

On A Methods Of Using Weighted Simulation Improving Reliability To Redundant Fiber Optic Communication Systems

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Abstract: The article describes the methodology of weighted modelling to increase the reliability of redundant fibre-optic communication systems. In a specific example, a network graph of a many-node fibre-optic communication system is considered. Applying general ideas to determine the reliability characteristics of systems consisting of a large number of different types of elements with different functional relationships between them is quite a difficult task. The materials presented in this article are intended to solve this problem. Along with their relative simplicity, they are highly accurate. The numerical examples in this chapter show that the use of these methods for highly reliable systems can reduce the variance of the estimate by several orders of magnitude compared to the direct modelling method, and thus reduce the time required for calculations on electronic computers by several orders of magnitude. The purpose of the study is to increase the reliability of fibre-optic communication systems. The research methodology is based on models with a limited number of monotonous failure chains, which is available for visual enumeration of the reliability of highly reliable systems. As a result, it is proposed to obtain an approximate formula for assessing the reliability of a highly reliable system, both by modelling and analytically, and calculations using it can be performed using the quadrature method or moment methods. This allows you to build a model according to the block principle, including full-scale blocks or records of the results of their tests, simplifies the interpretation of the results, and creates convenience in software implementation.

Keywords: Model, graph, node, failure flow, failure rate, approximation, a stream of events, density, a random value, polynomial.

1. Introduction

Modern telecommunication systems are very complex. In this regard, some serious problems arise for system developers, in particular, with the qualitative and quantitative analysis of the effectiveness of communication systems functioning in the initial stages of design. Increasingly high demands are placed on the reliability of a telecommunication system; therefore, the requirements for the adequacy of mathematical models and the accuracy of calculations are increasing. The main mathematical methods for analyzing the reliability of time-functioning systems are the Markov process method and the semi-Markov process method. Both methods, which are similar in their analytical content, allow us to solve a large number of various problems in the theory of reliability: determining the characteristics of redundant systems, analyzing control models, preventive maintenance, troubleshooting, etc. [1-4] However, with an increase in the number of process states, using these methods, great analytical difficulties arise. It is possible to solve such a problem as, for example, determining the distribution of the uptime of a redundant communication system in many ways, but not in all cases, and with the increasing complexity of the systems, the proportion of solvable among them becomes less and less. These methods not only give a computational gain but also make it possible to reveal the qualitative nature of the change in the reliability of communication systems.

1.1. The relevance of the work

In recent years, in all developed countries of the world, highly reliable data transmission equipment is widely used in the information transmission system. Based on this, this research option is relevant and timely.

1.2. Formulation of the problem

Using weighted modelling to redundant communication systems to ensure high reliability of telecommunication systems as a whole.

To solve this problem: we consider the real problems of reliability analysis of redundant systems, taking into account the nature of the failure flow. In the Markov model, where for k failures, the intensity of further failures is λ_k , the dependence of λ_k on k can be taken into account in many ways. One of the simplest methods is as follows. Let We realize the failure times t_1, \dots, t_{r-1} , as for the simplest flow with a parameter. If now for somewhere - independent random variables uniform in $(0, 1)$, then the test is considered ineffective; repeat the

test until the first successful test. The coefficient of increase in the average number of tests due to the repetition of the process

$$K = \prod_{i=1}^{r-1} \frac{\lambda}{\lambda_i} \quad (1)$$

However, it should be borne in mind that modelling may be only a small part of the entire implementation, and then even the coefficient values are acceptable.

About 10.

2. Materials and methods

An alternative method is to reject the implementation if at least one of the inequalities is $\omega_1 \geq e^{-\lambda_1 t_1}$, $\omega_2 \geq e^{-\lambda_2(t_2-t_1)}$, ..., $\omega_{r-1} \leq e^{-\lambda_{r-1}(t_{r-1}-t_{r-2})}$ met with an initial uniform sample in (0, 1). Coefficient K here is much smaller: it is approximately equal to, $1 + (y/2r)(\lambda_1 + \dots + \lambda_{r-1})$ where, At the same time, checking the effectiveness of the test becomes somewhat more complicated. In a sequential modelling method, $y = t_{r-1}$ it is easy to take into account the dependent flow. So, the first failure is realized in accordance with the density of the second-in accordance with the density, $\lambda_1 e^{-\lambda_1 t} / [1 - e^{-\lambda_1 y}]$, and so on. The weight is $\lambda_2 e^{-\lambda_2(t-t_1)} / [1 - e^{-\lambda_2(y-t_1)}]$ the product of the denominators of these fractions. For $\lambda \rightarrow 0$, the weight is $\lambda_1 \dots \lambda_{r-1} y^{r-1} / (r-1)!$ equivalent, and the moments t_i is obtained as ordered independent uniform random variables in the interval (0, y). Indeed, the joint density is proportional to; for small λ_y the exponent is approximately replaced by zero.

As you know, any stream of homogeneous events are defined by distributions $F(t_1)$, $F(t_2|t_1)$, $F(t_3|t_1, t_2)$, etc. If the stream events themselves are in the interval under consideration (0, y if these distributions are unlikely, then it makes no sense to approximate these distributions very precisely; however, it is desirable to preserve the General behaviour of the function [3-7]. So if t_j if there is $\lambda^2 t e^{-\lambda t}$ a moment when an element with unloaded duplication fails, $1 - e^{-\lambda y} (1 + \lambda y) \sim (\lambda y)^2 / 2$ then the density is t_j there is; thus, for small λy enter the weight, and the value of t_j we model it as $y \max \{\omega_1, \omega_2\}$, where ω_1, ω_2 are independent and uniform in (0, 1).

$$\lambda(t_1) = F'(t_1) / \bar{F}(t_1), \lambda(t_2|t_1) = F'(t_2|t_1) / \bar{F}(t_2|t_1), \dots, \text{denote where } = 1 - F.$$

These functions may also depend on some additional parameters: the operating mode and load level at the moment, the interval during which a particular mode is valid, and many others [6-7]. In most cases, the procedure for calculating the failure rate $\lambda(\dots)$ is preferable to using conditional distribution functions $F(\dots)$. With the help of the latter, only the weight is determined, and then only in the case when the approximate

$$F(t_i | \dots) \approx \lambda(\dots)(y - t_{i-1})$$

formula has the form:

When calculating reliability using formulas, only a few of its characteristics can be obtained explicitly. Statistical modelling immediately opens up a wide range of opportunities in this regard. In this article, we will only consider the functionals of a process on a failure chain (not necessarily monotonous).

