlang Service Time-A Numerical Approach" , Operation Research ., 15 , 286-303.
13. Singla, Neelam. "Busy Period Analysis Of A Markovian Feedback Queueing Model With Servers Having Unequal Service Rate." International Journal Of Applied Mathematics & Statistical Sciences (Ijamss) 6.1 (2017) 55-64
14. Ghimire, Sushil, Et Al. "Multi-Server Batch Service Queuing Model With Variable Service Rates." International Journal Of Applied Mathematics & Statistical Science (Ijamss) 6.4 (2017): 43-54.
15. Mohamad, H. A., And Sattar Naser Ketab. "Oscillation And Nonoscillation Properties Of Solutions Of Third Order Linear Neutral Differential Equations." International Journal Of Applied Mathematics & Statistical Sciences (Ijamss) 5.3 (2016): 31-38.
16. Shaalini, J. Vinci, And A. Emimal Kanaga Pushpam. "Application Of Rkhem Method For Solving Delay Differential Equations With Constant Lags." International Journal Of Applied Mathematics & Statistical Sciences (Ijamss) 5.3 (2016) 39-46
17. Muxamediyeva, D. K. "Properties Of Self Similar Solutions Of Reaction-Diffusion Systems Of Quasilinear Equations." International Journal Of Mechanical And Production Engineering Research And Development (Ijmperd) 8.2 (2018) 555-566
18. Jawahar, G. Gomathi. "Qualitative Analysis On Second Order Neutral Delay Difference Equations." International Journal Of Mechanical And Production Engineering Research And Development (Ijmperd) 9.2 (2019) 659-664