Investigation of Middle School Students' Solution Strategies in Solving Proportional and Non-proportional Problems *

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Abstract: The purpose of this study was to investigate middle school students' solution strategies in solving different types of proportional (i.e., missing value, numerical comparison and qualitative reasoning problems) and non-proportional problems and to compare if differences existed between sixth and eighth grades students' solution strategies. Data were collected from 101 sixth grade (n=44) and eighth grade (n=57) students from three different public middle schools. The students were asked to solve ten open-ended items that included seven proportional problems and three non-proportional problems. Descriptive data analysis methods were used to analyze data. The results revealed that the students' solution strategies differed based on problem type and grade level. The eighth grade students used cross-multiplication as a leading strategy whereas the sixth grade students used factor of change strategy. Moreover, the results showed that students commonly used incorrect proportional strategies to solve non-proportional problems.

Keywords: Middle school students, solution strategies, proportional problems, ratio and proportion, proportional reasoning

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1. Introduction

Ratio and proportion are two concepts that fall under the general umbrella of proportional reasoning. Ratio is a comparison of quantities, and proportion is the equality between ratios that convey the same relationship (Lamon, 1995). The relationships between quantities in ratios or proportions are multiplicative in nature (Cramer, Post & Currier, 1993). In order to understand this relationship proportional reasoning is needed. Thus, a conceptual understanding in the ratio and proportion concepts requires being a proportional reasoner (Ben-Chaim, Fey, Fitzgerald, Benedetto & Miller, 1998; Lo & Watanabe, 1997). Proportional reasoning comprises a network of understandings and relationships, and it plays an important role in solving ratio and proportion problems. According to Lamon (2007) proportional reasoning is defined as "detecting, expressing, analyzing, explaining and providing evidence in support of claims about proportional relationships" (p. 647). Similarly, Lesh, Post and Behr (1988) defines proportional reasoning as "a form of mathematical reasoning that involves a sense of co-variation and of multiple comparisons, and the ability to mentally store and process several pieces of information." (p. 93). Moreover, it has been referred to as the capstone of the elementary curriculum and the cornerstone of algebra and beyond (Lesh et al., 1988). Proportional reasoning is a measure of the understanding of mathematical ideas in the middle school mathematics curriculum. It also provides a mathematical basis for more complex concepts in high school (Lamon, 2012). However, research showed that students' proportional reasoning abilities were generally problematic (Ayan & Işıksal-Bostan, 2018, 2019; Atabaş & Öner, 2016; Ben-Chaim, Keret, & Ilany, 2012; Behr, Lesh, Post, & Silver, 1983; Cramer & Post, 1993; Cramer, Post, & Behr, 1989; Hart, 1988; Lamon, 2007; Özgün-Koca & Kayhan-Altay, 2009; Toluk-Uçar & Bozkuş, 2018). First of all, students frequently used limited number of strategies and mostly formal strategies, which do not highlight multiplicative relationships, to set up and solve proportional problems (Ayan & Işıksal-Bostan, 2019; Ben-Chaim et al., 2012; Cramer & Post, 1993; Özgün-Koca & Kayhan-Altay, 2009; Toluk-Uçar & Bozkuş, 2018). Moreover, students generally had difficulties in distinguishing proportional from non-proportional situations (Ayan & Işıksal-Bostan, 2018, 2019; Atabaş & Öner, 2016; Toluk-Uçar & Bozkuş, 2018). Proportional reasoning is the mathematical basis of a wide range of topics in the middle school mathematics curriculum (Lesh et al., 1988). Therefore, middle school students (grades 5-8 in Turkey), are critical stakeholders whose conceptions of ratio and proportion need to be studied.

1.1. Literature Review

In the literature, three different proportional problem types have been defined to evaluate the ability of proportional reasoning. (Cramer et al., 1993; Heller, Post, Behr, & Lesh, 1990; Post, Behr, & Lesh, 1988). These types of problems are missing value problems, numerical comparison problems, and qualitative reasoning problems. A missing value problem can be defined as a problem that "provides three of the four values in the proportion $a/b = c/d$ and the goal is to find the missing value" (Lamon, 2007, p. 637). Additionally, in a

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numerical comparison problem, all of the four values that form two ratios (a, b, c, and d) are provided and the aim is "to determine the order relation between the ratios $a/b = c/d$" (Lamon, 2007, p. 637). In other words, the numerical comparison problems require the comparison of two ratios in order to determine whether the two ratios are equal or which ratio is greater or smaller than the other one (Ben-Chaim et al., 2012). Moreover, qualitative reasoning problems do not include numerical values; however, they "require the counterbalancing of variables in measure spaces" (Cramer et al., 1993, p. 166). There are two different types of qualitative reasoning problems; these are qualitative comparison problems and qualitative prediction problems. To illustrate, the problem, "Mary ran more laps than Greg. Mary ran for less time than Greg. Who was the faster runner?" (Cramer et al., 1993, p. 166) is a typical qualitative comparison problem. Furthermore, the problem, "If Devan ran fewer laps in more time than she did yesterday, would her running speed be (a) faster, (b) slower, (c) exactly the same, (d) not enough information to tell." (Cramer et al., 1993, p. 166) is an example of qualitative prediction problem. The both of the problem types involve qualitative comparisons that do not depend on numerical values (Ben-Chaim et al., 2012).

Problems, which have non-proportional relationships between variables but give the impression of requiring proportional reasoning, are called as non-proportional problems (Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005). In order to solve the non-proportional problems, a proportional strategy is definitely incorrect but another strategy could be used to solve the problem. Van Dooren et al. (2005) categorized non-proportional problems as additive problems, constant problems and linear problems. Additive problems have a constant difference between the two variables, so a correct approach is to add this difference to a third value. An example of additive problems is "Sue and Julie were running equally fast around a track. Sue started first. When she had run 9 laps, Julie had run 3 laps. When Julie completed 15 laps, how many laps had Sue run?" (Cramer et al., 1993). There is a constant difference between number of laps Sue ran and number of laps Julie ran; therefore, the relationship between quantities in the situation could be represented as $Sue's\ laps = Julie's\ laps\ + 6\ laps$. In the constant problems, there is no relationship between the two variables. The value of the second variable does not change, so the correct answer is mentioned in the word problem. To illustrate, the problem, "Mama put 3 towels on the clothesline. After 12 hours they were dry. Grandma put 6 towels on the clothesline. How long did it take them to get dry?" is a constant problem and the correct answer is 12 hours because the time required for the towels to try does not change (Van Dooren et al., 2005). In linear problems, the linear function underlying the problem situation is of the form $f(x) = ax + b\ with\ b \neq 0$. For instance, in the problem situation "A taxicab charges 10 TRY plus 2 TRY per kilometer. The cost for one kilometer is 12 TRY; the cost for 2 kilometers is 14 TRY" the mathematical relationship between quantities is linear but not proportional because the relationship is expressed algebraically as $cost = 2 \times kilometers\ + 10$; in other words, it is defined by both multiplication and addition (Cramer et al., 1993). In the problem situation, the relationship between quantities can be represented as a function in the form $f(x) = ax + b$, (with $b \neq 0$), which implicates a linear, but non-proportional relationship, rather than a function in the form $f(x) = cx$, which implicates a linear and proportional relationship.

