Performance Analysis of a Standby System using Exponential-Rayleigh-Weibull Distributions

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Abstract: All through the life-cycle of a standby system, it is very challenging to keep a standby unit workable. It may be fatal if the standby found non-workable when needed. This paper evaluates the performance of a standby system working under two primary constraints by underlining the condition of the spare unit in standby mode. The first constraint is the maximum redundancy time for the standby and the second is maximum operation time for the operating unit. The standby unit fails or exceeds the maximum time threshold after which the decision about its repair/replacement is subject to the inspection. While after surpassing the maximum operating time limit preventive maintenance is carried out for the operating unit. To study the long-run performance or life-cycle of the system various performance indices have been analyzed using the theory of discrete-state continuous-time semi-Markov regenerative processes. Exponential, Rayleigh and Weibull probability distributions are used to study the system performance numerically.


1. Introduction

In everyday life, we come across many systems equipped with redundant units to facilitate the smooth functioning and ensure higher availability and system reliability. For instance, the spare engine in a jet fighter, alternate power supply in an Intensive Care Units, redundant safety installation in an atomic power plant, parallel lines in a communication network, parallel service counter in a bank and many more. Though the provision of a spare component guarantees reliability, availability and even safety in some cases but side by side put challenges to the budgetary resources. Despite all financial obligations, however, there are many safety installations where risk cannot be taken with reliability, in any case. Therefore, the provision of standby remains popular among safety and reliability professionals [see [1], [2], [3], [4], [5]]. An essential thing about a standby system is that it can be restored if and only if the spare unit found perfectly operable. In all the above studies, it is commonly assumed that the standby unit always found operable when needed. Is it practically correct? There is no doubt that the active operating load on standby unit in a cold-standby system is not equal to that of an operating unit, but this factor alone cannot be assumed responsible for the current state of standby. Indeed, the state of standby depends firstly on the local environment and secondly on the ways of its handling [6, 7]. So depending upon such factors the standby unit may or may not found operable. If not then there would be adverse consequences. Therefore, the study of standby systems with the possibility of standby failure becomes very much significant. Earlier, this issue has been discussed hardly [8], [9], [10], Furthermore, no preventive measures are taken before the system failure. It is either repaired or replaced at its failure. In particular, the preventive maintenance plan can better improve the system performance [11], [12], [13]. Keeping this fact in view this paper analyzes a stochastic model of a cold standby system incorporating the idea of pre-failure preventive maintenance of the atomic unit after crossing a pre-specified time limit, termed as maximum operation time. This paper presents the analysis of a two-unit cold standby system using the theory of discrete-state continuous-time Markov regenerative processes [14], [15], [16]. The system works under two constraints namely maximum redundancy time for the standby unit and maximum operation time for the operative unit. Upon crossing a pre-defined threshold time limit the standby fails and passes through inspection for deciding about its repair or replacement whereas the operative unit be given preventive maintenance aiming at enhancing system performance. The practical importance of theoretical results is shown in a particular case using Weibull distribution [17, 18].

2. Acronyms and Notations

\(E/\bar{E}\) : The set of regenerative/ Non-regenerative states
\(U/\bar{U}\) : The set of up-states/ down states
3. **System Description And Assumptions**

1. The system consists of two identical units. There are two modes of the units- operable (Normal) or non-operable (failed).
2. At time \( t=0 \), the system starts with one unit in active operation mode and another in cold standby.
3. The standby unit instantly switches into operation at the failure of operating unit, putting the failed directly under repair.
4. At the instant the unit in cold standby mode crosses a pre-specified maximum redundancy time fails just taken for inspection for deciding about repair or replacement.
5. The unit in operation taken under preventive maintenance just after crossing a pre-specified time limit, called maximum operation time.
6. The random variables associated with the model follow arbitrary probability distributions.
7. All the repairs and switching are perfect.

