An EPQ Model of Stock Dependent Demand Subject to Epidemic with Stochastic Lockdown Time

Ruchi Sharma¹, G.S. Buttar²

¹Department of Mathematics, Chandigarh University, Gharuan, Mohali, India.
²Associate Professor, Department of Mathematics, Chandigarh University, Gharuan, Mohali, India.

Article History: Received: 11 January 2021; Accepted: 27 February 2021; Published online: 5 April 2021

Abstract: The model created considers the effect of the epidemic on the classical Economic Production Quality (EPQ) model for a production unit exposed to stochastic lockdown time. Expected production time is evaluated utilizing continuous probability density function. The investigation is done to decide the ideal arrangement for the production system which limits the expected total cost per unit time exposed to certain conditions. Here EPQ model is created by taking lockdown time due to epidemic as stochastic. Machine breakdown affects the manufacturer but disaster like epidemic affects the manufacturer as well as the customer (or in other words, demand). During the production uptime, demand depend upon stock and decline in selling price, but in case of disaster (epidemic) selling price has no consideration and demand depends only on stock. The model is discussed by means of a numerical example and a case study.

Keywords: Inventory; Economic Production Quantity; flexible production system; Optimization; stochastic lockdown, relaxation in lockdown

1. Introduction

Classical Economic Production Quantity (EPQ) model expect that production units are totally flexible and reliable. This notion, though, doesn’t qualify for some actual systems. Indeed, even the most excellent and the most advanced manufacturing systems go through the circumstance of unexpected emergency like machine breakdown, and time taken in repair or replacement rely upon availability of machine part and/or mechanic. Likewise, in the case of epidemic, the resuming of work relies upon the intensity of epidemic and availability of medicine. Production capacity of the production unit may suffer due to epidemic in terms of lockdown imposed and unavailability of skilled or unskilled workers. Generally, the manufacturing system is considered as adaptable to deliver according to the demand. The manufacturing may stop at any arbitrary time and the lockdown period is additionally thought to be stochastic. The intention of this investigation is to decide the predictable optimum production run time with the end goal of reducing the overall cost per unit time.

There has been many proposed model considering the unexpected situations that lead to halting the manufacturing. Jawla et al. (2020) considered an EPQ model to examine the preservation technology impact with machine breakdown by assuming multivariate demand rate with crispy and fuzzy situation. Posoltanet al. (2020) considered the EPQ Model by taking stochastic machine breakdown and repair time with stochastic deterioration products and they all discussed the total cost comparison for different uptime. Fang & Yeh (2020) established an EPQ model by considering stochastic demand with unequal product life cycle. Sarka et al. (2020) optimize the cost of an EPQ Model with deterioration and stock dependent demand. Cárdenas-Barrón et al. (2020) suggested an EOQ model to optimize the retailer profit with and without shortages taking non linear demand and holding cost. Sharma & Singh (2020) considered EOQ model for imperfect items by considering collection and repair work. Oztür (2019) dealt with stochastic machine breakdown with two cases during production and after production to optimize the expected cost, discarding the imperfect products, and others are sold at reduced prices. Benkherouf et al. (2017) considered the EOQ model by taking two substitutable products and assuming both demand are varying with time. Lyonget al. (2017) considered the EPQ model with stochastic machine breakdown, stochastic repair time and deterioration to optimize the production cost. Singh et al. (2014) consider the EPQ Model by taking stochastic machine breakdown and stochastic repair time. They discussed profit for different production uptime. Wanget al. (2014) studied a problem of lot size with periodic-reviewed, random yield, due to disruptions breakdown in manufacturing. Dem and Singh (2012) considered the EPQ model having multivariate demand for delicate products. Widyada (2011) proposed an EPQ model with stochastic machine breakdown and random repair time. They examined that random repair model gives the better cost as compared to predetermined repair time. Giri and Chakraborty (2011) developed a model to reduce the cost considering supply chain management between vendor and buyer. They applied screening after every representation. Sana (2010) optimized the profit for imperfect production by considering consistency and manufacturing rate. Hou (2006) proposed an optimum model with inflation and shortages where demand depends upon stock. Giriet al. (2005) proposed EMQ model to examine the manufacturing rate and manufacturing lot size with machine breakdown and common repair time to minimize probable total cost.
2. Assumptions and Notations:

