Preservice Elementary Mathematics Teachers' Geometric and Algebraic Proof Process with Dynamic Geometry Software

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Abstract

Dynamic Geometry Software (DGS) has recently been used in mathematics courses. It helps students understand the mathematical concepts and methods easily and "*provides an environment in which students can experiment freely, hence they can easily check their intuitions and conjectures in the process of looking for patterns, general properties, etc*." (Marrades & Gutierrez, 2000). Battista and Clements (1995) claimed that students should learn the proof of any theorem by using visual material. According to Jones (2005), DGS is an important tool for students and teachers to make conjectures and control them and also understand the relationship between concepts. In addition to students, teachers can use it to teach mathematical concepts. For instance, proof is a difficult issue to be explained by using paper-pencil methods. According to Nordström's (2004) research, teachers have difficulties while explaining the formal proofs in the textbooks. For this reason, mathematics teachers should know how to use DGS. Since, DGS provides visual and it helps be turned abstract mathematical concepts into concrete (Pandiscio, 2002).

Preservice elementary mathematics teachers should learn how to use DGS to improve their future students' motivation in mathematics classes. They can use these programs to take students' interest on mathematical concepts and to provide efficient learning environment.

In this study, it was aimed to determine the preservice elementary mathematics teachers' algebraic proof processes by the use of dynamic geometry software. For this purpose, a course was designed in accordance to DGS. During this course participants solved algebraic problems related to algebraic proofs by using DGS for 10 weeks. During the course, participants prepared reflection papers about their proof processes and the effects of DGS to their way of proving. Moreover, the researchers had interviews selected participants about geometric and algebraic proofs with DGS.

According to findings, it can be claimed that preservice elementary mathematics teachers found Dynamic Geometry Software beneficial for algebraic proof works. Moreover, their attitudes towards to proof can be changed positively with the effect of DGS.

Key Words: Algebraic proof, dynamic geometry software, geometric proof, preservice mathematics teacher, proof process.

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1. Introduction

Computer-based learning environment has become an important issue in mathematics education (Jonassen, Carr and Yueh, 1998; Özyıldırım, Akkuş-İspir, Güler, İpek & Aygün, 2009). The National Council of Teachers of Mathematics (NCTM) has given a great importance to technological tools in mathematics classrooms. It was stated that if these technological tools especially computers are used efficiently and properly to teach mathematical concepts, it will enable to have a rich learning environment to improve students' mathematical thinking (NCTM, 2000). Dynamic learning environment, as a computer-based learning environment, has been provided the opportunity for efficient learning environments to students (Clements, 2003; Gillis, 2005; Karadağ, 2008).

Algebra and geometry are the two main fields of mathematics curricula of schools (Atiyah, 2001). Dynamic Geometry Software (DGS) reinforces students to discover mathematical concepts by doing practice (Baki, 2000; Hohenwarter, 2004; Sträßer, 2002). Using DGS supports both learners and teachers to learn and teach essential conditions of geometric concepts (Kokol-Voljc, 2007). All levels of students were provided to gain better understanding of mathematics by the use of DGS (Hohenwarter, Preiner and Yi, 2007). Therefore, mathematics teachers should learn the use of DGS to teach mathematical concepts. Some of the mathematical concepts, which are abstract, such as geometrical proofs and algebraic proofs are difficult to be explained to students by classical paper and pencil teaching method. In addition, mathematics educators and researchers pointed out students' difficulties with mathematical proofs (Hanna, 1991; Harel & Sowder, 1998; Moore, 1994; Raman, 2003; Selden & Selden, 1995; Usiskin, 1980; Weber, 2001). In spite of these difficulties of understanding the proofs, mathematical proofs are crucial elements of geometry and algebra (Chao and Jiansheng, 2009). From this point of view it was found important to investigate preservice elementary mathematics teachers' proof processes. Hence, the aim of this study is to determine preservice elementary mathematics teachers' proof processes with regard to the fundamental geometric and algebraic theorems by using DGS.