The indicator of falling into the off-call state is calculated by iterating through the States received sequentially by the system, and accessing the table or procedure for calculating the health function. The time τ of staying in the back state is calculated as a function of t_i, η_i and successive States of the system. Statistics τ unbiased estimates of the distribution moments τ are given, and the event indicator $\{\tau \Delta\}$ is an indicator of system failure during temporary redundancy. The mathematical expectation τ is essential for calculating the non-stationary and stationary availability coefficients of the system.

The work performed during the lifetime of the failure chain is calculated as the sum of the length of time spent in various States multiplied by the system performance in them. It is easy to determine the loss of productivity α as the difference between the work during this time, whether the system is very serviceable, and the actual work [8-11]. An indicator of failure in fibre-optic systems with temporary redundancy is when the value α reaches a certain critical level.

Let the system perform object management, and in the state of operability, the average mismatch characteristic or some other indicator of management quality is σ_0 . In the off-base state i the quality indicator changes according to the differential equation $\sigma'(t) = f_i(\sigma, t)$. For $\sigma(t) = \sigma_i$. Control crashes. First, we implement a chain of failures, then solve the differential equation in consecutive intervals, where the state of the system is unchanged, fixing the failure at the output $\sigma(t)$ to a critical level. This model allows generalization to the multidimensional

parameter $\sigma(t)$, which is essential when managing a multi-parameter object. Let's examine the partially accessible fibre-optic communication system shown in Fig. 1.

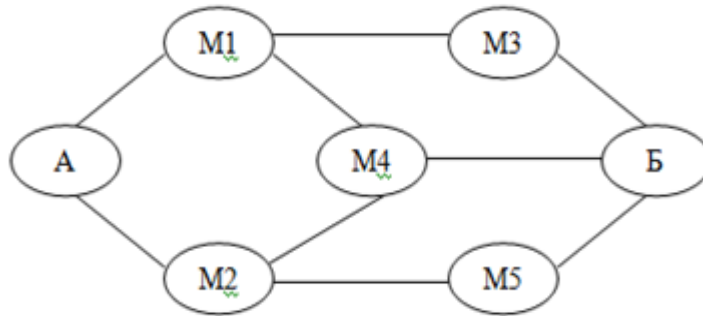


Fig. 1. Count much nodal fibre optic communication system

Communication between points A and B is carried out through M1-M5 routers, and paths $A \rightarrow M1 \rightarrow M3 \rightarrow B$, $A \rightarrow M1 \rightarrow M4 \rightarrow B$, $A \rightarrow M2 \rightarrow M4 \rightarrow B$, and $A \rightarrow M2 \rightarrow M5 \rightarrow B$. The M1 and M2 routers have 20 inputs each, M4 20 inputs each, and M3 and M5 10 inputs each. A failure is an event that occurs when, at some point in the segment, $[0, T]$ does not provide a simultaneous connection of 36 subscribers of point A with 36 different subscribers of point B. Failures of router inputs occur according to an exponential law with the parameter λ , recovery is independent, the recovery duration has a distribution function $B(x)$. Let's estimate the probability $q(T)$ failure of the fibre-optic communication system on the segment $[0, T]$ with the following data: $T=1$; $\lambda=0.02$; $B(x)=x$, $0 \leq x \leq 1$.

Let's assume $\delta_i(t)$ – several failed inputs i -th routers at the time of t . It is easy to see that if the system is working properly, and if a failure is possible (for example, in the case of $\delta_1(t) + \delta_2(t) = 5$). Therefore, we consider the back-end of a chain of length 5 or more. For the calculation, given that the recovery time is comparable to T (its average value is $T/2$), we will consider the system non-recoverable. However, in this case, the probability of minimal failure chains approximates $q(T)$ not accurate enough. For 10 % accuracy of calculations, we choose the weights so that the chains last up to 9 failures.

Let describe an example that allows you to more clearly imagine the structure of the process [12-15]. $\xi(t), t \geq 0$.

Let there be some system that is faulty if N or more of its elements are faulty. We denote $\nu(t)$ – the number of faulty elements at time t ; $\tau_i = \inf t : \nu(t) = i, i = 1, 2, \dots; \xi(t), t \geq 0$, – a right continuous Markov process describing the behaviour of the system.

Random process $\xi(t)$ describes the behaviour of the system in which unwanted events can occur; successive moments of the occurrence of these events, and the N -th event leads to a system failure; $P(T) = P\{\tau_N < T\}$ – the probability of a system failure in $[0, T]$. Let us denote by $\xi_i(t), i = 1, 2, \dots$,

random processes describing the behaviour of the system between moments τ_{i-1} and $\tau_i, \tau_0 = 0$. Thus, the

random process $\xi_i(t)$ can be represented as

$$\xi(t) = \xi_i(t - \tau_{i-1}), t \in [\tau_{i-1}, \tau_i),$$

Moreover $x_1 = \xi_1(0)$ – initial state of the process $\xi_i(t)$, but x_i is determined according to a given distribution over $\xi_{i-1}(\tau_{i-1} - \tau_{i-2} - 0)$ and τ_{i-1} .