The assumptions made to develop the current model are as follows:

- Rate of production is function of demand $P = lD(q)$, $l > 1$
- The demand capacity of the item is thought to be reliant on stock and decrease in selling cost in the interval $[0, \mu]$, $D(q) = (\alpha + \beta q) p$

Where:

$\beta$ is the shape factor and is used to calculate sensitivity of demand to vary the level of available inventory, $\alpha$ denotes deterministic factor and $p$ represents decline in selling cost.

- After $t = \mu$, relaxation start demand rate $D(q) = \alpha + \beta q$ is a function of stock displayed

Notations used in the model are given below:

- $q(t)$ available inventory echelon of items,
- $D(q)$ represents demand rate,
- $P$ is the production rate, $P = lD(q)$, $l > 1$.
- $M$ is the set up cost,
- $S$ is the selling cost per article,
- $p$ is the decrement in selling price,
- $h$ holding cost per unit item/time,
- $T_1$ represents time when production halts,
- $T_{relax}$ represents time when lockdown occurs,
- $\mu$ represents time when relaxation is given in lockdown,
- $T_2$ time when inventory of items disappears and deficiencies begin to gather causing loss sales,
- $E(T)$ expected period of production cycle,
- $E(PDC)$ expected production cost,
- $E(H)$ expected holding cost in production cycle,
- $E(TC)$ expected total cost,
- $E(TAC)$ expected total average cost/time beginning the production time.

1. Model Formulation:

![Fig.1 Model Formulation](image)

In this projected model, as shown in Fig.1, a system is considered in which manufacturing process is assumed to be flexible and manufacturing is done according to demand rate. The consistency of the production is assumed to be an exponentially declining function of time due to epidemic, as a result probability density function forepidemic is assumed as:

$$f(T_1) = ke^{-kT_1}$$

The demand function of the product is assumed to be dependent on available stock and reduction in selling price in the interval $[0, \mu]$,

$$D(q) = (\alpha + \beta q)p$$

The production cycle starts with nil inventories at time $t = 0$. During time span $0 \leq t \leq T_1$ inventory increases even after fulfilling market need. In case of unforeseen conditions like epidemic, production may stop prior to time $T_1$. 

2860
An EPQ Model of Stock Dependent Demand Subject to Epidemic with Stochastic Lockdown Time

During the epidemic, production halts at $t = T_1$ due to lockdown, where $T_1$ is the lockdown time which is less than $T_1$. During the relaxation in the lockdown period, which is denoted by $\mu$, the inventory level decline due to online demand in the interval $[T_1, \mu]$ The relaxation in lockdown starts at $t = \mu$ and the inventory decline and reaches zero at $t = T_2$. As lockdown time is likewise stochastic, production may not generally be conceivable and loss sale may happen. When lockdown time is below the cycle length, the inventory can be presented as:

$$\frac{dq}{dt} = P - D(q), \quad q(0) = 0, \quad 0 \leq t \leq T_1$$  \hspace{1cm} (1)

$$\frac{dq}{dt} = -(\alpha + \beta q)p, \quad q(T_1^+) = q(T_1^-), \quad T_1 \leq t \leq \mu$$  \hspace{1cm} (2)

$$\frac{dq}{dt} = -(\alpha + \beta q), \quad q(T_2) = 0, \quad \mu \leq t \leq T_2$$  \hspace{1cm} (3)

Using eqs. (1), (2) & (3) inventory levels are obtained as follows:

$$q(t) = \frac{a(e^{(l-1)\beta p} - 1)}{\beta}, \quad 0 \leq t \leq T_1$$

$$q(t) = \frac{a(e^{(T_2-T_l)\beta} - 1)}{\beta}, \quad T_1 \leq t \leq \mu$$

$$q(t) = \frac{a(e^{(T_2-T_l)\beta} - 1)}{\beta}, \quad \mu \leq t \leq T_2$$

Applying Taylor series expansion and continuity condition, we have

$$T_2 = I(T_1 p + \mu(1-p))$$  \hspace{1cm} (4)