4. Mathematical Description Of The Model

In this study, a stochastically failing and renewable system is considered. Figure 1 shows the conceptual layout of the system design. Let \( S_i: i=0,1,2,\ldots,16 \), denote the \( i^{th} \) state of the system. At the time \( t=0 \), the system starts operation in the state \( S_0 \). All the possible states are classified into two mutually exclusive and exhaustive categories; regenerative and non-regenerative states. It is observed that \( S_0,S_1,S_2,S_3 \) and \( S_4 \) are regenerative states. If \( \tau_i, \tau_1, \tau_2, \ldots \) be the time points at which system enters into \( S_j \in E \), set of regenerative states and \( X_n \) be the state visited at instant \( \tau_n \) i.e. just after the transition at \( \tau_n \), then \( \{X_n,\tau_n, n \in E, \text{ set of regenerative states} \} \) represent Markov renewal stochastic process. The transition probability matrix of the associated embedded Markov chain is given by \( P = [P_{ij}] = Q \phi(\infty) = Q(\infty) \). Here \( Q \phi(t) \) is the semi-Markov kernel over \( E \) such that \( Q_{ij}(t) = P[X_{n+1} = j, \tau_{n+1} - \tau_n \leq t | X_n = i] \), with non-zero elements \( P_{ij} \), as follows:

\[
\begin{align*}
p_{01} &= \int z(t)\overline{O}(t)\overline{S}(t)dt, \\
p_{02} &= \int s(t)\overline{O}(t)\overline{O}(t)dt, \\
p_{03} &= \int o(t)\overline{S}(t)\overline{O}(t)dt, \\
p_{10} &= \int g(t)\overline{O}(t)\overline{O}(t)dt, \\
p_{15} &= \int z(t)\overline{O}(t)\overline{O}(t)dt, \\
p_{16} &= \int o(t)\overline{O}(t)\overline{G}(t)dt, \\
p_{21} &= \int ah(t)\overline{O}(t)\overline{O}(t)dt, \\
p_{24} &= \int bh(t)\overline{O}(t)\overline{O}(t)dt, \\
p_{22} &= \int o(t)\overline{O}(t)\overline{O}(t)dt, \\
p_{23} &= \int m_1(t)\overline{O}(t)\overline{M}_1(t)dt, \\
p_{30} &= \int z(t)\overline{O}(t)\overline{H}(t)dt, \\
p_{31} &= \int o(t)\overline{H}(t)\overline{O}(t)dt, \\
p_{36} &= \int f(t)\overline{O}(t)\overline{M}(t)dt, \\
p_{40} &= \int o(t)\overline{M}_1(t)\overline{O}(t)dt, \\
p_{49} &= \int f(t)\overline{O}(t)\overline{O}(t)dt, \\
p_{41} &= \int g(t)dt, \\
p_{63} &= \int g(t)dt, \\
p_{66} &= \int m_1(t)dt, \\
p_{71} &= \int m_1(t)dt, \\
p_{83} &= \int m_1(t)dt, \\
p_{93} &= \int f(t)dt, \\
p_{101} &= \int f(t)dt, \\
p_{113} &= \int ah(t)dt, \\
p_{114} &= \int bh(t)dt, \\
p_{1215} &= \int bh(t)dt, \\
p_{133} &= \int ah(t)dt, \\
p_{143} &= \int f(t)dt, \\
p_{151} &= \int f(t)dt, \\
p_{161} &= \int g(t)dt, \\
p_{115} &= \int P_{15}P_{51}, \\
p_{136} &= \int P_{16}P_{63}, \\
p_{2112} &= \int P_{21}P_{12}P_{12}P_{15}P_{15}, \\
p_{2112} &= \int P_{21}P_{12}P_{12}P_{16}P_{16}, \\
p_{23113} &= \int P_{23}P_{11}P_{13}P_{13}, \\
p_{23114} &= \int P_{23}P_{11}P_{14}P_{14}, \\
p_{317} &= \int P_{37}P_{71}, \\
p_{338} &= \int P_{38}P_{83}, \\
p_{439} &= \int P_{49}P_{93}, \\
p_{4110} &= \int P_{41}P_{10}P_{10} \end{align*}
\]
Evidently, 

\[ p_{01} + p_{02} + p_{03} = 1, \quad p_{10} + p_{15} + p_{16} = 1, \quad p_{21} + p_{24} + p_{211} + p_{212} = 1, \quad p_{30} + p_{37} + p_{38} = 1, \quad p_{40} + p_{49} + p_{4,10} = 1, \]

\[ p_{51} = p_{63} = p_{71} = p_{83} = p_{10,1} = 1 \] also \( p_{11,13} + p_{11,14} = 1, \quad p_{12,15} + p_{12,16} = 1, \quad p_{13,3} = p_{14,3} = p_{15,1} = p_{16,1} = 1 \)