Consider a recurrent algorithm for constructing trajectories of a random process $\xi(t)$, which is the basis for the weighted modelling method proposed below. Let $\xi_i(t), \dots, \xi_{N+1}(t), t \geq 0$, – independent right continuous Markov processes defined on the same probability space and taking values in X , where N is some fixed natural number; (X, A) is a measurable space. Then the recurrent algorithm for constructing trajectories of the process $\xi(t), t \geq 0$, is formulated as follows:

1. Initial state $\xi(0) = \xi_1(0) = x_1$ determined according to a given distribution $D(\cdot)$.
2. Selective trajectory is built $z_1(t), t \geq 0$, random process $\xi_1(t), t \geq 0$, with initial state x_1 .

3. The implementation of the random variable is constructed θ_1 with a given, generally speaking, improper distribution function $A_1(\nu; z_1(\cdot))$, $\nu \geq 0$, depending on the entire trajectory $z_1(t), t \geq 0$. Moment $\tau_1 = \theta_1$ is chosen as the moment when the first event occurs. Let suppose $\xi(t) = z_1(t), 0 \leq t < \tau_1$.

4. If then the trajectory $\xi(t), t \geq 0$, built.

Let $\tau_1 < \infty$. Then according to the given distribution $Q_1(\tau_1, z_1(\theta_1 - 0); \cdot)$ state is determined $x_2 = \xi(\tau_1) = \xi(0)$.

5. Let the moment τ_i occurrence of the i -th rare event ($i = 1, \dots, N$) built and known state $x_{i+1} = \xi(\tau_i) = \xi_{i+1}(0)$. then a sample trajectory is constructed $z_{i+1}(t), t \geq 0$, random process $\xi_{i+1}(t), t \geq 0$, with an initial state x_{i+1} .

6. If $i = N$, then it is assumed by definition $\tau_{N+1} = \infty$ and $\xi(t) = z_{N+1}(t - \tau_N), t \geq \tau_N$. If $i < N$, then the implementation of the random variable is constructed θ_{i+1} with a given, generally speaking, improper distribution function $A_{i+1}(\nu; z_{i+1}(\cdot)), \nu \geq 0$. Moment $\tau_{i+1} = \tau_i + \theta_{i+1}$ is selected as the moment of occurrence ($i + 1$)-of the event. Suppose

$$\xi(t) = z_{i+1}(t - \tau_i), \tau_i \leq t < \tau_{i+1}.$$

7. If $\tau_{i+1} = \infty$, then the trajectory $\xi(t), t \geq 0$, is built. Let $\tau_{i+1} < \infty$. Then according to the given distribution $Q_{i+1}(\tau_{i+1}, z_{i+1}(\theta_{i+1} - 0); \cdot)$ state is determined $x_{i+2} = \xi(\tau_{i+1}) = \xi_{i+2}(0)$ and the described algorithm is repeated.

So if $\tau_{i-1} < \infty$ ($1 \leq i \leq N + 1$), then

$$\xi(t) = \xi_i(t - \tau_{i-1}), \tau_{i-1} \leq t < \tau_i, \tau_0 = 0, \tau_{N+1} = \infty.$$

This article describes a weighted modelling method that allows you to calculate with high accuracy the characteristics of systems represented in the form

$$P(T) = M \{ f(\tau_N; \xi_{N+1}(\cdot)) I(\tau_N < T) \},$$

Where T is some fixed number, $T \in (0, \infty)$; $I(B)$ – event indicator B; $f(t; z(\cdot))$ – nonnegative, bounded and measurable function of the corresponding variables.

The need to create special methods for accelerated modelling of the quantity $P(T)$ is justified by the fact that for real systems failure is a rare event (i.e. the probability of an event $\{\tau_N < T\}$ small, for example, has the order $10^{-4} - 10^{-6}$), and therefore direct simulation methods cannot be used. At the same time, the complexity of real systems is often an insurmountable obstacle to the use of analytical and asymptotic methods. Therefore, in several cases, the only way leading to finding certain characteristics of complex systems is to create accelerated modelling methods that have the advantages of both analytical (high accuracy) and statistical (universality) methods.

Here are the two most important interpretations of the quantity $P(T)$. If $f(t; z(\cdot)) \equiv 1$, then $P(T)$ – the probability of a system failure in between $[0, T]$. Let

$$f(t; z(\cdot)) = \begin{cases} \int_0^{T-t} I(z(u) \in E) du, & \text{если } t < T \\ 0 & \text{otherwise,} \end{cases}$$

Where E is the set of system failure states. Then $P(T)$ – average system failure time in the interval $[0, T]$.

Note that the structure of the process $\xi(t), t \geq 0$, is close to switching processes [3].

The recurrent way of constructing process trajectories $\xi(t)$ underlies the method of weighted modelling of the quantity $P(T)$. Let us formulate this method as an algorithm for constructing an unbiased estimate in one implementation $P(T)$.

1. According to the distribution $D(\cdot)$ build the state x_1 of the process $\xi_1(t)$ in the moment $t = 0$.
2. Build a sample trajectory $z_1(t), t \geq 0$, process $\xi_1(t), t > 0$, with initial state x_1 .
3. Calculate $J_1 = A_1(T; z_1(\cdot))$. If $J_1 = 0$, it is believed and the score is built. Otherwise, the random variable is implemented χ_1 distributed in $[0, T]$ with distribution function $A_1(y; z_1(\cdot)) / A_1(T; z_1(\cdot))$, $y \in [0, T]$.

Let $\chi_1 = t_1$ (moment of arrival of the first event).

4. According to the distribution $Q_1(t_1, z_1(t_1 - 0); \cdot)$ build process state $\xi(t)$ in the moment t_1 : $\xi(t_1) = \xi_2(0) = x_2$.

