On t-Neighbourhoods in Trigonometric Topological Spaces

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Abstract: In this paper we introduce a new type of neighbourhoods, namely, t-neighbourhoods in trigonometric topological spaces and study their basic properties. Also, we discuss the relationship between neighbourhoods and t-neighbourhoods. Further, we give the necessary condition for t-neighbourhoods in trigonometric topological spaces. .

Keywords: t-open; t-closed; t-neighbourhood

1. Introduction

In this paper, we introduce t-neighbourhoods in Trigonometric topological spaces. These spaces are based on Sine and Cosine topologies. In a bitopological space we have consider two different topologies but in a trigonometric topological space the two topologies are derived from one topology. From this, we observe that trigonometric topological space is different from bitopological space.

Section 2 deals with the preliminary concepts. In section 3, t-neighbourhoods are introduced and study their basic properties.

2. Preliminaries

Throughout this paper X denotes a set having elements from $[0, \frac{\pi}{2}]$. If (X,τ) is a topological space, then for any subset A of X, $X\setminus A$ denotes the complement of A in X. The following definitions are very useful in the subsequent sections.

Definition: 2.1 [2] Let X be any non-empty set having elements from $[0, \frac{\pi}{2}]$ and τ be the topology on X. Let SinX be the set consisting of the Sine values of the corresponding elements of X. Define a function f_s:X→SinX by $f_s(x)$=Sin x. Then f_s is a bijective function. This implies, $f_s(\phi)=\phi$ and $f_s(X)$=Sin X. That is, Sin $\phi=\phi$.
Let τ_s be the set consisting of the images (under f_s) of the corresponding elements of τ. Then τ_s form a topology on $f_s(X)$=SinX. This topology is called a Sine topology (briefly, Sin-topology) of X. The space $(SinX,\tau_s)$ is said to be a Sine topological space corresponding to X.
The elements of τ_s are called Sin-open sets. The complement of Sin-open sets is said to be Sin-closed. The set of all Sin-closed subsets of SinX is denoted by τ_s^c.

Definition: 2.2 [2] Let X be any non-empty set having elements from $[0, \frac{\pi}{2}]$ and τ be the topology on X. Let CosX be the set consisting of the Cosine values of the corresponding elements of X. Define a function f_c:X→CosX by $f_c(x)$=Cos x. Then f_c is bijective. Also, $f_c(\phi)=\phi$ and $f_c(X)$=CosX. This implies, Cos$\phi=\phi$.

Let τ_{cs} be the set consisting of the images (under f_c) of the corresponding elements of τ. Then τ_{cs} form a topology on CosX. This topology is called Cosine topology (briefly, Cos-topology) of X. The pair $(CosX,\tau_{cs})$ is called the Cosine topological space corresponding to X. The elements of τ_{cs} are called Cos-open sets. The complement of the Cos-open set is said to be Cos-closed. The set of all Cos-closed subsets of Cos X is denoted by τ_{cs}^c .

Definition: 2.3 [2] Let X be a non-empty set having elements from $[0, \frac{\pi}{2}]$. Define $T_u(X)$ by $T_u(X)$=SinX∪CosX and $T_i(X)$ by $T_i(X)$=SinX∩CosX.

Definition: 2.4 [2] Let X be a non-empty set having elements from $[0, \frac{\pi}{2}]$ and τ be the topology on X. We define a set $\mathcal{T}$={ϕ, U∪V∪T_i(X) : U∈τ_s and V∈τ_{cs}}. Then $\mathcal{T}$ form a topology on T_u(X). This topology is called trigonometric topology on T_u(X). The pair (T_u(X),$\mathcal{T}$) is called a trigonometric topological space. The elements of $\mathcal{T}$ are called trigonometric open sets (briefly, t-open sets). The complement of a trigonometric open set is said to be a trigonometric closed (briefly, t-closed) set. The set of all trigonometric closed sets is denoted by $\mathcal{T}^c$.

3. t-neighbourhoods

In this section we study about t-neighbourhoods in Trigonometric topological spaces. Throughout this section T_u(X) denotes the trigonometric topological space with trigonometric topology $\mathcal{T}$.