For viability of situation cycle length $T_2$ must be larger than relaxation lockdown time $\mu$ which implies $I(T_1 - \mu) > 0$. Total inventory in the absolute production cycle is

$$E(LS) = \frac{\mu}{\mu} \left[ \frac{a}{\beta} \left( \frac{e^{(l-1)\beta p} - 1}{(l-1)\beta p} + \frac{(p-1)e^{(l-1)\beta p} - p}{\beta p} (T_1p + \mu(1-p)) \right) \right]$$  \hspace{1cm} (5)

If lockdown occurs at $T_1$ then total inventory using (5) is given below

$$E(l) = \frac{\mu}{\mu} \left[ \frac{a}{\beta} \left( \frac{e^{(l-1)\beta p} - 1}{(l-1)\beta p} + \frac{(p-1)e^{(l-1)\beta p} - p}{\beta p} (T_1p + \mu(1-p)) \right) \right]$$  \hspace{1cm} (6)

Using probability density function of epidemic $(T_1) = ke^{-kT_1}, T_1 > 0$.

The expected inventory is calculated as:

$$E(L) = \frac{\mu}{\mu} \left[ \frac{a}{\beta} \left( \frac{e^{(l-1)\beta p} - 1}{(l-1)\beta p} + \frac{(p-1)e^{(l-1)\beta p} - p}{\beta p} (T_1p + \mu(1-p)) \right) \right]$$  \hspace{1cm} (7)

Expected lost sale cost

$$E(\mu) = \frac{\mu}{\mu} \left[ \frac{e^{(l-1)\beta p} - 1}{(l-1)\beta p} + \frac{(p-1)e^{(l-1)\beta p} - p}{\beta p} (T_1p + \mu(1-p)) \right]$$  \hspace{1cm} (8)

Lost sale take place when there is no relaxation in lockdown and it exceeds $T_2$, assuming that lockdown time $t$ is a arbitrary variable and is evenly distributed over the time $[0, c]$. The probability density function $f(t)$ for the lockdown period is given by

$$f(t) = \frac{1}{c}, \quad 0 \leq t \leq c$$

$$= 0 \quad \text{or else}$$

Expected lost sale cost

$$E(\mu) = \frac{\mu}{\mu} \left[ \frac{e^{(l-1)\beta p} - 1}{(l-1)\beta p} + \frac{(p-1)e^{(l-1)\beta p} - p}{\beta p} (T_1p + \mu(1-p)) \right]$$  \hspace{1cm} (9)

2861
Where $A = c - \mu(1 - p)$

Now production cost = $c_p \int_0^T P_d t = c_p \int_0^T I D(q) dt = c_p I \alpha \left(\frac{e^{(l-1)\beta T_1} - 1}{(l-1)\beta p} \right) \tag{9}$

When Lockdown occurs at $t = T_1$, then (9) becomes

$PDC = c_p \alpha \left(\frac{e^{(l-1)\beta T_1} - 1}{(l-1)\beta p} \right) \tag{9}$, $T_1 < T_1$

$= c_p \alpha \left(\frac{e^{(l-1)\beta T_1} - 1}{(l-1)\beta p} \right) \tag{9}$, $T_1 > T_1$

Expected production cost

$E(PDC) = \int_{T_1=0}^{T_1} c_p \alpha \left(\frac{e^{(l-1)\beta T_1} - 1}{(l-1)\beta p} \right) k \alpha \left(\frac{e^{-k T_1 d T_1}}{e^{-k T_1 d T_1}} \right) k e^{-k T_1 d T_1} = c_p \alpha \left(\frac{e^{(l-1)\beta T_1} - 1}{(l-1)\beta p} \right) k e^{-k T_1 d T_1}$

The expected lost cost sum of set up cost, expected holding cost, expected lost sale cost and production cost.