5. Suppose the moment t_{i-1} advance $(i-1)$ -of the event is constructed ($i = 2, \dots, N-1$), moreover $\xi(t_{i-1}) = \xi_i(0) = x_i$. Then build the sample trajectory $z_i(t), t \geq 0$, process $\xi_i(t), t \geq 0$.

6. Calculate $J_i = A_i(T - t_{i-1}; z_i(\cdot))$. If $J_i = 0$, it is supposed $P_1(T) = 0$ and the score is built. Otherwise, the random variable is implemented χ_i , distributed in $[t_{i-1}, T]$ with distribution function $A_i(y - t_{i-1}; z_i(\cdot)) / A_i(T - t_{i-1}; z_i(\cdot))$, $y \in [t_{i-1}, T]$.

As a moment t_i advance of i the event is chosen $t_i = \chi_i$.

7. According to the distribution $Q_i(t_i, z_i(t_i - t_{i-1} - 0); \cdot)$ build process state $\xi(t)$ in the moment t_i : $\xi(t_i) = \xi_{i+1}(0) = x_{i+1}$.

8. Let the moment t_N the advance of N the event is constructed, moreover, $\xi(t_N) = \xi_{N+1}(0) = x_{N+1}$. Then the sample trajectory is built $z_{N+1}(t), t \geq 0$, process $\xi_{N+1}(t), t \geq 0$. As $P_1(T)$ is chosen $P_1(T) = f(t_N; z_{N+1}(\cdot)) J_1 \dots J_N$.

The accuracy of an unbiased estimate is determined by its variance. Let us denote by $\hat{\alpha}_1$ evaluation in one implementation for $P(T)$, obtained by direct simulation. It's obvious that $D \hat{P}_1(T) \leq \hat{\alpha}_1$. Really,

$$\begin{aligned} D \hat{P}_1(T) &= M \left\{ \left[f(\tau_N; \xi_{N+1}(\cdot)) \right]^2 J_1^2 \dots J_N^2 \right\} - \\ &- \left[M \hat{P}_1(T) \right]^2 \leq M \left\{ \left[f(\tau_N; \xi_{N+1}(\cdot)) \right]^2 J_1 \dots J_N \right\} - \\ &- \left[P(T) \right]^2 = M \left\{ \left[f(\tau_N; \xi_{N+1}(\cdot)) \right]^2 I(\tau_N < T) \right\} - \\ &- \left[P(T) \right]^2 = D \hat{\alpha}_1. \end{aligned}$$

Even very simple and very rough estimates show that for highly reliable systems, the described method allows one to achieve a very significant gain invariance (and, consequently, to sharply reduce the amount of computer time). So, for highly responsible technical systems, the failure of the next element is a rare event. Therefore, it is

$$\varepsilon_i = \sup_{z_i(\cdot)} A_i(T; z_i(\cdot)), \quad 1 \leq i \leq N$$

quite natural to assume that

$$D \hat{P}_1(T) = M \left\{ \left[f(\tau_N; \xi_{N+1}(\cdot)) \right]^2 J_1^2 \dots J_N^2 \right\} -$$

Using this equation, we obtain the obvious relations

$$-\left[M \hat{P}_1(T)\right]^2 \leq \varepsilon_1 \dots \varepsilon_N M \left\{ \left[f(\tau_N; \xi_{N+1}(\cdot)) \right]^2 J_1 \dots J_N \right\} - [P(T)]^2 = \varepsilon_1 \dots \varepsilon_N \left\{ M \left[f(\tau_N; \xi_{N+1}(\cdot)) \right]^2 J_1 \dots J_N \right\} - [P(T)]^2 - [P(T)]^2 (1 - \varepsilon_1 \dots \varepsilon_N) \leq \varepsilon_1 \dots \varepsilon_N D \hat{\alpha}_1.$$

For highly reliable systems $\varepsilon_1 \dots \varepsilon_N \ll 1$ and the gain, invariance is significant. So if $P(T)$ – the probability of a system failure in between $[0, T]$, then $f(t; z(\cdot)) \equiv 1$. If, also, for the system failure, a simultaneous failure of three elements is required (i.e., $N = 3$) and the total failure rate of all elements does not exceed $0.1 \cdot T^{-1}$, so $\varepsilon_i < 0,1$, $i = 1, 2, 3$, and the gain invariance is at least 1000 times.

When studying specific classes of systems, it is possible to prove an important qualitative property of the estimate $\hat{P}_1(T)$: with an increase in system reliability, the relative root meansa square error of the estimate $\hat{P}_1(T)$ is limited.

It should be borne in mind that the variance estimate for $D \hat{P}_1(T)$ strongly overestimated. A much more

accurate estimate for $D \hat{P}_1(T)$ (which can be used for both highly reliable and unreliable systems) is the sample variance. The examples are given below, as well as the calculation of the reliability of real technical systems, show that for highly reliable systems it is possible to reduce the cost of computer time by several orders of magnitude in comparison with the method of direct simulation [11-14].

The described method serves as a basis for determining the reliability characteristics of various classes of systems. For this, it is necessary for the system under study to set random processes $\{\xi_i(t), t \geq 0\}$, function $\{A_i(t; z_i(\cdot))\}$ and distribution $D(\cdot)$, $\{Q_i(t; x; \cdot)\}$. Now we will consider exactly how to solve this issue for weighted modelling of redundant systems with recovery.

One of the most important classes of systems is redundant systems with recovery. To determine the reliability of a typical system of this class, we use a weighted modelling method based on the general ideas of the method described above.