Definition: 3.1 Let T_u(X) be a trigonometric topological space. A subset N of T_u(X) is said to be a t-neighbourhood (briefly, t-nbd) of y∈T_u(X) if there exists a t-open set M such that y∈M⊆N.

Definition: 3.2 Let T_u(X) be a trigonometric topological space. A subset N of T_u(X) is said to be a t-neighbourhood (briefly, t-nbd) of a subset A of T_u(X) if there exists a t-open set M such that A⊆M⊆N.

Example: 3.3 Let X=$\{\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}\}$ with topology τ=$\{\phi, \{\frac{\pi}{6}\}, \{\frac{\pi}{2}\}, \{\frac{\pi}{6}, \frac{\pi}{2}\}, X\}$. Then T_u(X)=$\{\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 1, 0\}$ and $\mathcal{T}$={ϕ, T_i(X), $\{\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\}$, $\{0, \frac{1}{\sqrt{2}}\}$, $\{\frac{1}{2}, \frac{1}{\sqrt{2}}\}$, $\{1, \frac{1}{\sqrt{2}}\}$, CosX, $\{\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\}$, $\{\frac{1}{2}, \frac{1}{\sqrt{2}}, 0\}$, $\{1, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\}$, $\{1, \frac{1}{\sqrt{2}}, 0\}$, SinX, $\{\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 0\}$, $\{1, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 0\}$, $\{1, \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{\sqrt{3}}{2}\}$, $\{1, \frac{1}{\sqrt{2}}, \frac{1}{2}, 0\}$, T_u(X)}. Let N=$\{\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\}$. Then N is a t-nbd of $\frac{\sqrt{3}}{2}$.

Proposition: 3.4 Let T_u(X) be a trigonometric topological space. If N is a proper subset of T_i(X), then N is not a t-nbd of any point of T_u(X).

Proof: Assume that N is a proper subset of T_i(X). Suppose that N is a t-nbd of y∈T_u(X). Then there exists a t-open set M such that y∈M⊆N. This implies, M is a proper subset of T_i(X). This contradicts the fact that every t-open set containing T_i(X). Therefore, N is not a t-nbd of any point of T_u(X).

Definition: 3.5 Let T_u(X) be a trigonometric topological space and N be a subset of X. Define the set N* by N*=SinN∪CosN∪T_i(X). Then N* is a subset of T_u(X).

Proposition: 3.6 Let T_u(X) be a trigonometric topological spaces and N,M be a subset of X. Then

1. If N is open in X, then N* is t-open in T_u(X),
2. If N⊆M, then N*⊆M*.

Proof: The proof follows directly from the definition.

Proposition: 3.7 Let T_u(X) be a trigonometric topological space. If N is a neighbourhood of x, then N* is a t-nbd of Sin x and Cos x.

Proof: Assume that N is a neighbourhood of x. Then there exists an open set M such that x∈M⊆N. This implies, Sin x∈Sin M⊆Sin N and Cos x∈Cos M⊆Cos N. This implies, Sin x∈Sin M∪Cos M∪T_i(X)⊆Sin N∪Cos N∪T_i(X) and Cos x∈Sin M∪Cos M∪T_i(X)⊆Sin N∪Cos N∪T_i(X). That is, Sin x∈M*⊆N* and Cos x∈M*⊆N*. Since M is open in X, we have M* is t-open. Therefore, N* is a t-nbd of Sin x and Cos x.

Proposition: 3.8 Let T_u(X) be a trigonometric topological space. If N is a neighbourhood of any point x∈X, then N* is a t-nbd of every point of T_i(X).

Proof: Assume that the subset N of X is a neighbourhoods of x. Then N* is a t-nbd of Sin x and Cos x. Then by Proposition 3.7, N* contains $T_i(X)$. Therefore, for each $y\in T_i(X)$, we have $y\in T_i(X)\subseteq N^*$. Hence N* is a t-nbd of every point of $T_i(X)$.

