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Research Article

$\psi-$operator Proximity in i-Topological Space

*A. **Yiezi Kadham Mahdi AL Talkany[1], Luay A.A.AL-swidi[2]***

[1] Department of mathematics ,*college of education for girle* university of kufa, Najaf , iraq
yieziK.alTalkany@uokufa.edu-iq,
[2] Department of mathematics ,college of education for pure science, Babylon university, Babylon , Iraq
pure.leal.abd@uobabylon.edu.iq

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Abstract: another form of ψ-operator defined in this paper by using employing two pillars they are i-topological spaces and the proximity spaces

Keywords: ψ-operator, proximity spaces, i-topological spaces, focal set , occlusion set

1. INTRODUCTION

In 1909, the researcher Riesz[1] presented the idea of proximity spaces in his theory "theory of enchainment", but this idea did not receive attention at that time. Then this idea was presented and developed by the Russian scientist Efremovic [3] and presented by the name of infinitesmma1 spaces in a series of research, and then he generalized the concept of proximity spaces by using the meaning of proximity neighborhood After that, the proximity spaces witnessed a clear development through the many and varied researches included the concept like [4,15,19,20].
As for the concept of ideal and ideal topology, it was introduced by the scientist *K. Kuratowski* in 1933[2], and this topic has witnessed many different researches that dealt with various aspects of this topic such as [23,13,12].
The i- topology, it is another form of topological spaces defined by the use of the family T with the ideal I , as it was defined by *Irina Zvina* in 2006[6].
The Ψ-operator was defined by T. Natkaniec [5] which is defined as the complement of the local function in ideal topological spaces , where different types and studies wer presented of ψ -operator and enrich this topic in the ideal topological spaces, and we will shed light on some of the researchers who worked in the fuzzy and soft topological spaces and proximity spaces And of them [9,10,17]

In this paper, we will study the effects that can have on the ψ - operator in the i- topological space using proximity spaces and using a set of principles that were previously studied

2. *Preliminaries*

Definition (2-1) [21]:

let (X,T,I) *be a space then a subset ,* A *of* X *is called focal set if there exist* $U\in T(x)\,,x\in X$ *such that* $U\alpha A$.

Definition (2-2) [21]:

let $(X,T,I\ )$ *is* $\mathfrak{i}$*-topology then , The focal closure of a set A is denoted by* $\mathcal{F}_{cl}\,(A)$ *and defined by* $\mathcal{F}cl(A)\ =\ A\cup\mathcal{F}d(A)$.

Definition (2-3) [21]:

let (X,T,I) *be a space and* $A\subseteq X$ *Then* $i-cl(A)$ *is the intersection of all* i*- closed sets suppress of A.*

Definition (2-4) [22]:

let (X,T,I) *is* i *–Topological space and* (X,δ) *is a proximity space and A* $\subseteq$ *X Then A is called Focal dense iff* $\mathcal{F}_{cl}$ *(A)* $=$ *X and is denoted by* $\mathcal{F}\mathrm{O}$*dense.*

Definition (2-5) [22]:

Let (X,T, I) is i-Topological space and (X,δ) is a proximity space then X is called Focal resolvable if there exist non empty disjoint $\mathcal{F}O$ dense sets A, B such that X $=$ A $\cup$ B

Definition (2-6): [21]

let (X,T,I) be a space and (X,δ) is a proximity space a point $x \in$ X is called occlusion point of a subset B of X if for each $U \in I_{\oint}(x), x \in U \text{ and } U \,\delta\, B$.

3. ψ-operator proximity

Definition (3-1) :

Let (X,T,I) is i – topological space and (X,δ) is a proximity space , then ψ-operator proximity defined by

$$\psi_\delta(A) = \{x \in X: \exists U \in I_{\oint}(x) \text{ and } u \ll A\}$$

.where the relation $u \ll A$ means that $u \,\bar{\delta}\, A$.

proposition (3-2) :

let (X,T,I_j) (resp. (X,T_j,I)), $j = 1,2$, are i – topological space such that $I_1 \subseteq I_2$ (resp. $T_1 \sqsubseteq T_2$) then $\psi_{\delta_{I_1}}(A) \subseteq \psi_{\delta_{I_2}}(A)$.

proposition (3-3) :

let (X,T,I) is i – topological space and (X,δ) a proximity space , all of the Suffix phrase are Verified :

1. $\psi_\delta(A) \subseteq \cup\{u \in I_{\oint}(x): u - A \in I\}$.
2. $\psi_\delta(A) \supseteq \cup\{u \in I_{\oint}(x): u \ll A\}$.
3. $\psi_\delta(A) \subseteq \cup\{u \in I_{\oint}(x): x \in A, u \approx A\}$.

Proposition (3-4) :

Let (X,T,I) is i – topological space and (X,δ) is a proximity space then $\psi_\delta(A) \subseteq Fcl(A)$ and the converse is not true.