Consider a system consisting of m independent elements and l repair devices ($1 \leq i \leq m$). In the moment $t = 0$ the system with a loaded reserve is switched on and all its elements are in good order. The uptime of the elements has distribution functions $F_i(y), 1 \leq i \leq m$. Let k - number of faulty elements at the moment If at this moment it failed i -й элемент and $k < l$, but immediately begins its recovery with the distribution function $G_i(y)$. If $k \geq l$, then the item is queued. The restoration of failed elements is carried out in the order of their failure. System failure occurs at the moment when the number of faulty elements becomes equal to r ($l + 1 \leq r \leq m$). Let ξ – the first moment of system failure in a given interval $[0, T]$:

$$P(T) = P\{\xi < T\}.$$

Suppose the functions $\{F_i(y)\}$ and $\{G_i(y)\}$ satisfy the conditions: $F_i(+0) = G_i(+0) = 0, i = 1, \dots, m$;

P {system in between $[t, t+h]$ will change its state two or more times} $= 0(h), h \rightarrow 0$, for any fixed $t \in [0, T]$.

We denote: for $i = 1, \dots, m$

$$v_i(t) = \begin{cases} 0, & \text{if the moment } i\text{-th element is healthy,} \\ -1 & \text{if the item is being restored,} \\ k, & \text{if the element is found } k - m \text{ by count} \\ & \text{in the recovery queue, } k = 1 \dots, m-1; \end{cases}$$

$$\gamma_i(t) = \begin{cases} 0, & \text{if } v_i(t) > 0, \\ \sup \{v: \text{for any } u \in (t-v, t) \ v_i(u) = \\ = v_i(t)\}, & \text{if } v_i(t) \leq 0, \end{cases}$$

t - time of stay i -element on recovery by the time t (if $v_i(t) = -1$) or the time of its continuous trouble-free operation to the moment t (if a $v_i(t) = 0$). The quantities $v_i(t)$ and $\gamma_i(t)$, $F_i(y)$, $1 \leq i \leq m$, completely determine the state of the system at the moment t and her further behaviour, i.e. the process $\xi(t) = (v_1(t), \dots, v_m(t); \gamma_1(t), \dots, \gamma_m(t))$, $t \geq 0$, is Markovian. Suppose this process is continuous on the right. Let X denote the set of all states of the process $\xi(t)$. Suppose $\xi(0) = (0, \dots, 0; 0, \dots, 0)$ with probability 1. Let N be a fixed natural number, $1 \leq N \leq r$, and every state of the system

$(v; \gamma) = (v_1, \dots, v_m; \gamma_1, \dots, \gamma_m)$ corresponds to the number of faulty elements in this state $|v| = \sum_{i: v_i \neq 0} 1$

Let suppose

$$E_i = \{(v; \gamma) = (v_1, \dots, v_m) : |v| \geq r - N + i\}$$

$$\tau_i = \inf \{t > 0 : \xi(t) \in E_i\}, \quad 1 \leq i \leq N.$$

In this way, τ_i , $1 \leq i \leq N$, – the first moment in time when the system is faulty $r - N + i$ elements, $\xi = \tau_N$.

We interpret the random processes for the system under consideration $\{\xi_i(t), t \geq 0\}$, functions $\{A_i(t; z_i(\cdot))\}$ and distribution $\{Q_i(t, x; \cdot)\}$. According to the general method outlined above, it is enough to have an algorithm for constructing trajectories of processes $\{\xi_i(t)\}$ only in between $[0, T]$. Let $\xi_i(0) = x \in E_i$. As trajectories of the process $\xi_i(t)$, $t \in [0, T]$, choose process trajectories $\xi(t)$, $t \in [0, T]$, built with the prohibition of entering the set of states E_i in the interim $[0, T]$. The algorithm for constructing a sample trajectory $z_i(t)$, $t \in [0, T]$ is formulated as follows:

1. Using the method of direct simulation, build the trajectory of the process $\xi(t)$, $t \geq 0$, with initial state x up to the first moment $t^{(1)}$, in which $|v(t^{(1)})| = r - N + i - 1$. If $t^{(1)} \geq T$, then the sample trajectory $z_i(t)$, $t \in [0, T]$ is constructed:

2. Let $t^{(1)} < T$, and $\xi(t^{(1)}) = (v_1^{(1)}, \dots, v_m^{(1)}; \gamma_1^{(1)}, \dots, \gamma_m^{(1)})$. Then $z_i(t) = \xi(t)$, $t \in [0, t^{(1)}]$. Next, the simulation determines the time $\beta^{(1)}$ until the end of the restoration of one of the faulty at the time $t^{(1)}$ items and number $j^{(1)}$ this element (if $t^{(1)} = 0$, $r = N$, then $\beta^{(1)} = T$).

3. It is calculated

$$\omega^{(1)} = \min_{k: v_k^{(1)} = 0} \left[\inf \{y : F_k(y) = 1\} - \gamma_k^{(1)} \right],$$

i.e. $t^{(1)} + \omega^{(1)}$ – the moment until which, with probability 1, one of the serviceable ones will fail at the moment $t^{(1)}$ elements. Let $\nu^{(1)} = \min \{T - t^{(1)}, \beta^{(1)}, \omega^{(1)}\}$. Prohibiting failures of serviceable elements in the half-

interval $[t^{(1)}, t^{(1)} + \nu^{(1)})$ (i.e., assuming that the elements cannot refuse in the indicated half-interval), we

$$z_i(t) = (v_1^{(1)}, \dots, v_m^{(1)}; \gamma_1^{(1)} + \delta(v_1^{(1)})(t - t^{(1)}), \dots,$$

assume $\dots, \gamma_m^{(1)} + \delta(v_m^{(1)})(t - t^{(1)}))$, $t \in [t^{(1)}, t^{(1)} + \nu^{(1)})$,

Where

$$\delta(v_k^{(1)}) = \begin{cases} 0, & \text{if } v_k^{(1)} > 0, \\ 1, & \text{if } v_k^{(1)} \leq 0. \end{cases}$$