Expected total cost $E(TC) = M + E(H) + E(LS) + E(PDC)$

The expected production time is the sum of expected production up time period, non production period and expected Lockdown time after $t = T_2$

$E(T) = \int_{T_1=0}^{T_1} T_2 e^{-k T_1 d T_1} + \int_{T_1=0}^{T_1} T_2 e^{-k T_1 d T_1} + \int_{T_1=0}^{T_1} (T - T_2) f(t) e^{-k T_1 d T_1}$

$= \mu(1 - p) + \frac{lp(1-e^{-k T_1})}{k} + \frac{k}{2c} \left(\frac{A}{k} + 1\right) + \frac{1}{2} \int_{T_1}^{T_1} \frac{(T - T_2) e^{-k T_1 d T_1}}{k} [1 - e^{-k T_1}]$

Now expected total cost per unit time can be calculated by ratio of the expected total cost per renewal cycle to the expected duration of a renewal cycle $E(TAC)$ (expected total average cost)

$E(TAC) = \frac{E(TC)}{E(T)}$

2. Optimal solution procedure:

The purpose of the model is to find out an optimal production up time $(T_1)$ so that it gives minimum $E(TAC)$. The necessary condition for $E(TAC)$ is to be minimum is

$\frac{d(E(TAC))}{dT_1} = 0$ and $\frac{d^2(E(TAC))}{dT_1^2} > 0$

Now for feasibility of model, lost sale occur only when $T_2$ is less than $c$ or we can say when lock down period exceeds $T_2$, subsequently investigating the pattern of $E(T)$ in the interval $0 \leq T_2 \leq c$

$\frac{dE(T)}{dT_1} = \left[-lpk - \frac{k^2}{2c} \frac{2Alp T_k}{2c} \frac{2Alp T_k}{c} \right] e^{-k T_1}$

$\frac{d^2E(T)}{dT_1^2} = \left[-lpk - \frac{k^2}{2c} \frac{2Alp T_k}{2c} \frac{2Alp T_k}{c} \right] e^{-k T_1}$

Assume $T_1 = 0$, then $\frac{d^2E(T)}{dT_1^2} = -lpk - \frac{2Alp k}{2c} < 0$

$\frac{d^2E(T)}{dT_1^2} < 0 \text{ if } \frac{kT_1^2 + \frac{1}{c}}{c} < lpk \left(\frac{A}{c} + 1\right) \text{ if } T_1 < \frac{A}{lp}$

where $A = c - \mu(1 - p)$

$E(T)$ is concave w.r.t. $T_1$ where $0 \leq T_1 \leq \frac{A}{lp}$

or $0 \leq T_1 \leq c$, therefore $E(T)$ is dipped when $0 \leq T_1 \leq c$

To prove $\frac{d^2E(TAC)}{dT_1^2} > 0 \text{ if } \frac{d^2E(T)}{dT_1^2} > 0 \text{ where } 0 \leq T_1 \leq c$

$\frac{dE(TAC)}{dT_1} = \frac{d^2E(T)}{dT_1^2} E(T) - E(T) \frac{d^2E(T)}{dT_1^2}$

As $\frac{d^2E(T)}{dT_1^2} < 0$ in the interval $0 \leq T_1 \leq c$, also $E(T) > 0, E(TC) > 0$

Therefore, in the interval $0 \leq T_1 \leq c$, $\frac{d^2E(TAC)}{dT_1^2} > 0 \text{ if } \frac{d^2E(T)}{dT_1^2} > 0$

$\frac{dE(TC)}{dT_1} = h \left[ \frac{1}{\beta} - \frac{1}{\beta} e^{(l-1)\beta p T_1} - \frac{1}{\beta} e^{-k T_1} \right] c_p \alpha \left(\frac{e^{(l-1)\beta T_1} - 1}{(l-1)\beta p} \right)$
An EPQ Model of Stock Dependent Demand Subject to Epidemic with Stochastic Lockdown Time

\[ \frac{d^2 E(TC)}{dT_1^2} = h \alpha l e^{-kT_1} \left( (l-1)p e^{(l-1)\beta p T_1} + lp(p-1) \right) + \frac{Sak}{c} \left[ l^2 p^2 T_1 e^{-kT_1} - Ap e^{-kT_1} \right] \]

+ \frac{Sak}{2c} \left[ A^2 e^{-kT_1} + l^2 p^2 T_1^2 e^{-kT_1} - 2ApT_1 e^{-kT_1} \right]