Review of the Related Literature

Proof in mathematics education was a research concept of many studies in the world. It was seen that the proof helped students learn effectively concepts and theorems in mathematics courses. Jones (2000) claimed that proof gave a meaning to geometry and proof should be used in mathematics classrooms at every level. There are many researches about proof in mathematics education gave similar results as Hanna, DeBruyn, Sidoli and Lomas (2002) who found out that students learned the theorems about triangles easily and effectively with the help of proof.

In many studies, it was claimed that visual representations of proofs were important for elementary students' learning. For instance, Arslan (2007) found that elementary school students learned the mathematical concepts more effective with the help of visual proofs.

Küchemann and Hoyles (2004) stated in their study that students liked showing theorems by the help of proofs and so they learned the theorems clearly.

In Turkey, there are some studies about proof; however most of the students had not enough knowledge for proving as the result of education system. For instance, Özer and Arıkan (2002) found out that by using mathematical materials students could not prove the theorems and also they had insufficient knowledge about proof methods and techniques. However, Ören (2007) claimed that girls chose informal proofs and boys chose proofs with help of materials. Therefore, she stated that when teaching proofs in mathematics, it is important to consider the gender and cognitive skills.

When the literature was examined about proof and preservice mathematics teachers, different studies were found. Jones (2000) found out that preservice mathematics teachers graduated without learning how a proof was done and improved. The preservice teachers needed to help mathematical knowledge for proving. Moreover, Stylianides, Stylianides and Philippou (2005) studied at difficulties on proving of preservice mathematics and elementary mathematics teachers. They stated that those preservice teachers did not know any concepts about proof and proving. Another research of them showed that preservice teachers had difficulties while proving with induction method (Stylianides, Stylianides and Philippou, 2005). This result was caused by insufficient fundamental knowledge of induction method and proof. Dane (2008) had the similar results in his research that preservice mathematics teachers had not enough knowledge about proof concepts and they had some misconceptions. He stated that elementary mathematics education programs should give importance the concept knowledge of proof.

Moralı, Uğurel, Türnüklü and Yeşildere (2006) applied a questionnaire on views about proof to preservice mathematics teachers. As the result of the data preservice teachers had not enough views on proof and they did not understand the importance of proof in mathematics an mathematics education. However, Yoo (2008) had interviews with the preservice mathematics teachers on proof. He stated that they realized the roles of proof in mathematics education.

De Villiers's (1999, 2000) studies showed that Dynamic Geometry Software (DGS) helped to be occurred the functions and effective learning of proof. He stated that proving with DGS provided students to make new conjectures, discover new formulas and improve thinking skills.

Furinghetti and Paola (2003) studied on how DGS affected the students' reasoning skills while proving. They maintained that DGS improved the reasoning skills of students and the students applied the methods of proof easily with DGS. The similar results were seen in Lew's (2006) study. He found that proving with DGS helped students make conjectures easily.

Pandiscio (2002) studied on preservice mathematics teachers proving with DGS. He stated that using DGS while proving helped the preservice teachers understand the problems, theorems and examine the theorems in detail. Moreover, DGS provides the preservice teachers to understand the functions of proof and improve their explaining and structuring skills (De Villiers, 2004; Christou, Mousoulides, Pittalis, and Pitta-Pantazi, 2004).

As seen in the literature, there are many researches on proof in mathematics education. Generally those researches were related proof functions, methods, students' proof skills and views on proof. Moreover, effects of using DGS while proving were examined by many researchers. However, in Turkey, there are a few studies related to this issue. Especially, it could not be found any research about geometric and algebraic proofs with DGS.

2. Method

2.1. Model of the Study

Case study model of qualitative research methods was used in this study. This model helps to examine the problem, to define the case and to identify the participants for the case (Yin, 2003). Since case studies might be base for new research studies, it is very important (Yıldırım and Şimşek, 2008; Yin, 2003).