A proportional reasoner can determine whether the quantities in a problem state are additional, multiplicative, or otherwise related (Lamon, 2007). However, research studies argued that students had difficulty in recognizing the mathematical relationships embedded in the two relationships and distinguishing one relationship from the other. Therefore, they tended to apply proportional strategies in non-proportional situations (Atabaş, 2014; Atabaş & Öner, 2016; Cramer et al., 1993; Van Dooren et al., 2005; Van Dooren, De Bock, Depaepe, Janssens, & Verschaffel, 2003). In fact, a proportional reasoner can explain the difference between functions of the form $y = mx$ and functions of the form $y = mx\ + b$; undoubtedly, in the latter function, y is not proportional to x (Lamon, 2007). The ability to give correct answers to proportion problems does not make sure that proportional reasoning is taking place because proportion problems might be solved by using algebraic algorithmic procedures (e.g., cross-multiplication), or by using mechanical knowledge about equivalent fractions or about numerical relationships without understanding of proportional relationships in the problem situation (Lamon, 2007). Lamon (2012) claims that a proportional reasoner exhibits greater efficiency in problem solving and uses a range of strategies, sometimes unique strategies, for dealing with problems. However, studies revealed that students and even teachers frequently use formal strategies, which are algebraic strategies in which rules and properties of algebra are used, to set up and solve proportion problems (Ben-Chaim et al., 2012; Cramer & Post, 1993; Pişkin-Tunç, 2016).

Studies in the literature indicate that there is a wide range of strategies to solve proportion problems (Baroody & Coslick, 1998; Ben-Chaim et al., 2012; Cramer & Post, 1993; Cramer et al., 1993; Kaput & West, 1994; Lamon, 2007, 2012). The most commonly used strategies are cross-multiplication, factor of change, building-up, and unit rate strategies. The strategies could be identified as formal (cross-multiplication) and informal (building-up, unit rate and factor of change) strategies. A formal strategy is an equation-based approach that involves "the syntactic manipulation of formal algebraic equations (e.g., cross-multiplication or formal division to help isolate a variable)" (Kaput & West, 1994, p. 244). Cross-multiplication is a formal strategy in which rules and properties of algebra are used. The strategy is a standard algorithm that involves setting up an equation of two ratios, one of which has an unknown quantity; cross-multiplying, and solving the equation for the unknown quantity (Van de Walle, Karp, & Bay-Williams, 2010). In other words, it is the combination of several quantitative operations (Kaput & West, 1994). Although it is an efficient strategy, it can cause confusion and errors (Cramer et al., 1993). Moreover, the ability to apply the cross-multiplication procedure does not include and guarantee proportional reasoning because accurate answers may be obtained without recognizing the structural similarities on both side of the proportion (Lamon, 2012). Furthermore, the cross-multiplication procedure has no physical referent, and therefore, it has less meaning (Cramer & Post, 1993). One of the most commonly used informal strategies is building-up strategy. In this strategy, one establishes a ratio and extends it to another ratio by using addition (Lamon, 2007, 2012). The main aim of the building-up strategy is to reason up to some desired quantity by using additive patterns (Lamon, 2007). Although it is a useful and an intuitive strategy that children use spontaneously in solving many proportion problems, it cannot be regarded as a process in which proportional reasoning takes place without additional information. The reason is that one who uses the strategy does not consider the multiplicative relationships between quantities (Lamon, 2007, 2012). Unit rate strategy is another commonly used informal strategy that "tells how many units of one type of quantity correspond to one unit of another type of quantity (Lamon, 2012, p.52). In other words, the strategy asks the question, "How many for one?" (Cramer & Post, 1993). The actual aim of the strategy is finding the multiplicative relationship between measure spaces through division (Cramer et al., 1993). Another most commonly used informal strategy is factor of change strategy that asks the question "How many times greater?" In this strategy, the aim is to find the multiplicative relationship between variables in a proportional situation (Cramer et al., 1993). Some strategies might yield an incorrect answer to a proportion problem. The most common used incorrect strategy is additive strategy (Ben-Chaim et al., 2012; Karplus, Pulos, & Stage, 1983). The strategy defined as "calculating the difference or sum of two parts of the ratio, and an attempt to divide the whole by the difference or the sum" (Ben-Chaim et al., 2012, p. 53). It is important to note that students often use incorrect additive strategies when the proportion problem has noninteger ratios (Karplus et al., 1983, Cramer et al., 1993; Post, Cramer, Behr, Lesh & Harel, 1993; Singh, 2000; Sowder, Philipp, Armstrong, & Schappelle, 1998).

Research studies revealed that students frequently rely on rote procedures such as the cross-multiplication strategy in order to solve missing value problems (Ayan & Işıksal-Bostan, 2019; Avcu & Avcu, 2010; Cramer & Post, 1993; Duatepe, Akkuş-Çıkla & Kayhan, 2005; Kayhan; 2005; Lamon, 2012; Pişkin-Tunç, 2016; Toluk-Uçar, & Bozkuş, 2018). Although the cross-multiplication strategy is an effective strategy, it can cause confusion and errors because it does not highlight the multiplicative relationships between variables (Cramer & Post, 1993; Cramer et al., 1993). Additionally, those who blindly apply an algorithm might have difficulties in determining whether relationships in a problem are proportional or not (Lamon, 2012). In a similar way, research studies indicate that there is a strong tendency to over generalize proportionality; in other words, many students incorrectly apply proportional reasoning in problems that have non-proportional relationships (Ayan & Işıksal-Bostan, 2018, 2019; Atabaş, 2014; Atabaş & Öner, 2016; Cramer et al., 1993; Van Dooren et al., 2003; Van Dooren et al., 2005; Toluk-Uçar & Bozkuş, 2018). Although the transition from additive to multiplicative thinking is an essential aspect of proportional reasoning, it has traditionally received little importance in the preparation of middle school mathematics teachers (Sowder et al., 1998). However, inaccurate additive strategies do not seem to disappear with maturation (Hart, 1988).