If $\nu^{(1)} = \omega^{(1)}$, then the failure of the next element with probability 1 will occur before the end of the recovery of one of the faulty at the moment $t^{(1)}$ elements. In this case, the trajectory $z_i(t), t \in [0, T]$ is constructed. If $\nu^{(1)} = T - t^{(1)}$, then the trajectory $z_i(t), t \in [0, T]$ is constructed. Let $\nu^{(1)} = \beta^{(1)}$. Suppose

$$z_i(t^{(1)} + \nu^{(1)}) = x^{(1)} = (v_1^{(1)}, \dots, v_m^{(1)}; \gamma_1^{(1)}, \dots, \gamma_m^{(1)}) | \nu(t^{(1)} + \nu^{(1)}) =$$

$$= r - N + i - 2,$$

Where

$$v_k^{(1)} = \begin{cases} 0, & \text{if } k = j^{(1)}, \\ -1, & \text{if } v_k^{(1)} = 1, \\ v_k^{(1)} - 1, & \text{if } v_k^{(1)} \geq 2, \\ v_k^{(1)} & \text{otherwise,} \end{cases}$$

$$\gamma_k^{(1)} = \begin{cases} 0, & \text{if } k = j^{(1)} \text{ or } v_k^{(1)} > 0, \\ \gamma_k^{(1)} + \nu^{(1)} & \text{otherwise } k = 1, \dots, m \dots \end{cases}$$

4. By the method of simulation, the trajectory of the process is built $\xi(t), t \geq t^{(1)} + \nu^{(1)}$, with the initial state $\xi(t^{(1)} + \nu^{(1)}) = x^{(1)}$ before the first $t^{(2)}$, in which $|\nu(t^{(2)})| = r - N + i - 1$. If $t^{(2)} \geq T$, then the sample trajectory $z_i(t), t \in [0, T]$, is constructed:
 $z_i(t) = \xi(t), t \in [t^{(1)} + \nu^{(1)}, T]$.

If $t^{(2)} < T$, it is supposed.

$$z_i(t) = \xi(t), t \in [t^{(1)} + \nu^{(1)}, t^{(2)}].$$

Further, starting from item 2, the algorithm repeats cyclically with the replacement of the superscript 1 by 2. Thus, with a fixed trajectory

$z_i(t), t \in [0, T]$, the interval $[0, T]$ can be divided into sections, during which the system is faulty even $r - N + i - 1$ elements and less than $r - N + i - 1$ elements. Therefore, with probability 1 $\xi_i(t) \in E_i$ for any $t \in [0, T]$. We denote:

K is the number of time intervals from $t \in [0, T]$, during which it is faulty exactly $r - N + i - 1$ elements;
 $t^{(k)}$ and $\nu^{(k)}$, $k = 1, \dots, K$, - the moment of the beginning and the duration of the k -th interval, respectively;
 $z_i(t^{(k)}) = (v_1^{(k)}, \dots, v_m^{(k)}; \gamma_1^{(k)}, \dots, \gamma_m^{(k)})$, $k = 1, \dots, K$.

Fixed trajectory $z_i(t), t \in [0, T]$, define the distribution function $A_i(t; z_i(\cdot))$ time θ_i before some event occurs. Such an event is a hit $\xi(t)$ in many states E_i . In order for the specified event not to occur in the interval $[0, t]$, it is necessary and sufficient that none of the elements fails at that moment ($\text{вз } [0, t]$) when the system is faulty $r - N + i - 1$ elements, i.e. $P\{\theta_i > t\}$ is equal to the probability that in none of the sections with lengths $\{\nu^{(j)}\}$, lying to the left t , there was no element failure. So if $K > 0$, then

$$A_i(t; z_i(\cdot)) = 1 - \prod_{k=1}^k \prod_{v_j^{(k)}=0} \frac{1 - F_j(\min\{\nu^{(k)}, t - t^{(k)}\} + \gamma_j^{(k)})}{1 - F_j(\gamma_j^{(k)})},$$

$$t \geq 0.$$

If $K = 0$, then $A_i(t; z_i(\cdot)) \equiv 0$. It remains to note that the distribution $Q_i(t, x; \cdot)$ is given by a natural algorithm for changing the state of the system in case of failure of an element that transfers it to E_i . Before proceeding to the presentation of the main result of this article of the method of weighted probability modelling $P(T)$, Let us formulate an auxiliary algorithm.

Let independent non-negative random variables with distribution functions $B_1(y), \dots, B_n(y)$ (it is assumed that $P\{\beta_i \neq \beta_j = 1\}$ for any $i \neq j$), and (χ, μ) – two-dimensional random variable taking values in $[0, T] \times \{1, \dots, n\}$ and having distribution

$$P\{\chi < y, \mu = j\} = P\{\beta_j < y, \beta_j < \beta_i, 1 \leq i \leq n, i \neq j \mid \min_{1 \leq i \leq n} \beta_i < T\}, \quad (2)$$

The algorithm formulated below allows one to build joint realizations of the number μ the element that failed first and the moment χ his refusal provided that in the interval $[0, T]$ one of the elements has failed.

$$q_1 = B_1(T), \quad q_i = B_i(T) \prod_{j=1}^{i-1} [1 - B_j(T)], \quad i = 2, \dots, n;$$

1. It is calculated:
 $q^* = P\left\{\min_{1 \leq i \leq n} \beta_i < T\right\} = 1 - \prod_{i=1}^n [1 - B_i(T)] = \sum_{i=1}^n q_i.$

2. Implement a random variable α , equal i to the probability q_i / q^* .

3. Implement a random variable $\beta^* \alpha$ with a distribution function $B_\alpha(y) / B_\alpha(T)$, $y \in [0, T]$.

4. If $\alpha < n$, then implement $n - \alpha$ independent random variables $\beta_{\alpha+1}, \dots, \beta_n$ with distribution functions $F_{\alpha+1}(y), \dots, F_n(y)$.