2.2. Working Group

The working group was made up of 39 preservice elementary mathematics teachers who have been trained at one of the state universities in Ankara. There were 25 female and 14 male participants who took a course related to DGS as part of course, namely computer based mathematics education (CBME) before. All participants knew how to use DGS such as Geometer's Sketchpad, GeoGebra, Cabri 3D and Cabri II. Moreover, the designed course for this study was an elective course and so the participants selected this course willingly.

2.3. Data Collection Tools

This is a qualitative research using the reflection papers and sketching of proofs with DGS as a method of data collection. The reflection papers were divided into two groups called geometric proof reflection papers and algebraic proof reflection papers. The sketches were also divided into two, one was geometric and the other was algebraic sketches.

Furthermore, semi-structured interviews were made with 14 selected participants to explore their ideas about proof and DGS. These participants were selected according to their reflection papers and their performance in the classroom. Each interview took about 60 minutes.

2.4. Procedure

A course, named geometry teaching, was designed in accordance to Dynamic Geometry Software (DGS) usage for a study. The participants were the preservice elementary mathematics teachers who have known how to use DGS; however they did not use the program to prove any geometric or algebraic theorems before. During this course, they solved problems related to geometric and algebraic proofs by using DGS. They solved five geometric and five algebraic problems related to proofs of some fundamental theorems of geometry and algebra. To state the preservice teachers' written and verbal skills of implying their arguments in the process of a mathematical proof, the researcher observed them and took field notes during the study. After every course session, participants wrote reflection papers about that session's proof problem. These reflection papers were written related to their sketches and procedures of proving with respect to five questions.

2.5. Data Analysis

Collected geometric and algebraic proof sketches on DGS, reflection papers, observation notes and interviews were analyzed. To analyze the data, the codes were determined as proof method and process, background information usage for geometric and algebraic proof, proof teaching, advantages of DGS to geometric and algebraic proofs and using DGS in mathematics education. The related codes were joined into two themes as geometric proofs with DGS and algebraic proofs with DGS. These themes were analyzed in detail and the findings were presented in below.

3. Findings

The findings of the study were inferred from reflection papers of participants, their sketches during the course sessions, observation notes and interview data. Two main themes were identified from these data as follows; *The Proof Processes of Participants* and *Geometric and Algebraic Proofs with DGS*. The findings will be presented with respect to the themes.

3.1. The Proof Processes of Participants

The participants used dynamic geometry software while doing the geometric proofs. By using the reflection papers, geometric proof sketches and observation notes, it was tried to introduce the preservice elementary mathematics teachers' proof processes, how they used shapes, constructed proofs and which information they needed to prove.

When reflection papers of the first geometric problem were examined, most of the participants used circle and arcs to show that a star's total of the corner angles are equal to 180°. This is not the only proof of the problem. There were any other methods to prove it. For instance, some participants proved by using the total angles of a triangle and some of them used convex polygons angles. However, participants did not explain the proof process clearly in their first reflection papers. They realized that "*DGS provided us to draw shapes well and visually effective learning.*"

The reflection papers of second and third proof problems were examined and it was claimed that the proof processes of the participants were more detailed than before. Furthermore, they added thinking and deciding to proof process. For instance a participant wrote the thinking and deciding step as *"before proving, first I thought that how I could prove and which method I could use. And also the selected method should be clear and then I decided to use circle, square in this circle and arc."* As this participant, most of the others began to write the steps of proof processes in details. Moreover, some of them stated that they tried many methods and selected a method which was more appropriate the problem.