The mean sojourn time in the state \( S_i \in E \) is given by \( \mu_i = E(t) = \int_0^\infty P(T > t) dt \), where \( T \) denotes the time to system failure, such that

\[ \mu_0 = \int_0^\infty \bar{O}(t) \bar{Z}(t) S(t) dt, \quad \mu_1 = \int_0^\infty \bar{O}(t) \bar{G}(t) \bar{Z}(t) dt, \quad \mu_2 = \int_0^\infty \bar{O}(t) \bar{H}(t) \bar{Z}(t) dt, \]

\[ \mu_3 = \int_0^\infty \bar{O}(t) \bar{M}(t) \bar{Z}(t) dt, \quad \mu_4 = \int_0^\infty \bar{O}(t) \bar{Z}(t) F(t) dt \]

The unconditional mean time taken by the system to transit to any regenerative state \( S_j \) when time is counted from an epoch of entrance into that state \( S_i \) is given by:

\[ m_{ij} = \int_0^\infty t \text{d} \{ Q_{ij}(t) \} \]

\[ m_{10} + m_{1,1.5} + m_{1,3.6} = \mu_1, \quad m_{21} + m_{24} + m_{21,12.15} + m_{21,12.16} + m_{23,11.13} + m_{23,11.14} = \mu_2, \]

\[ m_{30} + m_{3,7.1} + m_{3,3.8} = \mu_3, \quad m_{40} + m_{4,3.9} + m_{4,10.10} = \mu_4 \]

All the possible transition states of the system model are as follows:

The regenerative states:

\( S_0 = (N_o, C_s), \quad S_1 = (F_{ur}, N_o), \)

\( S_2 = (N_o, F_{ui}), \quad S_3 = (P_m, N_o), \)

\( S_4 = (N_o, F_{urp}) \)

The non-regenerative states:

\( S_5 = (F_{UR}, F_{wr}), \quad S_6 = (F_{UR}, WP_m), \)

\( S_7 = (P_m, F_{wr}), \quad S_8 = (P_m, WP_m), \)

\( S_9 = (WP_m, F_{URp}), \quad S_{10} = (P_w, F_{URp}), \)

\( S_{11} = (WP_m, F_{UI}), \quad S_{12} = (F_{wr}, F_{UI}), \)

\( S_{13} = (WP_m, F_{ur}), \quad S_{14} = (WP_m F_{urp}), \)

\( S_{15} = (F_{WR}, F_{urp}), \quad S_{16} = (F_{WR}, F_{ur}) \)

The expressions for all the measures of system performance can be expressed in terms of the transition probabilities and the mean sojourn times.
5. Reliability and Meantime to system failure (MTSF)

Let \( \bar{\phi}_i(t) \) be the cdf of time to system failure i.e. first passage time, starting from the state \( S_j \in E \) up to a failed state. Using the theory of regenerative processes, we have the following set of recursive relations for \( \bar{\phi}_i(t) \):

\[
\bar{\phi}_i(t) = \sum_{s \in \{1, 2, 3, 4\}} \left[ \sum_{j \in \{i, j, k, l\}} Q_{i,j,k}^j(t) \right] \phi_j(t) + \sum_{j \in \{i, j, k, l\}} Q_{i,j,k}^j(t); \quad i = 1, 2, 3, 4
\]

Here, \( S_j \) is an operative regenerative state to which the given regenerative state \( S_j \) can transit and \( S_k \) is a failed state to which the state \( S_j \) can transit directly. Further, \( S_{i,j,k,l,m,n} \) denotes the transition of the system from state \( S_i \in E \) to \( S_j \in E \) via failed states \( S_k, S_l, S_m \), etc.