Assume \( T_1 = 0 \)

\[ \frac{d^2 E(TC)}{dT_1^2} > 0 \]

if

\[ h \alpha l \beta \left( (l-1)p + lp(p-1)e^{-\mu \beta p} - \frac{kSaAlp}{c} + pl(l-1)\beta p \right) > 0 \]

Assume \( T_1 = \frac{A}{\beta} \)

\[ \frac{d^2 E(TC)}{dT_1^2} > 0 \]

if

\[ h \alpha l \beta \left( (l-1) + lp(p-1)e^{-\mu \beta p} \right) + cl(l-1)\beta p e^{(l-1)\beta p-kT_1} > 0 \]

Therefore, in the interval \( 0 \leq T_1 \leq c, \frac{d^2 E(TAC)}{dT_1^2} > 0 \)

3. Numerical Illustration:

In this section, numerical results are obtained by considering various parameter to illustrate the manner of expected production time \( E(T) \), and Expected Total Average Cost \( E(TAC) \). Calculations are performed using mathematics tool WolframMathematica 7. Various parametric values considered are given as follows:

\( \beta = 0.4, h = 1, M = 200, k = 0.1, \alpha = 20, c_p = 10, S = 30, c = 6, \mu = 4, l = 2, p = 3 \)

Effect of holding cost on optimal value of expected production time \( E(T) \), \( E(Q) \) and expected total average cost \( E(TAC) \) is shown in Table 1 and in Fig. 2.

**Table 1: Effect of holding cost on optimal value \( E(T) \), \( E(Q) \) and \( E(TAC) \)**

<table>
<thead>
<tr>
<th>( h )</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 )</td>
<td>2.06518</td>
<td>2.0856</td>
<td>2.10089</td>
<td>2.11188</td>
</tr>
<tr>
<td>( E(T) )</td>
<td>6.05362</td>
<td>6.17618</td>
<td>6.26772</td>
<td>6.33291</td>
</tr>
<tr>
<td>( E(HC) )</td>
<td>1679.65</td>
<td>2542.15</td>
<td>3412.29</td>
<td>4285.91</td>
</tr>
<tr>
<td>( E(LC) )</td>
<td>229.83</td>
<td>214.6</td>
<td>202.956</td>
<td>194.521</td>
</tr>
<tr>
<td>( E(PC) )</td>
<td>996.671</td>
<td>1008.23</td>
<td>1016.91</td>
<td>1023.151</td>
</tr>
<tr>
<td>( E(TAC) )</td>
<td>480.0683</td>
<td>609.5969</td>
<td>730.0496</td>
<td>869.0447</td>
</tr>
<tr>
<td>( E(Q) )</td>
<td>1679.65</td>
<td>1694.767</td>
<td>1706.145</td>
<td>1714.364</td>
</tr>
</tbody>
</table>
5.1 Observations:
(a) As holding cost increases with increase in lockdown time, production run time slightly increases so that demand can be satisfied, which increases the inventory level. Since the demand is based on inventory level, it leads to decrease in expected loss.

Effect of production cost in Expected production time $E(T)$, $E(Q)$, Expected loss cost $E(LC)$ and Expected total average cost $E(TAC)$ is shown in Table 2 and Fig. 3.

<table>
<thead>
<tr>
<th>$c_p$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$T_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(T)$</td>
<td>6.05362</td>
<td>6.0926</td>
<td>6.13055</td>
<td>6.16749</td>
</tr>
<tr>
<td>$E(LS)$</td>
<td>229.83</td>
<td>225.029</td>
<td>220.319</td>
<td>215.694</td>
</tr>
<tr>
<td>$E(HC)$</td>
<td>1679.65</td>
<td>1684.44</td>
<td>1689.12</td>
<td>1693.69</td>
</tr>
<tr>
<td>$E(PC)$</td>
<td>996.671</td>
<td>1100.37</td>
<td>1204.7</td>
<td>1309.63</td>
</tr>
<tr>
<td>$E(TC)$</td>
<td>2906.151</td>
<td>3009.839</td>
<td>3114.139</td>
<td>3219.01</td>
</tr>
<tr>
<td>$E(TAC)$</td>
<td>480.068</td>
<td>494.0155</td>
<td>507.9706</td>
<td>521.933</td>
</tr>
<tr>
<td>$E(Q)$</td>
<td>1679.65</td>
<td>1684.44</td>
<td>1689.12</td>
<td>1693.69</td>
</tr>
</tbody>
</table>