Proof :

Let $x \in \psi_\delta(A)$ so there exist $u \in I_{\oint}(x), u \ll A$, if possible that $x \notin Fcl(A)$ so there exist u $\in I_{\oint}(x)$ such that $x \in u,$ A $\cap\, u = \emptyset$,hence $A^c \in I_{\oint}(x)$,and this is contradiction , Then $x \in Fcl(A)$.

Proposition (3-5) :

Let (X,T,I) is i – topological space , and (X,δ) is a proximity space . Then $\psi_\delta(A) = \emptyset$ for each A of I .

Proof :

Suppose that $\psi_\delta(A) \neq \emptyset$ ***, then there exist*** $x \in \psi_\delta(A)$ ***so there is*** $U \in I_\oint(x)\,, U \cap A^C = \emptyset$ ***, and this is contradiction***

There are several properties of ψ ***-operator as in the following theorem***

Theorem (3-6) :

Let (X, T, I) *is i – topological space and* (X, δ) *is a proximity space then each of the following are holds :*

1. *If* $\psi_\delta(A) \neq \emptyset$ *then* $\psi_\delta(A) \in I_\oint(x)$.

2. *For each* $A \in T \,.\, \psi_\delta(A) \neq \emptyset$

3. *For each A in X,* $\psi_\delta(A)$ *is i – open set .*

4. $A \subseteq \psi_\delta(A)$ *for each i – open set A of X .*

5. *If* $A \subseteq B$ *then* $\psi_\delta(A) \subseteq \psi_\delta(B)$.

6. $\psi_\delta(A \cap B) = \psi_\delta(A) \cap \psi_\delta(B)$.

7. $\psi_\delta(A) \cup \psi_\delta(B) \subseteq \psi_\delta(A \cup B)$.

8. $\psi_\delta(\,\psi_\delta(A)) = \psi_\delta(A)$ *for any subset A of X.*

9. $X - \oint(X - A) \subseteq \psi_\delta(A)$.

10. *If* $A \subseteq X$ *and* $B \in I$ *then*

$\psi_\delta(A - B) = \psi_\delta(A) = \psi_\delta(A \cup B)$

.

11. $\psi_\delta(A) \subseteq \oint(A)$.

12. *If A is i – close set then* $\psi_\delta(A) - A = \emptyset$.

13. *If* $A \in T$ *, then* $A \subseteq \psi_\delta(A)$.

Proof :

6. *let* $x \in \psi_\delta(A) \cap \psi_\delta(B)$ *this mean that there exist* $U, W \in I_\oint(x)$ *, Such that* $U \ll A$ *and* $W \ll B$ *and by proposition (3-5) (2) [21]* $U \cap W \in I_\oint(x)$ *,and so* $U \cap W \ll A$ *and* $U \cap W \ll B$ *, there for* $U \cap W \ll A \cap B$ *, hence* $x \in \psi_\delta(A \cap B)$.

8. *let* $x \in \psi_\delta(\psi_\delta(A))$ *, then*

$x \in \cup \{u \in I_\oint(x), x \in \psi_\delta(A), u \approx \psi_\delta(A)\}$

and hence $x \in \psi_\delta(A)$ *.Conversely exist by (2) .*

10. *since* $A \cap B^c \subseteq A$ *so* $\psi_\delta(A-B) \subseteq \psi_\delta$ *, Now let* $x \in \psi_\delta(A)$ *if possible that x* $\notin \psi_\delta(A-B)$ *, then for each* $U \in I_\oint(x)$ *we have* $U\ \delta\ (A^c \cup B)$ *hence* $U\ \delta\ A^c$ *and this is contradiction, also if* $U\ \delta\ B$ *, but* $B^c \in I_\oint(x)$ *and this is contradiction from that we get* $x \in \psi_\delta(A-B)$

Now since $A \subseteq A \cup B$ *then* $\psi_\delta(A) \subseteq \psi_\delta(A \cup B)$*,let* $x \in \psi_\delta(A \cup B)$ *then there exist* $U \in I_\oint(x)$ *such that* $U\ \bar{\delta}\ (A^c \cap B^c)$ *but* $U \cap B^c \in I_\oint(x)$ *and* $U \cap B^c \subseteq A$*, so if possible* $x \notin \psi_\delta(A)$ *then for each* $U \in I_\oint(x)$ *,* $u\ \delta\ X-A$ *,and this is contradiction , From that we get* $\psi_\delta(A) = \psi_\delta(A \cup B)$.

Proposition (3-7) :

Let (X,T,I) *is i – topological space and* (X,δ) *is a proximity space if* $A-B \cup B-A \in I$ *then* $\psi_\delta(A) = \psi_\delta(B)$.

Proof :

Since $A-B \in I$ *or B* $-A \in I$ *, let* $A-B = H_1$ *and* $B-A = H_2$

Then

$$B = (A \cap B) \cup (B-A) = (A-H_1) \cup H_2$$

and by proposition (3-10) (10) we have $\psi_\delta(A) = \psi_\delta(A-H_1)$ *so* $\psi_\delta(B)\ \psi_\delta\big((A-H_1) \cup H_2\big) = \psi_\delta(A)$.