For \( i = 1 \), we have

\[
\bar{\phi}_1(t) = Q_{10}(t) \phi_0(t) + \sum_{j=3,6} Q_{1,j}(t)
\]

The left-hand side of this equation shows that the system remains operative \( S_j \in E \cap U \), until time \( t \). The first term of right-hand side indicates that the system transits from \( S_i \) to \( S_0 \), at a time less than time \( t \) and the system completes its operation until the time \( t \), starting from state \( S_0 \), whereas the second term implies that the system moves from a regenerative up-state \( S_i \) to failed states \( S_j \) and \( S_6 \). Similarly, all other possible equations can be obtained and explained using above general expression.

Taking LST of above relation (1) and solving for \( \bar{\phi}_0(s) \), omitting the argument \( s \) for brevity, we get:

\[
\bar{\phi}_0(s) = \frac{[\bar{Q}_{15} + \bar{Q}_{16}][\bar{Q}_{01} + \bar{Q}_{02}\bar{Q}_{21}] + \bar{Q}_{12}[\bar{Q}_{211} + \bar{Q}_{212}] + \bar{Q}_{02}\bar{Q}_{24}[\bar{Q}_{49} + \bar{Q}_{410}] + \bar{Q}_{03}[\bar{Q}_{37} + \bar{Q}_{38}]}{1 - \bar{Q}_{02}\bar{Q}_{24}\bar{Q}_{40} - \bar{Q}_{03}\bar{Q}_{30} - \bar{Q}_{10}[\bar{Q}_{01} + \bar{Q}_{02}\bar{Q}_{21}]}
\]

The reliability and mean time to system failure (MTSF) are given by

\[
\text{Reliability } R(t) = L^{-1} \left[ \frac{1 - \bar{\phi}_i(s)}{s} \right]
\]

\[
\text{MTSF } = \lim_{s \to 0} \frac{1 - \bar{\phi}_0(s)}{s} = \frac{N_1}{D_1}
\]

Where

\[
N_1 = \mu_0 + [p_{01} + p_{02}p_{21}] \mu_1 + p_{02} \mu_2 + p_{03} \mu_3 + p_{02}p_{24} \mu_4
\]

\[
D_1 = 1 - p_{01}p_{10} - p_{02}[p_{10}p_{21} + p_{24}p_{40}] - p_{03}p_{30}
\]

6. Analysis of Economic Measures

Let the system entered the regenerative state \( S_i \in E \), at \( t = 0 \). Considering \( S_j \in E \), as a regenerative state to which the given regenerative state \( S_j \in E \), reaches. Now using the above terms the recursive relations for various measures of system performance are given as follows:

a. System availability:

Let \( A_i(t) = P[S_i \in U, \text{at time} \ t \mid S_i \in E, \text{at} \ t = 0] \), then using the standard notations given in section 2, we have the following expression for the system availability in \( (0, t] \):

\[
A_i(t) = 1 - p_{01} + p_{02}[p_{10}p_{21} + p_{24}p_{40}] - p_{03}p_{30}
\]
\( A_i(t) = M_{i \in (U \cup E)}(t) + \sum_{i,j \in E} \left[ q_{i,j}(t) + \delta_{i,j} \right] \quad (i)A_j(t); i = 0,1,2,3,4 \quad (2) \)

**b. Busy period**

Let us define \( B_i^{IB/Rp/PM}(t) = P[\text{Server is busy in inspection/repair/replacement/PM at time } t | S_i \in E \text{ at } t = 0] \)