5. As a realization of a two-dimensional random variable (χ, μ) it is chosen:

$$\chi = \min(\beta_\alpha^*, \beta_{\alpha+1}, \dots, \beta_n); \quad (3)$$

$$\mu = \arg \min(\beta_\alpha^*, \beta_{\alpha+1}, \dots, \beta_n) \quad (4)$$

The random variables constructed according to (3) and (4) satisfy (2).

Indeed, for $y \in [0, T]$

$$P\{\chi < y, \mu = j\} = P\{\alpha = j, \beta_j^* < y, \beta_j^* < \beta_i,$$

$$i = j + 1, \dots, n\} +$$

$$+ P\{\alpha < j, \beta_j < y, \beta_j < \beta_\alpha^*, \beta_j < \beta_i, i = \alpha + 1, \dots, n, i \neq j\} =$$

$$= \frac{1}{q^*} q_j \int_0^y \prod_{i=j+1}^n [1 - B_i(u)] d B_j(u) / B_j(T) +$$

$$+ \sum_{s=1}^{j-1} \frac{q_s}{q^*} \int_0^y \frac{B_s(T) - B_s(u)}{B_s(T)} \prod_{\substack{i=s+1 \\ i \neq j}}^n [1 - B_i(u)] d B_j(u) = \frac{1}{q^*} \int_0^y \prod_{i=j+1}^n [1 - B_i(u)] \left\{ \prod_{i=1}^{j-1} [1 - B_i(T)] + \right.$$

$$\left. + \sum_{s=1}^{j-1} \prod_{i=1}^{s-1} [1 - B_i(T)] [B_s(T) - \right.$$

$$\left. - B_s(u) \right\} \prod_{i=s+1}^{j-1} [1 - B_i(u)] d B_j(u).$$

Here $\prod_{i=s}^k \dots = 1$ and $\sum_{i=s}^k \dots = 0$, if $k < s$. Let's transform the expression in curly braces:

$$\{ \dots \} = P \{ \beta_i > T, i = 1, \dots, j-1 \} + \sum_{s=1}^{j-1} P \{ \beta_i > T, i = 1, \dots, s-1, \beta_s \in (u, T], \beta_i > u, i = s+1, \dots, j-1 \} = P \{ \beta_i > u, i = 1, \dots, j-1 \} = \prod_{i=1}^{j-1} [1 - B_i(u)]$$

Hence,

$$P \{ \mathcal{X} < y, \mu = j \} = \frac{1}{q^*} \int_0^y \prod_{\substack{i=1 \\ i \neq j}}^n [1 - B_i(u)] dB_j(u) = \frac{1}{q^*} P \{ \beta_j < y, \beta_i > \beta_j, i = 1, \dots, n, i \neq j \} = \\ = P \{ \beta_j < y, \beta_i > \beta_j, i = 1, \dots, n, i \neq j \mid \min_{1 \leq i \leq n} \beta_i < T \}$$

3. Results

The above interpretations of random processes $\{ \xi_i(t), y \geq 0 \}$, functions $\{ A_i(t; z_i(\cdot)) \}$ and distributions $D(\cdot), \{ Q_i(t, x; \cdot) \}$ can be proposed the following weighted probability modelling method $P(T)$. Let us

formulate this method as an algorithm for constructing an estimate $\hat{P}_1(T)$.

1. Build a sample trajectory $z_1(t), t \in [0, T]$, process $\xi_1(t), t \in [0, T]$, at initial state $\xi_1(0) = x_1 = (0, \dots, 0; 0, \dots, 0)$. In the process of modelling, the following values are determined: K – number of time intervals from $[0, T]$, during which it is faulty exactly $r - N$ elements;

$t^{(k)}$ and $\nu^{(k)}, k = 1, \dots, K$, – the start time and duration respectively k -interval;

$z_1^{(k)} = z_1(t^{(k)}) = (\nu_1^{(k)}, \dots, \nu_m^{(k)}; \gamma_1^{(k)}, \dots, \gamma_m^{(k)}), k = 1, \dots, K$.

If $K = 0$, then $\hat{P}_1(T) = 0$ and the implementation is over.

2. Let $K > 0$. Then it is calculated

$$J_1 = A_1(T; z_1(\cdot)) = 1 - \prod_{k=1}^K \prod_{j: \nu_j^{(k)}=0} \frac{1 - F_j(\nu^{(k)} + \gamma_j^{(k)})}{1 - F_j(\gamma_j^{(k)})}. \quad (5)$$

3. If $J_1 = 0$, then $\hat{P}_1(T) = 0$. Otherwise, the random variable is implemented \mathcal{X}_1 , distributed in $[0, T]$ with distribution function

$$H(y) = \frac{1}{J_1} \left[1 - \prod_{\substack{k=1 \\ kt^{(k)} < y}} \prod_{j: \nu_j^{(k)}=0} \frac{1 - F_j(\min \{ \nu^{(k)}, y - t^{(k)} \} + \gamma_j^{(k)})}{1 - F_j(\gamma_j^{(k)})} \right]$$

Construction of the implementation of a random variable is carried out as follows. The function $H(y)$ can be represented as

$$H(y) = \sum_{k=1}^K a_k H_k(y), y \in [0, T]$$

Where

$$a_1 = \frac{1}{J_1} u_1, a_k = \frac{1}{J_1} u_k \prod_{i=1}^{k-1} u_i, k = 2, \dots, K, \sum_{k=1}^K a_k = 1,$$

$$u_k = 1 - \prod_{j: v_j^{(k)}=0} [1 - F_j(\gamma_j^{(k)} + v^{(k)})] / [1 - F_j(\gamma_j^{(k)})]$$

$$H_k(y) = \frac{1}{u_k} \left[1 - \prod_{j: v_j^{(k)}=0} \frac{1 - F_j(v_j^{(k)} + y - t^{(k)})}{1 - F_j(v_j^{(k)})} \right], y \in [t^{(k)}, t^{(k)} + v^{(k)}],$$

$$H_k(y) = 0, y < t^{(k)}, H_k(y) = 1, y \geq t^{(k)} + v^{(k)}, k = 1, \dots, K.$$