Proposition (3-8) :

Let (X,T,I) *is i – topological space and* (X,δ) *is a proximity space if* $A \cap B = \emptyset$ *for each subsets* A, B *of X , then* $\oint(A) \cap \psi_\delta(B) = \emptyset$ *and* $\oint(B) \cap \psi_\delta(A) = \emptyset$.

Proof :

Suppose that $\oint(A) \cap \psi_\delta(B) \neq \emptyset$ *, then there exist* $x \in \oint(A) \cap \psi_\delta(B)$ *Hence* $x \in \oint(A)$ *and this mean that for each* $u \in I_\oint(x)$ *,* $u\ \delta\ A$ *and then* $u\ \delta\ B^c$ *and* $x \in \psi_\delta(B)$ *this mean that there exist* $u \in I_\oint(x)$ $u \ll B$ *and this is contraction hence* $\oint(A) \cap \psi_\delta(B) = \emptyset$.

Proposition (3-9) :

Let (X,T,I) *is i – topological space and* (X,δ) *is a proximity space if* $A, B \in T$ *such that* $A \cap B = \emptyset$*, then* $\oint(A) \cap B = \emptyset$ *and* $\oint(B) \cap A = \emptyset$.

Proposition (3-10) :

Let (X,T,I) *is i – topological space and* (X,δ) *is a proximity space then A is Focal dense iff* $\psi_\delta(A^c) = \emptyset$.

Proof :

Let A is Focal dense hence $Fcl(A) = X$ *Suppose that* $\psi_\delta(A^c) \neq \emptyset$ *, then there exist* $x \in \psi_\delta(A^c)$ *and then there exist* $u \in I_\oint(x)$ *,* $u \ll A^c$ *so* $u\ \bar{\delta}\ A$ *and thenif but x*$\in Fcl(A)$ *hence if x*$\in Fd(A)$ *so for each focal*

set u of x and $x \in u$ *,* $u \cap A /_{x} \neq \emptyset$*, but* $u \bar{\delta} A$ *for some u and this is contradiction . we get a contradiction also if* $x \in A$ *and* $x \notin Fd(A)$ *hence there exist* $u \in I_{\oint}(x)$ *,* $x \in u,\ u \cap A /_{x} = \emptyset$ *,hence* $\psi_{\delta}(A^{c}) = \emptyset$*.*
Conversely, suppose that $Fcl(A) \neq X$ *then there exist* $x \in X$ *,* $x \notin Fcl(A)$*, this mean that there exist* $u \in I_{\oint}(x)$ *such that* $u \sqsubseteq A^{c}$ *, hence* $u \sqsubseteq \psi_{\delta}(u) \sqsubseteq \psi_{\delta}(A^{c})$ *so* $u = \emptyset$ *and this is contradiction . then* $x \in Fcl(A)$ *for each* $x \in X$*.*
Some of examples discussed some cases in this paper are below

Example (3-11) :
Let
$X = \{a, b, c\}$, $T = \{X, \emptyset, \{a\}, \{b\}\}$, $I = \{\emptyset, \{c\}\}$
and δ *defined on X as follow* $A\delta B$ *iff* $\cap B \neq \emptyset$ *.Then if* $A = \{a\}$ *then we have that* $\cup \{u \in I_{\oint}(x) : u - A \in I\} \nsubseteq \psi_{\delta}\{a\}$ *Also if* $B = \{a, b\}$ *then* $\psi_{\delta}(B) \nsubseteq \cup \{u \in I_{\oint}(x) : u \ll B\}$*.*

<u>C</u> Example (3-12) :
Let
$X = \{a, b, c\}$, $T = \{X, \emptyset, \{a, b\}, \{a, c\}\}$, $I = \{\emptyset, \{c\}\}$
and δ ***defined as follow*** $A\delta B$ ***iff*** $\cap B \neq \emptyset$ ***,then for A*** $= \{c\}$ *then* $\psi_{\delta}(A) = \emptyset$ ***and*** $Fcl\{c\} = \{c\}$ ***.clearly that*** $Fcl\{c\} \nsubseteq \psi_{\delta}(A)$
Example (3-13) :
Let
$X = \{a, b, c\}$, $T = \{X, \emptyset, \{a, b\}, \{a, c\}\}$, $I = \{\emptyset, \{c\}\}$
and (X, δ) ***defined by*** $A\delta B$ ***iff*** $A \cap B \neq \emptyset$ ***.If A*** $= \{c\}$, $B = \{b, c\}$, ***then*** $\psi_{\delta}(A) = \psi_{\delta}\{b.c\}$***, But*** $A - B \cup B - A \notin I$*.*

4. conclusion :

1. In this paper the definition of the $\psi -$ *operator in the topological space was presented and it became clear to us the clear effect of proximity spaces on some characteristics and anthologies related to the operator.*
2. This definition can be applied to a group of subjects presented by a group of researchers such as[7,8,11,14,18].

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