Now using the simple probability rules we have following expressions for different busy periods of the server in \((0, t)\):

i. **Server busy period due to inspection**

\[
B_j^{I}(t) = W_{i \in E}^{I}(t) + \sum_{i,j \in E \atop k,l,m,... \in (U \cap E)} \left[ q_{i,j}(t) + \delta_{i,j,k} \right] \quad (i)B_j^{I}(t); i = 0,1,2,3,4 \quad (3)
\]

ii. **Server busy period due to repair**

\[
B_j^{R}(t) = W_{i \in E}^{R}(t) + \sum_{i,j \in E \atop k,l,m,... \in (U \cap E)} \left[ q_{i,j}(t) + \delta_{i,j,k} \right] \quad (i)B_j^{R}(t); i = 0,1,2,3,4 \quad (4)
\]

iii. **Server busy period due to replacement**

\[
B_j^{Rp}(t) = W_{i \in E}^{Rp}(t) + \sum_{i,j \in E \atop k,l,m,... \in (U \cap E)} \left[ q_{i,j}(t) + \delta_{i,j,k} \right] \quad (i)B_j^{Rp}(t); i = 0,1,2,3,4 \quad (5)
\]

iv. **Server busy period due to preventive maintenance**

\[
B_j^{PM}(t) = W_{i \in E}^{PM}(t) + \sum_{i,j \in E \atop k,l,m,... \in (U \cap E)} \left[ q_{i,j}(t) + \delta_{i,j,k} \right] \quad (i)B_j^{PM}(t); i = 0,1,2,3,4 \quad (6)
\]

c. **Expected number of remedial activities**

Let us first define the conditional expectation of the remedial activities done on the system i.e.

\[
I_j(t)/R_j(t)/R_j^{E}(t)/B_j^{PM}(t) = E[\text{No of insp./repairs/repl./PM of unit in } (0, t) | S_i \in E \text{ at time } t = 0]
\]

Now using some probabilistic rules we have the following expressions:

i. **Expected number of inspections of the unit**

\[
I_j(t) = \sum_{i,j \in E \atop k,l,m,... \in (U \cap E)} \left[ Q_{i,j}(t) + \delta_{i,j,k} \right] \quad (s)I_j(t); i = 0,1,2,3,4 \quad (7)
\]

ii. **Expected number of repairs of the unit**

\[
R_j(t) = \sum_{i,j \in E \atop k,l,m,... \in (U \cap E)} \left[ Q_{i,j}(t) + \delta_{i,j,k} \right] \quad (s)R_j(t); i = 0,1,2,3,4 \quad (8)
\]

iii. **Expected number of replacements of the unit**

\[
R_j^{C}(t) = \sum_{i,j \in E \atop k,l,m,... \in (U \cap E)} \left[ Q_{i,j}(t) + \delta_{i,j,k} \right] \quad (s)R_j^{C}(t); i = 0,1,2,3,4 \quad (9)
\]

iv. **Expected number of preventive maintenance of the unit**

\[
P_j^{M}(t) = \sum_{i,j \in E \atop k,l,m,... \in (U \cap E)} \left[ Q_{i,j}(t) + \delta_{i,j,k} \right] \quad (s)P_j^{M}(t); i = 0,1,2,3,4 \quad (10)
\]
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Here \( \delta_j = \begin{cases} 1 & \text{if there is a repair/ treatment from } S_i \text{ to } S_j \nonumber \\ 0 & \text{Otherwise} \end{cases} \)

And \( \delta_{i,j,k,l} = \begin{cases} 1 & \text{if there is a transition from } S_i \text{ to } S_j \text{ via } S_k \text{ to } S_l \nonumber \\ 0 & \text{Otherwise} \end{cases} \)

7. Steady-State Analysis

To obtain the steady results we use the method of Laplace/ Laplace-Stieltjes transform. The Laplace-Stieltjes transform of a real-valued function \( f \) is given by

\[
L[f(t)] = \int e^{-st} df(t) = f(s)
\]

The final value formula of Laplace transform states that

\[
limit_{t \to \infty} f(t) = limit_{s \to 0} s f(s)
\]