Further, implement σ , equally to k with probability a_k . Let $\sigma = k$. Then as χ_1 choose a random variable with distribution function $H_k(y)$. Let $\beta_i, i: v_i^{(k)} = 0, -$ independent random variables,

$$P\{\beta_i < y\} = B_i(y) = [F_i(y + \gamma_i^{(k)}) - F_i(\gamma_i^{(k)})] / [1 - F(\gamma_i^{(k)})].$$

Then

$$H_k(y) = P\left\{ \min_{j: v_j^{(k)}=0} \beta_j < y - t^{(k)} \mid \min_{j: v_j^{(k)}=0} \beta_j < v^{(k)} \right\}, y \in [t^{(k)}, t^{(k)} + v^{(k)}].$$

Number μ_1 , на котором достигается $\min_{j: v_j^{(k)}=0} \beta_j$ under the specified condition, and the value $\chi_1 - t^{(k)}$ this minimum is modelled in accordance with (2) and (3), namely:

$$\chi_1 = t^{(k)} + \min \left\{ \beta_\alpha^*, \min_{\substack{i: v_i^{(k)}=0 \\ \alpha+1 \leq i \leq m}} \beta_i \right\};$$

$$\mu_1 = \arg \min \left\{ \beta_\alpha^*, \min_{\substack{i: v_i^{(k)}=0 \\ \alpha+1 \leq i \leq m}} \beta_i \right\},$$

where α - a random variable equal to j the probability $p_i = q_j / q^* (1 \leq j \leq m)$;

$$q^* = \sum_{j=1}^m q_j, q_1 = s_1; q_j = s_j \prod_{i=1}^{j-1} (1 - s_i), j = 2, \dots, m;$$

$$s_i = \begin{cases} 0, & \text{если } v_i^{(k)} \neq 0, \\ B_i(v^{(k)}), & \text{если } v_i^{(k)} = 0, 1 \leq i \leq m, \end{cases}$$

and β_α^* at $\alpha = j$ is a random variable with distribution function $B_j^*(y) = \frac{B_j(y)}{B_j(v^{(k)})} =$

$$= \frac{F_j(\gamma_j^{(k)} + y) - F_j(\gamma_j^{(k)})}{F_j(\gamma_j^{(k)} + v^{(k)}) - F_j(\gamma_j^{(k)})}, y \in [0, v^{(k)}]$$

4. Let the moment $\chi_1 \in [t^{(k)}, t^{(k)} + v^{(k)})$ failure of μ_1 the element is constructed. Then as $\xi(\chi_1) = \xi_2(0)$ is chosen

$$x_2 = \xi_2(0) = (v_1^{\wedge}, \dots, v_m^{\wedge}; \gamma_1^{\wedge}, \dots, \gamma_m^{\wedge}),$$

Where

$$v_i^{\wedge} = \begin{cases} -1, & \text{if } i = \mu_1 \text{ and } r - N + 1 \leq l, \\ r - N + 1 \leq l, & \text{if } i = \mu_1 \text{ и } r - N + 1 \leq l, \\ v_i^{(k)} & \text{otherwise,} \end{cases} \gamma_i^{\wedge} = \begin{cases} 0, & \text{if } i = \mu_1 \text{ or } v_i^{(k)} > 0, \\ \gamma_i^{(k)} + \chi_1 - t^{(k)} & \text{otherwise,} \end{cases}$$

$i=1, \dots, m$.

Further in between $[0, T - \chi_1]$ a sample trajectory is built $z_2(t)$ at initial state $\xi_2(0) = x_2$. In this case, new values of the quantities $K, \{t^{(k)}, \nu^{(k)}, z_2^{(k)}, k=1, \dots, K\}$. Then, using the formula (4), calculate J_2 . If $J_2 = 0$, then the implementation is over and $\hat{P}_1(T) = 0$. And if $J_2 > 0$, then build the implementation of a two-dimensional random variable (χ_2, μ_2) , where $\chi_2 \in [0, T - \chi_1]$ – the moment of failure of the next element, and μ_2 – the number of this item. Having determined the state of the process $\xi_2(t)$ at the moment $\chi_2(\xi_2(\chi_2)) = \xi(\chi_1 + \chi_2)$, a selective process trajectory is built $\xi_3(t)$ in between $[0, T - \chi_1 - \chi_2]$ on the condition $\xi_3(0) = \xi_2(\chi_2)$. Further, the described algorithm is repeated until, until it is calculated J_N . As $\hat{P}_1(T)$ is selected

$$P_1(T) = J_1 J_2 \dots J_N \quad (6)$$

Having made a sufficient number of realizations, construct an estimate for $P(T)$ with the specified reliability

and accuracy. An unbiased estimate $\hat{P}_1(T)$ follows from the general algorithm given above [5].

The accuracy of the estimates obtained essentially depends on the parameter $N \in \{1, \dots, r\}$. If it is chosen $N = 0$, then the method formulated above will turn into a conventional method of direct simulation. It is easy

to show that the variance $D\hat{P}_1(T)$ decreases monotonically with increasing N and is minimal at $N = r$.

4. Conclusions

Based on the above example, it can be judged that one of the ways to speed up the simulation is the method of "weighted" modelling. In the practice of modelling random processes that describe the behaviour of complex systems, various concretizations of the method of weighted modelling have been applied, taking into account the specifics of a particular class of problems.

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