Most of the participants realized that background knowledge and theorems had important role while proving. They explained the background knowledge and theorems in their proof processes. A participant stated as follows "*...a center angle is equal to the measure of arc which is seen and the diagonals of square whose corners on the circle are perpendicular (90°) since the diagonals are also diameter of the circle."*

The participants stated that they tried to prove the problems for their future students who could understand easily. Especially they selected the terms from background knowledge of six to eight grades students. It was seen on the fourth and fifth geometric problem reflection papers clearly. They used such terms as *inverse angle, translate, rotate, isosceles and etc.* For instance a participant wrote the fifth geometric proof on their reflection paper as "*segments which were drawn from center to circle, are equal since all those segments are radius of the circle. And isosceles triangle has two equal angles..."*

The participants did not prove only geometric problems but also they proved algebraic problems. Most of the participants used geometric shapes to prove the algebraic problems. For the first and second algebraic problem, they thought that they could use square and other geometric shapes to illustrate the algebraic expressions. For instance they used *square* to illustrate the x^2. One of the participant explained the second algebraic proof as *"if $(a/2)^2$ be the area of a square, whose one side is a/2 units, is taken out of the $[x+(a/2)]^2$ be the area of a square, whose one side is $(x+ a/2)$ units, then the rest area is equal to (x^2+ax). So I have $x^2+ax = [x+(a/2)]^2 - (a/2)^2$ this equality."*

In addition to square expressions, there were cubic expressions in the algebraic expression. When the reflection papers were examined, it was seen that the geometric objects were used to prove and illustrate the cubic algebraic expressions. Moreover, the fourth and fifth algebraic problems included trigonometric expressions. Those expressions covered both algebra and geometry. The participants stated that trigonometric equations were be able to prove by using geometry since the explanation was clearer with geometric shapes and objects. For instance, a participant claimed as "*this algebraic statement is difficult to explain. But with DGS, it turns as a visual and it is clearly seen that statement is true."*

Generally, the participants wrote the last reflection papers more detailed than first ones. They prepared the reflection papers clear and easy with respect to thinking, deciding, applying steps, background knowledge, axioms and theorems.

3.2. Geometric and Algebraic Proofs with DGS

The participants used dynamic geometry software while doing the geometric and algebraic proofs. By using the reflection papers, geometric proof sketches and observation notes, it was tried to introduce the preservice elementary mathematics teachers' proof processes, how they used shapes, did proofs and which information they used to prove.

The second problem of the geometric problems was proved by using circle and Pythagoras Theorem. One of the proof sketches was seen in the Figure 1.

Problem: *If ABC is a right angled triangle, a+b is hypotenuse and h is height of the triangle. Then, show that* $h^2 = a.b$.

Proof:

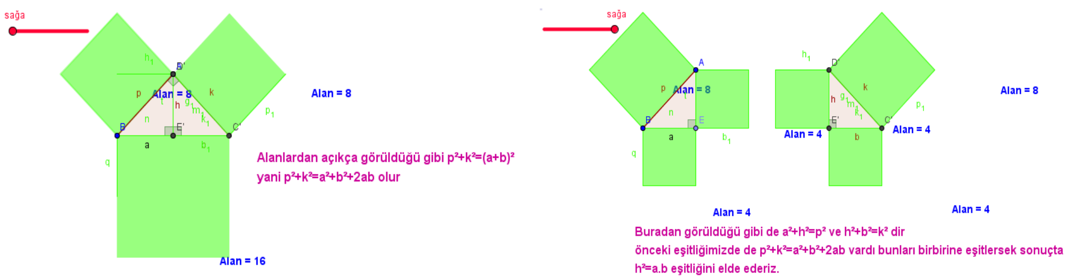

Figure 1. A dynamic proof of the second geometric problem

As in the figure 1, the animation procedure helped to show proof clearly and understandably. Participants wrote that *the DGS provides colored visual which helps students learn with enjoy.*

The menu of the DGS did not help the participants by drawing the parallelogram easily. However, some of the participants used parallel lines to draw a parallelogram and then showed the opposite angles were equal by using parallel lines, inverse angle theorem and supplementary angles theorem. One of the examples of sketches (the fourth geometric problem) is as shown in the Figure 2.