Using these formulae, we obtained the expressions for following system performance indices:

\[
A_0 = \lim_{t \to \infty} A_i(t) = \lim_{s \to 0} s[L[A_i(t)] = \frac{N_2}{D_2}; \quad B_0^i = \lim_{t \to \infty} B_0^i(t) = \lim_{s \to 0} s[L[B_0^i(t)] = \frac{N_3^i}{D_2};
\]

\[
B_0^R = \lim_{t \to \infty} B_0^R(t) = \lim_{s \to 0} s[L[B_0^R(t)] = \frac{N_3^R}{D_2}; \quad B_0^Rp = \lim_{t \to \infty} B_0^Rp(t) = \lim_{s \to 0} s[L[B_0^Rp(t)] = \frac{N_3^{RP}}{D_2};
\]

\[
B_0^PM = \lim_{t \to \infty} B_0^PM(t) = \lim_{s \to 0} s[L[B_0^PM(t)] = \frac{N_3^{PM}}{D_2}; \quad I_0 = \lim_{t \to \infty} I_0(t) = \lim_{s \to 0} s[L[I_0(t)] = \frac{N_4}{D_2};
\]

\[
R_0 = \lim_{t \to \infty} R_0(t) = \lim_{s \to 0} s[L[R_0(t)] = \frac{N_4^R}{D_2}; \quad R_0^C = \lim_{t \to \infty} R_0^C(t) = \lim_{s \to 0} s[L[R_0^C(t)] = \frac{N_4^C}{D_2};
\]

\[
P_0^M = \lim_{t \to \infty} P_0^M(t) = \lim_{s \to 0} s[L[P^M_0(t)] = \frac{N_4^PM}{D_2};
\]

\[
N_2 = [\mu_0 + \mu_2P_{p2} + \mu_4P_{p4}P_{p2}][P_{p10}(1 - p_{33.8}) + p_{13.6}P_{p30}] + \mu_1[p_{p2}(p_{21}(p_{30}P_{p41,10} - p_{40}P_{p31,7}) + p_{30}(p_{21} + p_{21,12,15} + p_{21,12,16}) + p_{p10}P_{p30} + p_{31,7})] + \mu_3[p_{p2}(p_{10}(p_{21,11,13} + p_{23,11,14} + p_{24,43.9}) - p_{24}P_{p40}P_{p31,6} + p_{13,6} + p_{p0}P_{p10}]
\]

\[
D_2 = [\mu_0 + \mu_2P_{p2} + \mu_4P_{p4}P_{p2}][p_{p10}(1 - p_{33.8}) + p_{13,6}P_{p30}] + \mu_1[p_{p2}(p_{21}(p_{30}P_{p41,10} - p_{40}P_{p31,7}) + p_{30}(p_{21} + p_{21,12,15} + p_{21,12,16}) + p_{p10}P_{p30} + p_{31,7})] + \mu_3[p_{p2}(p_{10}(p_{21,11,13} + p_{23,11,14} + p_{24,43,9}) - p_{24}P_{p40}P_{p31,6} + p_{13,6} + p_{p0}P_{p10}]
\]

\[
N_3 = \frac{W_1^{R^*}}{0}[1 - p_{33.8}]P_{p2} + p_{21} + p_{21,12,15} + p_{21,12,16} + p_{p0} + W_1^{R^*}(0)p_{31,7}P_{p2} + p_{24}P_{p43,9} + p_{23,11,13} + p_{23,11,14} + p_{p0}]
\]

\[
N_3^{RP} = \frac{W_4^{R^*}}{0}[p_{p0}P_{p4}(1 - p_{33.8}) + p_{13,6}P_{p30}], \quad N_3^I = \frac{W_2^I}{0}[p_{p0}(1 - p_{33,8}) + p_{13,6}P_{p30}]
\]

\[
N_3^{PM} = \frac{W_3^{PM^*}}{0}[1 - p_{11,5}]P_{p2} + p_{23,11,13} + p_{23,11,14} + p_{24,43,9} + p_{p0} + W_3^{PM^*}(0)p_{13,6}P_{p2} + p_{24}P_{p4,10,15} + p_{23,12,16} + p_{23,12,16} + p_{p0}]
\]

\[
N_4 = [1 - p_{33,8}]P_{p2} + p_{p0}[1 - p_{21} + p_{21,12,15} + p_{21,12,16} + p_{p0} + p_{p0}P_{p4}P_{p41,10} + p_{21,12,16} + p_{23,11,13}] + p_{p0}P_{p4}P_{p41,7}P_{p43,9} - p_{p0}P_{p4}P_{p41,10} + p_{21,12,16} + p_{23,11,13} + p_{p0}P_{p4}P_{p41,10}P_{p43,9}
\]