A study by Avcu and Avcu (2010) was conducted to investigate sixth grade students' solution strategies in ratio and proportion problems. Students in the study utilized six different strategies which were cross-product algorithm, equivalent fractions, factor of change, equivalent class, unit rate, and building-up strategies. Nevertheless, the results showed that the most commonly used strategy was cross-product algorithm. In a similar way, Duatepe et al. (2005) investigated elementary school students' solution strategies in solving different types of problems that had proportional and non-proportional relationships. The findings of the study revealed that the most frequently used strategy in solving missing value problems was cross-multiplication strategy; in solving quantitative comparison problems (i.e., numerical comparison problems) was unit rate; in solving non-proportional problems was additive strategy, and in solving inverse proportion problems was inverse proportion algorithm. Additionally, it was concluded that problem types affected students' solution strategies. Similar results were reported by Pelen and Artut (2016) who investigated seventh grade students' problem-solving performances on some proportional and non-proportional problems. They found that problem types affected students' problem-solving performances. In the study by Kayhan (2005), which investigated solution strategies of sixth and seventh grade students on items required proportional reasoning skills in terms of their grade level, gender and problem types, similar results were obtained. The results demonstrated that the students used different strategies for different problem types. Moreover, the results showed that the most frequently used strategy in solving missing value problems were cross-multiplication and unit rate strategies, and in solving quantitative comparison problems were equivalence class and additive strategies.

Cramer and Post (1993) investigated students' facility with proportional reasoning. Some proportional problems were asked to solve seventh and eighth grade students. The problems were missing value, numerical comparison, qualitative prediction, and qualitative comparison problems in different context such as speed, scaling, mixture, and density. The results revealed that success rates of missing value and numerical comparison problems were low. When students correctly solved these problem types, they used four distinct solution strategies that were unit rate, factor of change, equivalent fraction, and cross-multiplication. The researchers found that while the seventh graders mostly used the unit rate strategy, the eighth graders mostly used the cross-product algorithm. One of the interesting results of the study was that in a non-proportional problem, the seventh-grade students, who had not been taught the cross-product algorithm, were more successful than the eighth graders, who had been taught the algorithm. This means that over-reliance on the cross-multiplication strategy can cause confusion. While the eighth graders incorrectly applied the cross-product algorithm to solve the problem, seventh graders solved it by using other problem-solving strategies. Studies revealed that the context of the problem influenced difficulty of the problem (Cramer & Post, 1993; Heller, Ahlegren, Post, Behr & Lesh, 1989). Heller et al. (1989) investigated the impact of problem context on the performance of seventh-grade students in solving proportional reasoning problems. They concluded that when students encountered with a less familiar problem context, the difficulty of the problem increased.

Understanding multiplicative relationships and distinguishing them from additive relationships is an important part of proportional reasoning (Sowder et al., 1998). Lamon (2007) suggests that a proportional reasoner can discriminate situations in which proportionality is an appropriate mathematical model from situations in which proportionality is not appropriate. However, research studies show that there is a strong tendency to over generalize proportionality; that is, many students incorrectly apply proportional reasoning in situations that have non-proportional relationships (Atabaş, 2014; Cramer et al., 1993; Toluk-Uçar & Bozkuş, 2016; Van Dooren et al., 2005; Van Dooren et al., 2003). For instance, a study, which was conducted with 4^{th}, 5^{th}, 6^{th} and 7^{th} grade students, by Toluk-Uçar and Bozkuş (2016) found that students had some difficulties in distinguishing problems with multiplicative relationships (proportional) from problems with additive relationships (non-proportional). Additionally, results showed that students tended to use multiplicative (proportional) strategies in additive (non-proportional) situations and vice versa. Lamon (2012) argues that proportional reasoners can determine whether a situation is proportional or not, and so, they will not blindly apply an algorithm if the quantities in the situation are not proportional. The rule based algorithm applied in proportional problems is often cross-multiplication strategy. Although it is an efficient strategy, it can cause confusion and errors (Cramer et al., 1993). In addition, the ability to apply the cross-multiplication procedure does not include and guarantee proportional reasoning due to the fact that it does not highlight multiplicative relationships between variables (Cramer & Post, 1993; Cramer et al., 1993; Lamon, 2012). Furthermore, those who blindly apply an algorithm might have difficulties in determining whether a situation is proportional or not (Lamon, 2012). Research studies conducting with students supported the claim (Cramer & Post, 1993; Toluk-Uçar, & Bozkuş, 2016; Toluk-Uçar, & Bozkuş, 2018). According to the mathematics curriculum in Turkey, in the sixth grade level only ratio concept is taught and rule based procedures such as cross-multiplication or inverse proportion algorithm are started to teach in the seventh grade level (Ministry of National Education [MNE], 2018). Therefore, it is important to investigate the differences existed between sixth and eighth grades students' solution strategies. In this sense, the purpose of this study was to investigate and compare middle school students' solution strategies in solving different types of proportional (i.e., missing value, numerical comparison and qualitative reasoning problems) and non-proportional problems. Thus, research questions that were addressed in the present study are as follows:

a. What types of solution strategies do sixth and eighth grade students use when solving proportional and non-proportional problems?
b. What are the differences existed between sixth and eighth grades students' solution strategies in solving proportional and non-proportional problems?

2. Method

The current study was conducted with 101 (52 boys, 49 girls) sixth and eighth grade students from three different public middle schools. In order to investigate solution strategies of middle school students in solving different types of proportional and non-proportional problems a descriptive research design was used. Descriptive research design aims to describe a phenomenon and its characteristics in its naturally occurring setting without any intervention or manipulation of variables. This research design is more concerned with "what" rather than "how" or "why" something has happened, so observation and survey tools are often used to collect descriptive data (Borg & Gall, 1989; Gall, Gall, & Borg, 2007). In this research type, the data may be collected qualitatively, but it is frequently analyzed quantitatively, using frequencies, percentages, or averages (Nassaji, 2015). In a similar way, frequencies and percents of strategies used by students for each problem were used to analyze data.

2.1. Participants

Participants of the study were selected by using convenience sampling. In the convenience sampling method, the researcher selects participants who are close and easy to access. This sampling method gives speed and practicality to research (Yıldırım & Şimşek, 2013). For this reason, three different public middle schools in Western Black Sea Region, Turkey, at which the researcher was a responsible instructor of teaching practice course were selected. As mentioned earlier, since in the sixth grade level only ratio concept is taught and rule based procedures are started to teach in the seventh grade level, the seventh graders did not participate in this study. In the school A and B, there were three classes in the sixth grade and the eighth grade levels, and in the school C, there were one class in the sixth grade and eighth grade levels. Only one class was selected from each middle schools' grade levels. There were 19 sixth graders and 20 eighth graders from school A, 17 sixth graders and 16 eighth graders from school B, and 18 sixth graders and 21 eighth graders from school C. A total of 101 middle school students, 52 boys and 49 girls, and 44 sixth graders and 57 eighth graders participated in this study from three different middle schools in the spring semester.