Problem: *Show that the opposite angles of a parallelogram are equal. If |AB| // |CD| and |AC| // |BD|, then∠ (A) =∠ (D) and∠ (B) =∠(C).*

Proof:

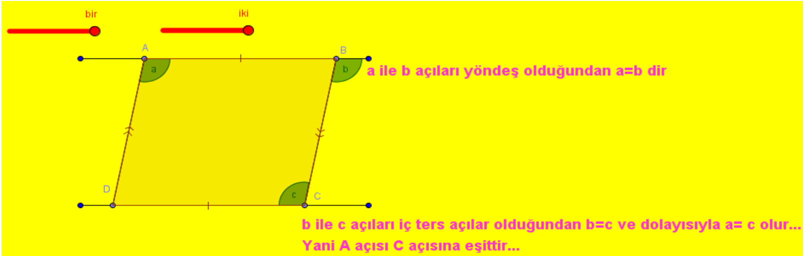

Figure 2. A dynamic proof of the equality of opposite angels of a parallelogram

Most of the preservice teachers stated that *seeing the whole shape and theorems all in one will be helped the students understand the theorem and formulas easily and clearly*.

The second algebraic problem was a bit difficult to see the clue than the others. However, most of the preservice teachers proved this statement by using rectangles and squares clearly and easily. A sketch with animations, in the Figure 3 and 4, is well to explain the students this statement clearly.

Problem: *Show that the statement is true.* $x^2 + ax = (x+a/2)^2 - (a/2)^2$

Proof:

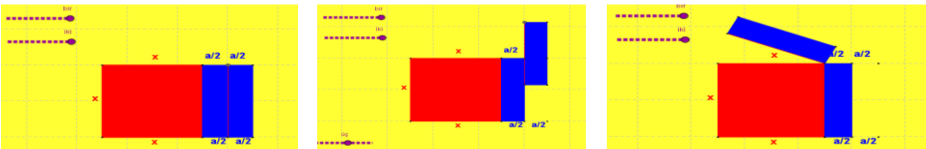

Figure 3. A dynamic proof of the problem given above step by step

One of the participants stated his ideas as; *although explaining such an algebraic proof is difficult with paper-pencil, DGS helps to make algebra visual and translate abstract into concrete. In addition, animation of DGS provides clear understanding of the proof.*

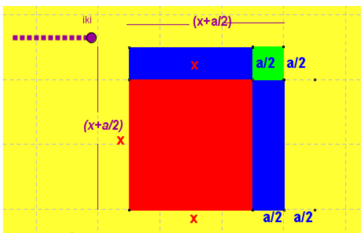

Figure 4. The complete form of the Figure 3

Almost all participants proved the Sine Theorem (fifth algebraic problem) by using properties of circles, angles, triangles and trigonometric ratios. A participant indicated that *DGS provided me to make visualization the things which I created in my mind*, as shown in the Figure 5.

Problem: *Show that the Sine Theorem is correct.*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$

Proof:

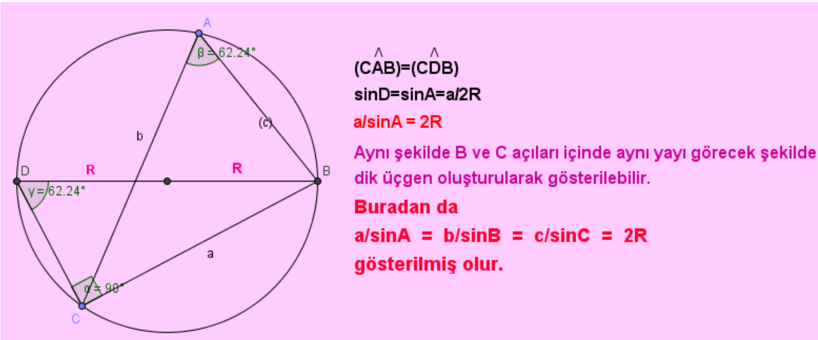

Figure 5. A dynamic proof of Sine Theorem

As seen in the examples of the proofs, which were done by participants, both geometric and algebraic proofs were easily done by using DGS. Therefore, as most of the participants stated, DGS provided to explain the proofs clearly and easily students to understand.