\[
N_4^C = p_{p0}(1 - p_{33,8} + p_{13,6}P_{p30})P_{p2} + p_{23,11,14} + p_{23,11,14} + N_4 = p_{p0}(1 - p_{33,8} + p_{13,6}P_{p30})
\]
\[ N_4^{PM} = [1 - p_{11.5}] \{ p_{02} \{ p_{23.11.13} + p_{23.11.14} + p_{24.43.9} \} + p_{03} \} + p_{13.6} \{ p_{02} \{ p_{21} + p_{21.12.15} + p_{21.12.16} + p_{24.41.10} \} + p_{01} \} \]

Now we can obtain the profit incurred to the system in steady-state viz.

\[ \text{Profit} = (\text{total revenue generated}) - (\text{total expenses incurred}) \]

\[ P_0 = \left( K_0 A_0 \right) - \left( K_1 B_0 + K_2 B_0^R + K_3 B_0^R + K_4 L_0 + K_5 R_0 + K_6 R_0^C + K_7 B_0^PM + K_8 P_0^M \right) \]

\[ K_0 = \text{Revenue per unit up-time of the system} \]

\[ K_1 = \text{Cost per unit time for which server is busy in the inspection of the standby} \]

\[ K_2 = \text{Cost per unit time for which server is busy due to repair} \]

\[ K_3 = \text{Cost per unit time for which server is busy due to replacement} \]

\[ K_4 = \text{Cost per unit inspection} \]

\[ K_5 = \text{Cost per unit repair} \]

\[ K_6 = \text{Cost per unit replacement} \]

\[ K_7 = \text{Cost per unit time for which server is busy due to preventive maintenance} \]

\[ K_8 = \text{Cost per unit preventive maintenance} \]

8. **Graphical Illustrations**

   a. **Case-1: Weibull Distribution**

   Taking shape parameter \( \eta = 0.5 \), the pdf for different random variables becomes as follows:

   \[ z(t) = \frac{\lambda}{2\sqrt{t}} \exp(-\lambda \sqrt{t}), \quad s(t) = \frac{\mu}{2\sqrt{t}} \exp(-\mu \sqrt{t}), \quad o(t) = \frac{\xi}{2\sqrt{t}} \exp(-\xi \sqrt{t}), \]

   \[ g(t) = \frac{\beta}{2\sqrt{t}} \exp(-\beta \sqrt{t}), \quad f(t) = \frac{\gamma}{2\sqrt{t}} \exp(-\gamma \sqrt{t}), \quad h(t) = \frac{\alpha}{2\sqrt{t}} \exp(-\alpha \sqrt{t}), \quad m(t) = \frac{\nu}{2\sqrt{t}} \exp(-\nu \sqrt{t}), \]

   ![MTSF vs Failure Rate](image1)

   **Fig. 1: Effect of various parameters on MTSF (\( \eta = 0.5 \))**

   ![Availability vs Failure Rate](image2)

   **Fig. 2: Effect of various parameters on Availability (\( \eta = 0.5 \))**
Performance Analysis of a Standby System using Exponential-Rayleigh-Weibull Distributions

Fig. 3: Effect of various parameters on Profit ($\eta = 0.5$)

Fig. 4: Effect of various parameters on Profit ($\eta = 0.5$)

b. Case-2: Exponential Distribution
Taking shape parameter $\eta = 1.0$, the pdf for different random variables becomes as follows

\[ z(t) = \lambda \exp(-\lambda t), \quad s(t) = \mu \exp(-\mu t), \quad o(t) = \xi \exp(-\xi t), \quad g(t) = \beta \exp(-\beta t), \]
\[ f(t) = \gamma \exp(-\gamma t), \quad h(t) = \alpha \exp(-\alpha t), \quad m_1(t) = \nu \exp(-\nu t). \]

where $t \geq 0$ and $\eta, \lambda, \mu, \nu, \beta, \gamma, \alpha, \xi > 0$

Fig. 5: Effect of various parameters on MTSF ($\eta = 1$)