2.2. The instrument

The middle school students were wanted to solve ten open-ended items which included seven proportional problems and three non-proportional problems. The items in the instrument were selected and/or adapted from literature (Cramer et al., 1993; Hillen, 2005; Karplus et al., 1983) and written by the researcher. The aim of the instrument was to investigate middle school students' solution strategies to different problem types and ability to distinguish proportional from non-proportional problem statements. To ensure the validity of the instrument, a mathematics teacher and an instructor in mathematics education were asked to judge whether the items of the instrument were matched with the research questions and aim of the study. Then, the items were revised until there was an agreement. Next, the instrument was pilot tested with one sixth grade class and one eighth grade class to ensure that the problem statements were easy to understand, that the problems were relevant and to measure the time needed to complete the instrument. Afterwards, the items were revised and the problem 7 was changed with another non-proportional problem. Two problems were omitted and a problem was added. Table 1 shows the final version of the instrument.

Table 1. The match between the problems in the instrument and problem types

Problem	Problem Type
Problem 1: Car A is driven 180 km in 3 hours. Car B is driven 400 km in 7 hours. Which car was driven faster? Why? (Karplus et al., 1983, p. 220).	Numerical comparison problem
Problem 2: Find the kilometers of a car driven in 12 hours if the car drives 175 kilometers in 3 hours. Explain your answer.	Missing value problem
Problem 3: Merve and Doruk want to paint their rooms with the same color. Merve mixes 3 boxes of yellow paint with 6 boxes of white paint. How many boxes of white paint should Doruk mixed with 7 boxes of yellow paint to get the same color? Explain your answer.	Missing value problem
Problem 4: Esra ran more laps than Gonca. Esra ran for less time than Gonca. Who was the faster runner? Why? (a) Esra, (b) Gonca, (c) Same, (d) Not enough information to tell (Adapted from Cramer et al., 1993).	Qualitative reasoning problem
Problem 5: A mother makes fruit juice by mixing apple and orange every day for her daughter. If the mother used less orange and less apple than she did yesterday, would her fruit juice drink taste: (a) More orange than yesterday's, (b) More apple than yesterday's, (c) The same as yesterday's, (d) Not enough information to tell. Explain your answer. (Adapted from Hillen, 2005).	Qualitative reasoning problem
Problem 6: Nil and Mina ran at the same speed in a marathon. Nil started running before Mina. If Nil completed 9 laps, Mina completed 3 laps; how many laps did Nil run when Mina completed 15 laps? Explain your answer (Adapted from Hillen, 2005).	Non-proportional problem
Problem 7: Cem is 12 years old and his brother is 15 years old. How old is his brother when he is twice his age? Explain your answer.	Non-proportional problem
Problem 8: If a satin skirt dries in 100 minutes, how long does it take to dry 5 satin skirts at the same time? Explain your answer.	Non-proportional problem
Problem 9: A photographer wants to enlarge a photograph with 2 cm width and 3 cm length. Find the width of the enlargement photograph if the length was 9 cm. Explain your answer (Adapted from Hillen, 2005).	Missing value problem
Problem 10: When two friends went to the market; they saw that 2 liters of orange soda was 6 TRY and 6 liters of lemonade was 15 TRY and they decided to buy orange soda. Did they make the most economical choice? Why?	Numerical comparison problem

In the instrument, there were three missing value problems, two numerical comparison problems, two qualitative reasoning problems and three non-proportional problems. Problems were presented to students in different real-world contexts: speed, mixture, age, time, scaling and shopping. The instrument was administered to the participants in their math class session that lasted about 40 minutes.

2.3. Data Analysis

Descriptive data analysis methods were utilized to analyze data gathered from the instrument. In the descriptive data analysis, data can be arranged according to predetermined themes, or can be presented by taking into account the questions or dimensions used in interview or observation process (Yıldırım & Şimşek, 2013). Additionally, Nassaji (2015) asserted that in the descriptive research, the data may be collected qualitatively, but it is frequently analyzed quantitatively, using frequencies, percentages, or averages to determine relationships. Similarly, in order to analyze students' solutions to missing value (problem 2, 3 and 9) and numerical comparison problems (problem 1 and 10), frequencies and percentages of strategies used for each problem were identified. Data were arranged according to some predetermined codes based on literature. These codes were proportional solution strategies that were "cross-multiplication", "factor of change", "unit rate" and "building-up", and "inaccurate additive strategy". Moreover, analysis of solutions showed that there were some answers with unclear solutions that were not classified under a predetermined strategy in the coding list. Additionally, there were some unanswered problems. Therefore, frequencies and percents of the unclear solutions and unanswered problems were also presented. In addition, students' solutions to qualitative problems (problem 4 and 5) were classified as "including multiplicative comparison" and "not including multiplicative comparison". In these problems, qualitative comparisons that did not depend on numerical values were classified under "including multiplicative comparison" code as "qualitative multiplicative comparison" and quantitiative comparisons by giving numerical examples were classified under the same code as "quantitative multiplicative comparison". Furthermore, students' solutions to non-proportional problems (problem 6, 7 and 8) were identified as "non-proportional strategy" and "inaccurate proportional strategy". For each problem, frequencies and percentages were used to analyze students' solutions. Moreover, in the tables, frequency and percentages of sixth and eighth grade students' solutions were presented separately in order to examine the differences existed between sixth and eighth grades students' solution strategies. Coding of the data was independently conducted by the researcher and a second coder who was a fourth grade preservice mathematics teacher. The second coder was informed about the proportional reasoning literature and coding list of the study. The researcher and the second coder compared their results for consistency. Once coding was completed individually, ninety percent agreement on the codes was reached. Then, they discussed the discrepancies until a consensus was reached.

3. Findings

In this section, findings are presented with sample solutions of students.

3.1. Solution strategies of missing value problems

The instrument had three missing value problems, which were problem 2, problem 3 and problem 9 as seen in the Table 1. Table 2 shows frequencies and percentages of strategies used by students to solve problem 2, problem 3 and problem 9 with regard to grade level. Data analysis of missing value problems revealed that the students used four distinct proportional solution strategies and an inaccurate additive strategy to solve the problems. In addition, there were some inaccurate answers with unclear solutions, which were classified as "not clear" in the Table 2. Moreover, some students did not answer the problems, in other words; they left the problems blank. These answers classified as "no solution" in the Table 2.