Fig. 2: Effect of holding cost on optimal value $E(T)$, $E(Q)$ and $E(TAC)$

Fig. 3: Effect of production cost on $E(T)$, $E(Q)$, $E(LC)$ and $E(TAC)$
(b) As the production cost increases due to increase in lockdown time, production run time also increases to satisfy the demand which depend upon the stock and leads to decrease in expected loss sale. For the case considering long epidemic time, Table 3 shows variation of expected loss cost and expected total average cost. Figures 4&5 show the variation of E(SL) and E(TAC) with k and Variation of $T_1$ and E(T) with k respectively.

| Table 3: Variation of expected loss cost and expected total average cost |
|------------------|------------------|------------------|
| k               | 0.01             | 0.1              |
| $T_1$           | 3.30563          | 2.06518          |
| E(T)            | 11.2885          | 6.05             |
| E(SL)           | -135.92          | 229.83           |
| E(TAC)          | 491.645          | 480.0683         |

Fig. 4: Variation of E(SL) and E(TAC) with k

(c) Large value of k implies chance of epidemic increase, production run time decreases and expected lockdown also decreases or early lockdown is the best policy i.e. production run time decreases.
3. Case Study:
A case study to validate the finding is conducted. The total sales of Tata cars were observed in pre and post lockdown periods. Post lockdown the manufacturing plants of Tata motors are being operated on a maximum of 50 % capacity and the manufacturing is done based on demand. The following table shows the growth in sales of the car in post lockdown period in comparison with the same month of previous year i.e. 2019.

<table>
<thead>
<tr>
<th>Month</th>
<th>Total Units Sold in 2020</th>
<th>Total Unit Sold in 2019</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>May</td>
<td>3153</td>
<td>10126</td>
<td>-74.65%</td>
</tr>
<tr>
<td>June</td>
<td>11419</td>
<td>13351</td>
<td>-14%</td>
</tr>
<tr>
<td>July</td>
<td>15001</td>
<td>10485</td>
<td>+43%</td>
</tr>
<tr>
<td>August</td>
<td>18583</td>
<td>7316</td>
<td>+154%</td>
</tr>
<tr>
<td>September</td>
<td>21199</td>
<td>8097</td>
<td>+162%</td>
</tr>
<tr>
<td>October</td>
<td>23600</td>
<td>13169</td>
<td>+79%</td>
</tr>
</tbody>
</table>

Source: [https://gaadiwaadi.com](https://gaadiwaadi.com)

It can be observed that the sale of Tata cars has increased significantly post lockdown even when the company is not offering any reduction in price or any discounts. The customer is purchasing as per his demand rather than discounts and other offerings.

4. Conclusion

The proposed model considers the effect of epidemic on classical EPQ model for a production unit exposed to stochastic lockdown time. Expected production time is evaluated utilizing continuous probability density function. The investigation is done for optimal inventory management of production system to limits the expected total cost per unit time exposed to uncertain conditions. Machine breakdown affects the manufacture but disaster like epidemic affects the manufacturer as well as the demand. During the production uptime, demand depends upon stock and decline in selling price, but in case of disaster (epidemic) selling price has no consideration and demand depends only on stock.

During epidemic, if situation is under control, i.e., when epidemic cases are not increasing, production must go on for quite a while to increase the inventory, even if it leads to increase in holding cost or production cost. As holding cost and production cost increase with increase in lockdown time, production run time slightly increases so that demand can be satisfied. It increases the inventory level. Since the demand is based on inventory level, it leads to decrease in expected loss. However, in the event of increase in epidemic cases, production run time decreases and expected lockdown also decreases or early lockdown would be the best policy i.e., production run time should decrease to prevent the loss.

References

An EPQ Model of Stock Dependent Demand Subject to Epidemic with Stochastic Lockdown Time