Analysis of Fuzzy Non-preemptive Priority Queuing Model with Unequal Service Rate

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Abstract: This article provides an effective method to analyze the performance measures of non-preemptive fuzzy priority queues with unequal service rates. Here the arrival rate and the service rate are in fuzzy numbers. Using a new ranking method, the fuzzy values are reduced to the crisp values. For that cause, both the Triangular Fuzzy Number (TFN) and Trapezoidal Fuzzy Number (TpFN) are chosen to establish the proposal's effectiveness. An illustration is given to find the efficiency of the performance measures of the fuzzy queuing model.

Keywords: Fuzzy Queuing Model, Triangular Fuzzy Number (TFN), Trapezoidal Fuzzy Number (TpFN).

1. Introduction

Nowadays, the concept of Queuing theory has pervasive applications in the real-time hypothesis. Overall, the Priority Queues has a wide range of applications like communication networks, transport sector, healthcare management, etc. At the same time, the concept of Fuzzy Queues is also extensively debated. Li and Lee (1989), Negi and Lee (1992), Kao and Wilson (1999), Chen (2005, 2006) are some of the researchers who developed the concept of Fuzzy Queues. Section 3 proposes a new ranking method in this paper, and section 4 intends to apply the proposal effectively.

2. Preliminaries

2.1 Fuzzy Set

A Fuzzy set $\tilde{F}$ is defined by $\tilde{F} = \{(x, \mu_{\tilde{F}}(x)): x \in X, \mu_{\tilde{F}}(x) \in [0,1]\}$. In the pair of an ordered set $(x,\mu_{\tilde{F}}(x))$, the first element x belongs to the Universe X, and the second element $\mu_{\tilde{F}}(x)$ belongs to the interval [0, 1], then the set $\tilde{F}$ is called a fuzzy set and $\mu_{\tilde{F}}(x)$ is called the Membership function.

2.2 Crisp Set

For any crisp set $\tilde{F}$, it is defined as the characteristic function $\mu_{\tilde{F}} \to \{ 0,1 \}$, i.e., the characteristic function takes either of the value 0 or 1 in the classical set.

2.3 Triangular Fuzzy Number

The Triangular Fuzzy Number (TFN) represented three points: $\tilde{F} = (\tilde{f_1}, \tilde{f_2}, \tilde{f_3})$. This representation is interpreted as membership functions and satisfies the following conditions

$$\mu_{\tilde{F}}(x) = \begin{cases} 0 & \text{for } x < f_1 \\ \dfrac{(x - f_1)}{f_2 - f_1} & \text{for } f_1 \le x \le f_2 \\ \dfrac{(f_3 - x)}{f_3 - f_2} & \text{for } f_2 \le x \le f_3 \\ 0 & \text{for } x > f_3 \end{cases}$$

(i) $\tilde{f_1}$ to $\tilde{f_2}$ is a monotonically increasing continuous real-valued function.
(ii) $\tilde{f_2}$ to $\tilde{f_3}$ is monotonically decreasing continuous real-valued function
(iii) $\tilde{f_1} \le \tilde{f_2} \le \tilde{f_3}$.

2.4 Trapezoidal Fuzzy Number

The Trapezoidal Fuzzy Number (TpFN) is represented with three points: $\tilde{F} = (\tilde{f_1}, \tilde{f_2}, \tilde{f_3}, \tilde{f_4})$. This representation is interpreted as membership functions and satisfies the following conditions.

$$\mu_{\tilde{F}}(x) = \begin{cases} 0 & \text{for } x < f_1 \\ \dfrac{(x - f_1)}{f_2 - f_1} & \text{for } f_1 \le x \le f_2 \\ 1 & \text{for } f_2 \le x \le f_3 \\ \dfrac{(f_3 - x)}{f_3 - f_4} & \text{for } f_3 \le x \le f_4 \\ 0 & \text{for } x > f_4 \end{cases}$$

(i) f_1 and f_2 is real-valued monotonically increasing continuous real-valued function
(ii) f_3 and f_4 is real-valued monotonically decreasing continuous real-valued function
(iii) $f_1 < f_2 \le f_3 < f_4$