Table 2. Distribution of students' strategies for solving missing value problems

Strategy	**Grade Level**	**Problem 2**		**Problem 3**		**Problem 9**	
		f	**%**	**f**	**%**	**f**	**%**
Cross-multiplication	6th grade	0	0	1	2	0	0
	8th grade	29	51	24	42	26	46
Factor of change	6th grade	13	30	12	27	19	43
	8th grade	11	19	5	9	13	23
Unit Rate	6th grade	17	39	0	0	1	2
	8th grade	4	7	6	11	3	5
Building-up	6th grade	4	9	1	2	0	0
	8th grade	1	2	1	2	0	0
Inaccurate Additive Strategy	6th grade	4	9	21	48	13	30
	8th grade	3	5	4	7	2	4
Not Clear	6th grade	2	5	1	2	2	5
	8th grade	0	0	2	4	1	2
No solution	6th grade	4	9	8	21	9	20
	8th grade	9	16	15	26	12	21

The problem 2 is a typical missing value problem that students generally encounter in math lessons. The most frequently used strategy was "unit rate strategy" for 6th graders (39%) and "cross-multiplication strategy" for 8th graders (51%). As expected, none of the 6th graders used "cross-multiplication" in order to solve the missing value problem whereas the most of the 8th graders used it to solve the same problem. The reason was that the 6th graders had not learned ratio and proportion issue, in which formal strategies such as cross-multiplication were taught. Therefore, they were used to solving missing value problems by using informal strategies such as unit rate, factor of change. Moreover, the 8th graders, who had been taught the cross-multiplication strategy, generally have concerns about solving problems quickly because of time limitations in the statewide exams. Thus, they probably preferred to solve proportion problems by using cross-product algorithm. The second commonly used strategy was "factor of change strategy" for both 6th (30%) and 8th (19%) graders. Although "building-up strategy" was the least used proportional strategy for both grade levels, the 6th graders (9%) used the strategy more than the 8th graders (2%). For instance, the Figure 1 demonstrates a sixth grade student's solution of the problem 2 in which she used building-up strategy.

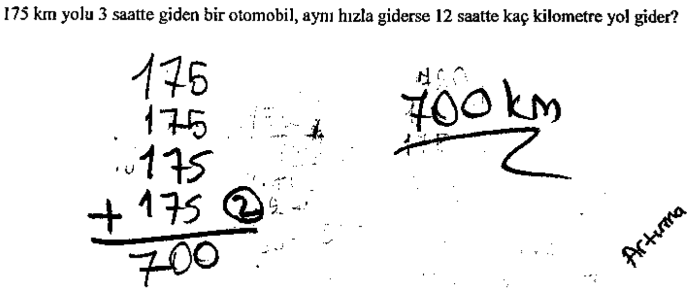

Figure 1. A sixth grade student's solution to Problem 2 by using building-up strategy

As seen in the Figure 1, the student used building-up strategy in such a way that she established a ratio (3 hours for 175 kilometers) and extends it to another ratio by using addition (adding up 175 kilometers for each 3 hours). Although it is a useful and an intuitive strategy, it cannot be regarded as a process in which proportional reasoning takes place without additional information because one who uses the strategy does not consider the multiplicative relationships between quantities (Lamon, 2007, 2012).

Problem 3 (see Table 1) is a missing value problem that gives the number of boxes in the first mixture and the number of yellow boxes in the second mixture, and asks to find the number of white boxes in the second mixture needed to get the same color. The most commonly used strategies were "inaccurate additive strategy" for 6th graders (48%) and "cross-multiplication strategy" for 8th graders (42%). The additive strategy, which was a non-proportional strategy, yielded an incorrect answer for the missing value problem. For instance, the Figure 2 shows a sixth grade student's solution of the problem 3 in which the student used inaccurate additive strategy.

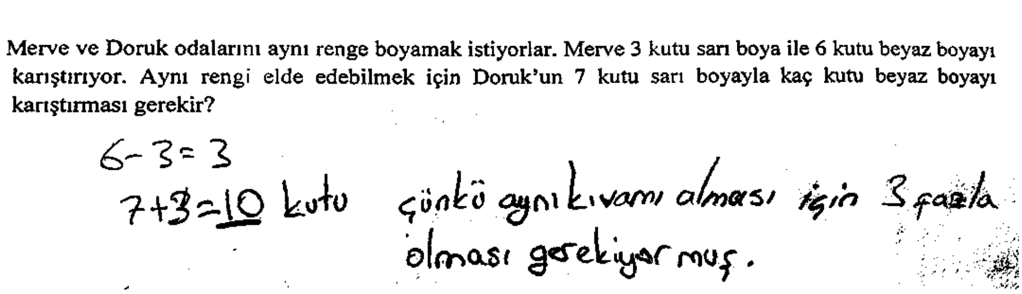

Figure 2. A sixth grade student's solution to Problem 3 by using inaccurate additive strategy (English translation: 10 boxes because it takes three more to get the same consistency.)

In the Figure 2, the student calculated the difference between Merve's yellow paints (3 boxes) and white paints (6 boxes). Then, he added the difference (3) to Doruk's yellow paints (7 boxes) and finally found the number of white paints of Doruk (10 boxes). The student could not realize the multiplicative relationship between the variables. On the contrary, he supposed that the relationship between the numbers of paints in each situation was additive. According to the literature, students often use incorrect additive strategies when the proportion problem has noninteger ratios. Similarly, in the problem the ratio between yellow paints (3:7) and white paints (6:14) was not integer though the ratio between yellow and white paints in the first mixture (3:6) and in the second mixture (7:14) was integer. The second frequently used strategies of problem 3 were "factor of change strategy" for 6th graders (27%) and "unit rate strategy" for 8th graders (11%). Both of the strategies were informal strategies that highlighted the multiplicative relationships between the variables. However, some students from both grade levels (21% of 6th graders and 26% of 8th graders) could not answer the problem, in other words; they left the problem blank.

The problem 9 (see Table 1) was a missing value problem which had a context involving similar figures. The most frequently used strategies were “factor of change strategy” for 6th graders (43%) and “cross-multiplication strategy” for 8th graders (46%). Some students’ solutions to the problem are presented in the following figures.

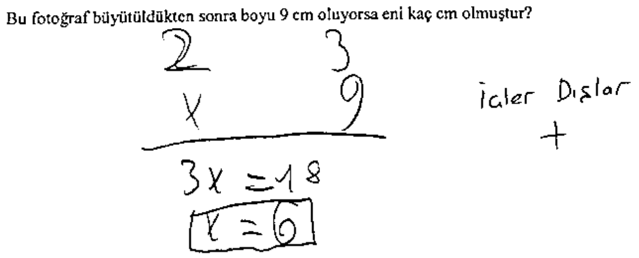

Figure 3. An eighth grade student’s solution to Problem 9 by using cross-multiplication strategy

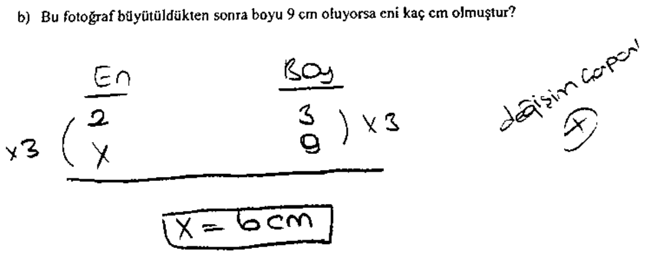

Figure 4. A sixth grade student’s solution to Problem 9 by using factor of change strategy

In the Figure 3, an eighth grade student solved the problem by using cross-multiplication although there was an integer scale factor. However, in the Figure 4, a sixth grade student recognized the integer scale factor (3) and solved the problem by using factor of change strategy, which was an efficient strategy for the problem. Similarly, some of the 8th graders (23%) recognized the integer scale factor and utilized factor of change in order to solve the problem. On the other hand, the second frequently used strategy was “inaccurate additive strategy” by 6th graders (30%). Some 6th graders could not distinguish proportional from non-proportional relationships and they supposed that the relationships between the lengths and widths of similar figures were additive.