4. Discussion

In this study, the data were gathered to examine the preservice elementary mathematics teachers' proof process with the use of DGS. Based on the findings of the data, it could be said that the participants improved their proof skills with the help of DGS and their proof processes also improve proof by proof. The literature supported these results (de Villiers, 2004; Furinghetti and Paola, 2003; Christou, Mousoulides, Pittalis, and Pitta-Pantazi, 2004; Lew, 2006). However, it could be said that they had few problems while transferring the proofs to DGS at the beginning. Moreover, some of the participants had language problems with some DGS programs. Since the language of some programs is English. However, the preservice teachers solved these problems by taking ideas of friends, trying and error method and with the help of course director. These findings were related to some researches in the literature (Cusi and Malara, 2007; Stylianides, Stylianides and Philippou, 2005). The researchers found out that insufficient knowledge caused those problems and they stated that preservice teachers should be given education to gain the proof skills.

The preservice teachers had some problems while proving with DGS. However, it could be said that they found solutions in a short time. It was taught that by the use of DGS provided these improvements. These findings were parallel to the literature (Baki, 2000; Clements, 2003; Cuoco and Goldenberg, 1997; de Villiers, 2002, 2004; Erbaş, Ledford, Polly and Orrill, 2004; Goldenberg, 2001; Kaput, Hegedus and Lesh, 2007; Morrow, 1997; NCTM, 2000; Pandiscio, 2002). Moreover, according to studies, a mathematics teacher should know proof process and have view about proof (Almeida, 2000; Üzel and Özdemir, 2009). For this reason, elementary mathematics education departments of universities should give courses about proof and DGS applications, since the results would be positive.

In this study, participants had positive view on DGS and proof. According to interview data, the preservice teachers stated that the importance of proof in mathematics education. Moreover, participants added to their views that teaching proof could provide effective learning and improve reasoning skills. While examining the research papers related to thoughts of preservice teachers about proof, some of the findings contradicted and some of them were similar to the findings of this study. Preservice teachers had negative views on proof (Almeida, 2000; Üzel and Özdemir, 2009) or had not clear views (Moralı and the others, 2006) that findings contradicted with findings of this study. On the other hand, Sarı, Altun and Aşkar (2007) stated in their research that successful preservice teachers had positive views on proof. Moreover, different studies stated that preservice teachers had positive views on proof (Conner, 2007; Nyaumwe and Buzuzi, 2007; Yoo, 2008).

DGS provided proofs to visual, concreteness and also it helped to give opportunities, improve reasoning skills and do implications to the participants (Hoyles and Jones, 1998; De Villiers, 1999, 2002; Marrades and Guitierrez, 2000; Hadas, Hershkowitz and Schwarz, 2000; Mariotti, 2000; Furinghetti and Paola, 2003; Lew, 2006). It could be said that the findings of this study supported to the related literature.

5. Conclusion and Suggestion

According to the findings, preservice elementary mathematics teachers' proof skills were improved by the help of Dynamic Geometry Software. They could prove both geometric and algebraic proofs with DGS. Most of the participants used the animations and color options of DGS to make proofs interesting and enjoyable as in the figures for their future students. They realized after starting to use DGS for proving geometric and algebraic theorems that if a concept was explained colored and animated with DGS, students would understand more easily and clearly then the classic paper and pencil method. Some of them had some difficulties to think the algebraic concepts with DGS at the beginning, and then they became familiar with DGS and proof and so they did proofs clearly and easily with the help of animations and colors. In addition, they mentioned that they will teach their future students with DGS in their future job.

After this study, it can be suggested that the DGS can be a teaching and learning tool for undergraduate abstract geometry and abstract algebra courses. Moreover, in-service mathematics teachers can learn and use proofs with DGS in their mathematics classrooms for explaining the fundamental theorems. Since technology is improving day by day and the usage of it in education system increasingly becomes widespread. Hence DGS is a useful tool for mathematics and proof is a fundamental issue in mathematics education, so both DGS and proof should be learned and used at all level of mathematics classrooms.

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