2.5 Models and Description
2.5.1 Non-pre preemptive priority with unequal service rates

Let us consider a single server two-class non-preemptive priority queue with different service rates. The inter-arrival rate of λ_1 and λ_2 are distributed independently. The service rates μ_1 and μ_2 are also distributed independently. FCFS queue discipline is followed, whereas the low priority customer gets prior service than the high priority customers. From the classical queueing theory,

$$L_q^{(1)} = \frac{\lambda_1 \left[\frac{\lambda_1}{\mu_1^2} + \frac{\lambda_2}{\mu_2^2}\right]}{(1 - \rho_1)}$$

$$L_q^{(2)} = \frac{\lambda_2 \left[\frac{\rho_1}{\mu_1} + \frac{\rho_2}{\mu_2}\right]}{(1 - \rho_1)(1 - \rho)}$$

where,

$\rho = \rho_1 + \rho_2$

$\rho_1 = \frac{\lambda_1}{\mu_1}$,

$\rho_2 = \frac{\lambda_2}{\mu_2}$

$\rho = \rho_1 + \rho_2 < 1$ is mandatory for its steady-state.

3. New Proposed Ranking Method

To change the fuzzy values into real crisp values, we use the following new proposed ranking method.

$$R(\tilde{F}) = \frac{(\tilde{f}_{min} + \tilde{f}_{max})}{2}$$

whereas $\tilde{f}_{min}$ and $\tilde{f}_{max}$ are the minimum and maximum values of the given fuzzy number.

4. Numerical Illustration

Let us assume a critical situation happens in Prabhu Medical Clinic in Chennai, where some outpatients have arrived in need of medical treatment as a matter of urgency. In case of emergency, the doctor allows patients immediately to receive his attention and treatment (non-preemptive priority only). We now calculate the average queue length of that two-class non-preemptive priority outpatients queue on this contingency.

4.1 Triangular Fuzzy Number

Let $\lambda_1 = [3, 5, 7]$ and $\lambda_2 = [4, 6, 8]$ are the arrival rate and $\mu_1 = [21, 23, 25]$ and $\mu_2 = [22, 24, 26]$ are two different service rates respectively.

$$R(\lambda_1) = \frac{7 + 3}{2} = 5$$

$R(\lambda_2) = \frac{4+8}{2} = 6$

$R(\mu_1) = \frac{21+25}{2} = 23$

$R(\mu_2) = \frac{22+26}{2} = 24$

$\rho_1 = \frac{\lambda_1}{\mu_1} = 0.2174$

$\rho_2 = \frac{\lambda_2}{\mu_2} = 0.2500$

$\rho = \rho_1 + \rho_2 = 0.4674$

4.1.1 Results

No. of. Priority	Average Queue Length (L_q)
First Priority	0.1268
Second Priority	0.2861

4.2. Trapezoidal Fuzzy Number

Let $\lambda_1 = [11, 13, 15, 17]$ and $\lambda_2 = [12, 14, 16, 18]$ are the arrival rate and $\mu_1 = [27, 29, 31, 33]$ and $\mu_2 = [28, 30, 32, 34]$ are two different service rates respectively.

$R(\lambda_1) = \frac{11+17}{2} = 14$

$R(\lambda_2) = \frac{12+18}{2} = 15$

$R(\mu_1) = \frac{27+33}{2} = 30$

$R(\mu_2) = \frac{28+34}{2} = 31$

$\rho_1 = \frac{\lambda_1}{\mu_1} = 0.4667$

$\rho_2 = \frac{\lambda_2}{\mu_2} = 0.4839$

$\rho = \rho_1 + \rho_2 = 0.9506$

4.2.1. Results

No. of. Priority	Average Queue Length (L_q)
First Priority	0.81826
Second Priority	17.7438

5. Conclusion

This paper examines the average queue length of the two-class non-preemptive priority queue with unequal service rate. The crisp values of the fuzzy arrival rate and the fuzzy service rate were calculated by a new ranking method. It is more efficient than other existing ranking method. It can be further applied for queuing models in the future.

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