3.2. Solution strategies of numerical comparison problems

The instrument had two numerical comparison problems, which were problem 1 and problem 10 as seen in the Table 1. Table 3 shows frequencies and percentages of strategies used by students to solve problem 1 and problem 10 with regard to grade level. Analysis of solutions revealed that the students used four distinct proportional solution strategies and an inaccurate additive strategy to solve the numerical comparison problems. Moreover, there were some wrong answers with unclear solutions, which were classified as “not clear” in the Table 3. Furthermore, some students did not write a solution or an answer for the problems and some of them could write the correct answer but not write a solution. These answers were classified as “no solution” in the Table 3.

Table 3. Distribution of students’ strategies for solving numerical comparison problems

Strategy	Grade Level	Problem 1		Problem 10	
		f	%	*f*	%
Cross-multiplication	6th grade	0	0	0	0
	8th grade	4	7	2	4
Factor of change	6th grade	0	0	5	11
	8th grade	0	0	9	16
Unit Rate	6th grade	25	57	27	61
	8th grade	33	58	26	46
Building-up	6th grade	1	2	1	2
	8th grade	0	0	0	0
Inaccurate Additive Strategy	6th grade	3	7	4	9
	8th grade	2	4	1	2
Not Clear	6th grade	6	14	0	0
	8th grade	1	2	1	2
No solution	6th grade	9	20	7	16
	8th grade	17	30	18	32

In the problem 1 (see Table 1), the work was to compare the ratios of kilometers to hours of each car to find out which car was faster. As seen in the Table 3, some students from both grade levels did not write a solution or an answer for the problem. However, some of these students (14% of 6th graders and 16% of 8th graders) could write the correct answer of the problem although they did not write a solution. But, it is difficult to say that the students determined the order relation between ratios since they did not explain their answers. The most frequently used strategy was "unit rate strategy" for both grade levels (57% of 6th graders and 58% of 8th graders). For example, the following figure demonstrates an eighth grade student's solution of the problem 1 in which the student used unit rate strategy.

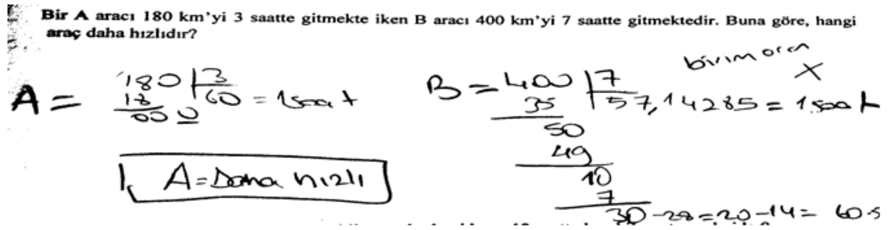

Figure 5. An eighth grade student's solution to Problem 1 by using unit rate strategy (English translation: A is faster.)

In the above solution, first of all, the student calculated the unit rate for the first situation as 60 kilometers per 1 hour (the result of the division of 180 kilometers by 3) and for the second situation as 57,14 kilometers per 1 hour (the result of the division of 400 kilometers by 7). Then, she compared the unit rates and correctly concluded that Car A was driven faster. While none of the sixth graders used "cross multiplication strategy", some of the eighth graders (7%) used the strategy. Some of these students (5%) could find the correct answer, but a student (2%) could not correctly solve the problem by using cross-multiplication. Some students (7% of 6th graders and 4% of 8th graders) solved the problem by using inaccurate additive strategy, although they were not as many as in the case of missing value problems.

The problem 10 (see Table 1) was a numerial comparison problem which wanted to compare two ratios in order to determine the most economical choice. As seen in the Table 3, some students from both grade levels did not write a solution or an answer for the problem. However, some of these students (5% of 6th graders and 4% of 8th graders) could write the correct answer of the problem although they did not write a solution. Yet, it is difficult to say that the students determined the order relation between ratios since they did not explain their answers. The most frequently used strategy was "unit rate strategy" for both grade levels (61% of 6th graders and 46% of 8th graders). Most of the students used the strategy in order to solve the numerical comparison problem. The second frequently used strategy of the problem was "factor of change strategy" for both grade levels (11% of 6th graders and 16% of 8th graders). While none of the sixth graders used "cross multiplication strategy", some of the 8th graders (4%) used the strategy. These students could correctly solve the problem. In addition, some students (9% of 6th graders and 2% of 8th graders) solved the problem by using inaccurate additive strategy, although they were not as many as in the case of missing value problems.

3.3. Solution strategies of qualitative reasoning problems

The instrument had two qualitative reasoning problems, which were problem 4 and problem 5 as seen in the Table 1. Table 4 presents frequencies and percentages of strategies used by students to solve problem 4 with regard to grade level. The students' solutions to the qualitative problems were classified as including multiplicative comparisons and not including multiplicative comparisons as seen in the Table 4.

Table 4. Distribution of students' strategies for solving qualitative reasoning problems

	Strategy	**Grade Level**	**Problem 4**		**Problem 5**	
			f	**%**	***f***	**%**
Including multiplicative comparison	Qualitative multiplicative comparison	6th grade	32	72	1	2
		8th grade	39	68	2	4
	Quantitative multiplicative comparison	6th grade	1	2	0	0
		8th grade	3	5	0	0
Not including multiplicative comparison	Inaccurate additive comparison	6th grade	0	0	42	96
		8th grade	0	0	44	77
	Could not make any comparison	6th grade	11	25	1	2
		8th grade	15	27	11	19

The problem 4 (see Table 1) was a typical qualitative comparison problem that required making qualitative multiplicative comparisons that do not depend on numerical values. Data analysis revealed that most of the students (72% of 6th graders and 68% of 8th graders) could make qualitative comparisons that did not depend on numerical values. These students could correctly find that Esra was the faster runner. In addition, some students from both grade levels (2% of 6th graders and 5% of 8th graders) made quantitiative multiplicative comparisons and correctly solved the problem by giving numerical examples, in other words; they converted the qualitative reasoning problem to a numerical comparison problem. For instance, an eighth grade student explained her answer as follows: "Let's assume Gonca ran 4 laps in 20 minutes and Esra ran 6 laps in 10 minutes since Esra ran in a shorter amount of time she was the faster runner." Although she made a multiplicative comparison to solve the qualitative problem, she could not make qualitative comparisons that did not depend on numerical values because she quantitatively compared the number of laps. Moreover, some students from both grade levels (25% of 6th graders and 27% of 8th graders) could not make multiplicative comparisons. Some of these students (23% of 6th graders and 11% of 8th graders) said that the problem could not be solved since there was not enough information such as number of laps or running time. The other students (2% of 6th graders and 16% of 8th graders) did not write an explanation for the problem. The rest of the students (2% of 6th graders and 4% of 8th graders) stated that they did not understand the problem.