Fig. 6: Effect of various parameters on Availability ($\eta = 1$)
c. Case-3: Rayleigh Distribution
When shape parameter $\eta = 2$ then the probability distributions reduce to Rayleigh with the pdfs given below:

- $z(t) = 2\lambda t \exp(-\lambda t^2)$, $s(t) = 2\mu t \exp(-\mu t^2)$, $o(t) = 2\xi t \exp(-\xi t^2)$, $g(t) = 2\beta t \exp(-\beta t^2)$,
- $f(t) = 2\sigma t \exp(-\sigma t^2)$, $h(t) = 2\alpha t \exp(-\alpha t^2)$, $m_1(t) = 2\nu t \exp(-\nu t^2)$,

where $t \geq 0$ and $\eta, \lambda, \mu, \xi, \beta, \gamma, \alpha, \nu > 0$
8. Discussion On Results

To carry out above graphical study all the random variables are supposed to follow Weibull distribution and the following data set is considered:

\[
\begin{align*}
\alpha &= 0.3, \quad \beta = 0.7, \quad \gamma = 0.14, \quad \delta = 0.07, \quad \mu = 0.1, \quad \nu = 0.25, \quad \xi = 0.041, \\
K_0 &= 30000, \quad K_1 = 100, \\
K_2 &= 500, \quad K_3 = 150, \quad K_4 = 800, \quad K_5 = 1000, \quad K_6 = 700, \quad K_7 = 300, \quad K_8 = 400.
\end{align*}
\]

The system performance is studied for three different values of the shape parameter of Weibull distribution that exhibit three distinct probability distributions as follows:

1) \(0 < \eta < 1; \eta = 0.5\), decreasing failure rate (DFR), fig.1-4.
2) \(\eta = 1\), constant failure rate (CFR), i.e. case of exponential distribution, fig.5-8.
3) \(\eta > 1; \eta = 2\), increasing failure rate i.e. case of Rayleigh distribution, fig.9-12.

In all three cases, we observed that MTSF, Availability and Profit declines with increasing value of failure rate. When we fixed the values of other parameters, the values of all these indices go up as value of any of the parameters viz. inspection rate; \(\alpha = 0.14\) to \(\alpha = 0.3\), repair rate; \(\beta = 0.12\) to \(\beta = 0.2\), replacement rate; \(\gamma = 0.2\) to \(\gamma = 0.4\) or the preventive maintenance rate; \(\nu = 0.25\) to \(\nu = 0.3\) increases. All these indices declines with increasing value of \(\mu\) (Maximum redundancy time) and rate at which operative unit goes under preventive maintenance (maximum operation time). Figure 4, 8 and 12 gives the cutoff points for profit against revenue per unit up time of system.

In case of \(\eta = 0.5\), the trend lines of \(\alpha = 0.3\) decline sharply than the lines corresponding to \(\beta = 0.2\) for \(\lambda > 0.02\). It means that replacement works better as compare to inspection after failure rate 0.02. In case of \(\eta = 1\), the trend lines of \(\alpha = 0.3\) decline sharply than the lines corresponding to \(\beta = 0.2\) for \(\lambda > 0.04\). In case of \(\eta = 2\), the trend lines of \(\alpha = 0.3\) decline sharply than the lines corresponding to \(\beta = 0.2\) for \(\lambda > 0.06\).

9. Conclusion and Future Directions

This paper analyzed a two-unit cold standby system with limits on maximum redundancy time for the standby and maximum operation time for the operative unit. The discrete state-continuous time Markov regenerative processes are used to develop and analyze the model. The standby unit went under inspection for checking the practicability of repair or replacement after surpassing the maximum redundancy time limit. On the other hand, preventive maintenance is used on the operative unit just after crossing the maximum operation time limit. The graphical trends obtained for various measures of system performance underlined the proven facts that system performance declines with failures and uprises with remedial activities, that advocate for the model’s practicality.

For future research, the prospective option may be the generalization of this model considering m-operative and n-standby components. The assumption of independence of all random variables maybe relaxed for more realism.

References

3. M. Manglik and M. Ram, “Reliability analysis of a two unit cold standby system using markov