The problem 5 (see Table 1) was a qualitative prediction problem that required making qualitative multiplicative comparisons that do not depend on numerical values. Analysis of data revealed that the most of the students (96% of 6th graders and 77% of 8th graders) used inaccurate additive strategy to solve the problem. These students frequently answered as "The same as yesterday's". They did not mention the ratios of the apples and oranges in the fruit juice. Moreover, they made additive comparisons. For example a students' explanation was, "The taste of the fruit juice would be the same as yesterday's taste. Because if the amount of apple and orange used for the fruit juice decreased, only the amount of fruit juice would change." The student made an additive comparison between yesterday's fruit juice and today's fruit juice instead of a multiplicative comparison. In addition, some of the students used inaccurate additive comparisons found the correct answer as "Not enough information to tell". Yet, they took into account the reduced amount of apple and orange (additive comparison) instead of the ratios of the apples and oranges in the fruit juices (multiplicative comparison). In other words, even if we knew the reduced amount of apple and orange in today's mixture, we could not solve the problem without finding and comparing the ratios of apples and oranges in both mixtures. A few students (2% of 6th graders and 4% of 8th graders) could made qualitative multiplicative comparisons. They considered the ratios of the apples and oranges in the fruit juices. Additionally, they provided valid explanations by making multiplicative comparisons between variables without giving numerical examples. Furthermore, some students from both grade levels (2% of 6th graders and 19% of 8th graders) calssified as they could not make any comparison because they did not write any explanation for the problem. Some of these students (12% of 8th graders) found the correct answer of the problem; but, since they did not provide an explanation, it was difficult to say that they made multiplicative comparisons.

3.4. Solution strategies of non-proportional problems

The instrument had three non-proportional problems, which were problem 6, problem 7 and problem 8 as seen in the Table 1. Table 5 shows frequencies and percentages of strategies used by students to solve non-proportional problems with regard to grade level. Students' solutions to non-proportional problems were identified as non-proportional strategy and inaccurate proportional strategy.

Table 5. Distribution of students' strategies for solving non-proportional problems

Strategy	**Grade Level**	**Problem 6**		**Problem 7**		**Problem 8**	
		f	***%***	***f***	***%***	***f***	***%***
Non-proportional strategy	6th grade	21	48	18	41	24	55
	8th grade	12	21	25	44	28	49
Inaccurate proportional strategy	6th grade	14	32	22	50	18	41
	8th grade	39	68	27	47	24	42
Not Clear	6th grade	3	7	2	5	1	2
	8th grade	4	7	4	7	1	2
No solution	6th grade	6	14	2	5	1	2
	8th grade	2	4	1	2	4	7

The problem 6 (see Table 1) was a non-proportional problem since the variables in the problem were related additively. Data analysis revealed that while nearly half of the 6th graders (48%) could realize additive relationship between the variables, only 21% of the 8th graders could realize the same relationship and used a non-proportional strategy. On the other hand, 68% of the 8th graders and %32 of the 6th graders did not realize the variables in the problem were related additively instead of multiplicatively, and so, they solved the problem by using a proportional strategy that did not work. That is, they could not distinguish proportional from non-proportional situations. An eighth grade student's solution was, $9/3 = x/15; 3x = 135 \Longrightarrow x = 45$. The student blindly applied cross-product algorithm although the problem situation did not involve a proportional relationship. In addition, some students from both grade levels (14% of 6th graders and 4% of 8th graders) did not solve the problem and some solutions (7% of 6th graders and 7% of 8th graders) were not clear.

The problem 7 (see Table 1) was an age problem. In this problem, the variables were related additively, and so, it was a non-proportional problem. Analysis of data revealed that most of the students (50% of 6th graders and 47% of 8th graders) used inaccurate proportional strategy to solve the problem. They did not realize the variables in the problem were related additively instead of multiplicatively, and so, they solved the problem by using a proportional strategy that did not work. The eighth grade students frequently solved the problem by setting up and solving a proportion: $24 \cdot 15 = 12 \cdot x \Longrightarrow x = 30$. They blindly applied cross-product algorithm although the problem situation did not involve a proportional relationship. Similarly, the sixth grade students mostly used a proportional strategy, factor of change. They assumed that 2 was an integer scale factor, and so, they multiply 15 by 2. Some students from both grade levels (41% of 6th graders and 47% of 8th graders) could realize additive relationship between the variables and could use a non-proportional strategy to solve the problem. Moreover, some solutions from both grade levels (5% of 6th graders and 7% of 8th graders) were not clear and some students (5% of 6th graders and 2% of 8th graders) did not solve the problem.

Since in the problem 8 (see Table 1), there was a constant relationship between variables, it was a non-proportional problem. Data analysis revealed that most of the students (55% of 6th graders and 49% of 8th graders) could realize the constant relationship between the variables. They frequently stated that time did not changed, and so, 5 satin skirts dried in 100 minutes, too. However, some students (41% of 6th graders and 42% of 8th graders) assumed that variables in the problem were related multiplicatively, and so, they solved the problem by using a proportional strategy that did not work. Most of these eighth grade students blindly applied cross-product algorithm and found the answer as 500 minutes. In a similar way, sixth grade students, who used inaccurate proportional strategy, frequently found the answer as 500 minutes by using factor of change strategy. Furthermore, some solutions from both grade levels (2% of 6th graders and 2% of 8th graders) were not clear and some students (2% of 6th graders and 7% of 8th graders) did not solve the problem.

4. Conclusion and Discussion

The results of the study revealed that the sixth grade students mostly preferred to use strategies highlighting multiplicative relationships such as factor of change, unit rate. In fact, the sixth-grade students in this study almost never used the cross-multiplication strategy in order to solve the missing value and numerical comparison problems. The most obvious explanation for this is the fact that sixth graders in this study were not formally introduced to the ratio and proportion where they usually learn and apply strategies such as the cross-multiplication. Therefore, they were used to solving proportion problems by using informal strategies. The results of this study confirm the results from the study by Cramer and Post (1993). The researchers suggested that the seventh graders mostly used the unit rate strategy since they had not been taught the cross-product algorithm. On the other hand, the study by Avcu and Avcu (2010) found some conflicting results with the current study. The researchers argued that the sixth grade students' most frequently used strategy was "cross-multiplication" in solving ratio and proportion problems. The reason for this conflicting result might be that different mathematics curricula were adopted by the time of data collection. For instance, when Avcu and Avcu (2010) conducted their study, the sixth grade mathematics curriculum contained both ratio and proportion concepts and suggested to teach cross-multiplication algorithm in a proportion (MNE, 2009). However, when the current study was conducted, the sixth grade mathematics curriculum contained only ratio concept and did not mention cross-multiplication algorithm (MNE, 2013).

In addition, the results of the study showed that the sixth grade students made use of inaccurate additive strategies to solve some proportion problems. In other words, they could not realize the multiplicative relationship between the variables in the problems. Thus, they utilized some additive strategies that yielded incorrect answers for proportion problems. Moreover, the results revealed that the sixth graders often used incorrect additive strategies when the problem had noninteger relationships between variables. Similarly, according to the literature, students often use incorrect additive strategies when the proportional problem has noninteger ratios (Cramer et al., 1993; Karplus et al., 1983; Post et al., 1993; Singh, 2000; Sowder, Philipp et al., 1998). In addition, the context of the problem influenced the sixth graders' solution strategies. For example; they frequently used inaccurate additive strategies when they encountered missing value problems with mixture and scaling contexts although they could use proportional strategies in problems with speed context. The reason might be that speed is a prevalent context for missing value problems. According to Heller et al. (1989), when students encountered with a less familiar problem context, the difficulty of the problem increased. In a similar way, the results of studies by Heller et al. (1989) and Cramer and Post (1993) indicated that the context of the problem influenced difficulty of the problem.

The results revealed that the eighth grade students used limited number of strategies. The students mostly preferred to use cross-multiplication strategy to solve the missing value problems and unit rate strategy to solve the numerical comparison problems. They mostly used cross-product algorithm to solve both proportional and non-proportional problems. Accordingly, research studies argued that students frequently used limited number of strategies and mostly formal strategies, which do not highlight multiplicative relationships, to set up and solve proportion problems (Ayan & Işıksal-Bostan, 2019; Ben-Chaim et al., 2012; Cramer & Post, 1993; Özgün-Koca & Kayhan-Altay, 2009; Toluk-Uçar & Bozkuş, 2018). The reason for overreliance on the *cross-multiplication* might be that the eighth graders generally have concerns about solving problems quickly because of time limitations in the statewide exams. However, although cross-multiplication is an efficient strategy, it might cause confusion and lead to error (Cramer & Post, 1993; Cramer et al., 1993). In addition, proportional problems may be correctly solved by applying cross product algorithm without understanding of proportional relationships between variables in the problem situation (Lamon, 2007). To illustrate, the eighth grade students commonly used incorrect proportional strategies to solve non-proportional problems. In other words, they did not realize the relationship between variables in the problem was additive or constant instead of multiplicative, and so, they solved the problem by using a proportional strategy that did not work. That is, they blindly applied cross-product algorithm although the problem situation did not involve proportional relationships. In conclusion, they had difficulty in distinguishing proportional from non-proportional situations. In a similar way, studies indicate that there is a strong tendency to over generalize proportionality. That is to say, many students incorrectly apply proportional reasoning in situations that have non-proportional relationships (Atabaş, 2014; Cramer et al., 1993; Pişkin-Tunç, 2016; Van Dooren et al., 2005; Van Dooren et al., 2003).

The results about the differences existed between sixth and eighth grades students' solution strategies suggested that the sixth graders used building-up strategy more than the eighth graders. The reason might be that building-up strategy is an intuitive strategy that young children use spontaneously in solving many proportion problems (Lamon, 2007, 2012). Although it is a useful and an intuitive strategy, it cannot be regarded as a process in which proportional reasoning takes place without additional information. The reason is that one who uses the strategy does not consider the multiplicative relationships between quantities (Lamon, 2007, 2012). In addition, while the sixth grade students mostly preferred to use strategies highlighting multiplicative relationships such as factor of change, unit rate, the eighth grade students mostly preferred to use algebraic strategies such as cross-multiplication strategy. Moreover, the results indicated that students from both grade levels had difficulty in differentiate proportional from non-proportional situations. For instance, the sixth graders commonly made use of inaccurate additive strategies to solve proportion problems. In other words, they could not realize the multiplicative relationship between the variables in the problems. On the other hand, the eighth grade students generally used incorrect proportional strategies to solve non-proportional problems. In other words, they overgeneralized proportionality. For instance, the eighth graders had difficulty to realize the constant relationship between the variables in the problem 8 since they blindly applied cross-product algorithm. It is interesting to note that the sixth graders were more successful than the eighth graders when the relationship between the variables was constant. The reason might be that although cross-multiplication is an efficient strategy, it can cause confusion and errors (Cramer & Post, 1993; Cramer et al., 1993). Considering the findings of the study, it may be suggested that students should be encouraged to use different solution strategies highlighting multiplicative relationships to solve proportional problems. In addition, mathematics teachers should put emphasis on solving different problem types including numerical comparison and qualitative reasoning problems. Moreover, teachers should pay more attention to the transition from additive to multiplicative thinking than it has traditionally received. That is, mathematical tasks that included not only proportional situations but also non-proportional situations in which variables had additive and constant relationships should be solved in mathematics classrooms. In addition, mathematics textbooks should pay attention to compare proportional and non-proportional situations.

The results of the current study indicated that students' solution strategies differed with respect to problem type. For instance, the eighth graders mostly preferred to use cross multiplication strategy in order to solve missing value problems whereas they mostly used unit rate strategy in order to solve numerical comparison problem. In a similar way, while the sixth graders frequently used factor of change to solve missing value problems, they commonly used unit rate strategy to solve numerical comparison problem. Furthermore, students were more successful in solving missing value problems than the other types of problems. Similarly, research studies reported that problem types affected students' solution strategies and problem-solving performances (Duatepe et al., 2005; Kayhan, 2005; Özgün-Koca & Kayhan-Altay, 2009; Pelen & Artut, 2016). Furthermore, the findings of the study revealed that students' success rates were changed according to the context of the problems. To illustrate, most of the students had difficulty in solving the problems with mixture context and they frequently used inaccurate strategies to solve these problems. Accordingly, Cramer and Post (1993) argued that the context of the problem influenced difficulty of the problem. In addition, the results showed that numerical structures of the problem also had an impact on students' solution strategies. To illustrate, the students preferred to use incorrect additive strategies in order to solve a missing value problem that had noninteger relationships between variables. Simiarly, according to the literature, students often use incorrect additive strategies when the proportion problem has noninteger ratios (Karplus et al., 1983, Cramer et al., 1993; Post et al., 1993; Singh, 2000; Sowder, Philipp et al., 1998). In brief, it was concluded that problem type, context and numerical structures of the problem were effective on which solution strategy students would choose and their success rates. This study only analysed the sixth and eighth grade students' written work. The studies that will investigate proportional reasoning of middle school students with all grades (5^{th} to 8^{th}) by using different data collection methods such as interviews and observations will be beneficial for understanding the nature of students' proportional reasoning.

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