Proposition: 3.9 Let $T_u(X)$ be a trigonometric topological space. Then $T_i(X)$ is a t-nbd of each of its points.
Proof: For each point $x\in T_i(X)$, there exists a t-open set $T_i(X)$ such that $x\in T_i(X)\subseteq T_i(X)$. Therefore, $T_i(X)$ is a t-nbd of each of its points.

Proposition: 3.10 Let $T_u(X)$ be a trigonometric topological space. Then N is a t-open set if and only if N is a t-nbd of each of its points.

Proof: Assume that N is t-open. Let $y\in N$. Then N is a t-open set and $y\in N\subseteq N$. This implies, N is a t-nbd of y. Since $y\in N$ is arbitrary, we have N is a t-nbd of each of its points. Conversely, assume that N is a t-nbd of each of its points. Then for each point y_i of N, there exists a t-open set M_i such that $y_i\in M_i \subseteq N$. This implies, N is the union of M_i. Therefore, N is t-open.

Remark: 3.11 If N is a t-nbd of some of its points, then N need not be a t-open set. For example, Consider $X=\{\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{2}\}$ with $\tau=\{\phi,X\}$. Then $T_u(X)=\{\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2},1,0\}$ and $\mathcal{T}=\{\phi,T_i(X),SinX,CosX,T_u(X)\}$. Let $N=\{\frac{1}{\sqrt{2}},1,0\}$ be a subset of $T_u(X)$. Then N is a t-nbd of $\frac{1}{\sqrt{2}}$. But it is not a t-open set.

Proposition: 3.12 Let $T_u(X)$ be a trigonometric topological space. If A is a t-closed subset of $T_u(X)$ and $y\notin A$, then there exists a t-nbd N of y such that $N\cap A=\phi$.

Proof: Let A be a t-closed set and $y\notin A$. Let $N=T_u(X)\setminus A$. Then N is a t-open set containing y. Since every t-open set is a t-nbd of each of its points, we have N is a t-nbd of y. Also, $N\cap A=\phi$.

Definition: 3.13 Let $T_u(X)$ be a trigonometric topological space and $y\in T_u(X)$. The set of all t-nbd of y is called the t-nbd system at y and is denoted by t-N(y).

Proposition: 3.14 Let $T_u(X)$ be a trigonometric topological space and $y\in T_u(X)$. Then $t\text{-}N(y)\neq\phi$ for every $y\in T_u(X)$.

Proof: Since $T_u(X)$ is the t-open set, we have $T_u(X)$ is the t-nbd of each of its points. Therefore, for every point y of $T_u(X)$, $t\text{-}N(y)\neq\phi$.

Proposition: 3.15 Let $T_u(X)$ be a trigonometric topological space and $y\in T_u(X)$. If $N\in t\text{-}N(y)$, then $y\in N$.

Proof: If $N\in t\text{-}N(y)$, then N is a t-nbd of y. This implies, $y\in N$.

Proposition: 3.16 Let $T_u(X)$ be a trigonometric topological space and $y\in T_u(X)$. If $N\in t\text{-}N(y)$ and $N\subseteq M$, then $M\in t\text{-}N(y)$.

Proof: Assume that $N\in t\text{-}N(y)$ and $N\subseteq M$. Then N is a t-nbd of y. Therefore, there exists a t-open set W such that $y\in W\subseteq M$. This implies, M is a t-nbd of y. Hence $M\in t\text{-}N(y)$.

Proposition: 3.17 Let $T_u(X)$ be a trigonometric topological space and $y\in T_u(X)$. If $N\in t\text{-}N(y)$ and $M\in t\text{-}N(y)$, then $N\cup M$, $N\cap M\in t\text{-}N(y)$.

Proof: Assume that $N\in t\text{-}N(y)$ and $M\in t\text{-}N(y)$. Then there exist t-open sets A and B such that $y\in A\subseteq N$ and $y\in B\subseteq M$. This implies, $y\in A\cap B\subseteq N\cap M$ and $y\in A\cup B\subseteq N\cup M$. Since A and B are t-open, we have $A\cap B$ and $A\cup B$ are t-open. Therefore, $N\cap M$ and $N\cup M$ are t-nbd of y. Hence $N\cup M$, $N\cap M\in t\text{-}N(y)$.

Proposition: 3.18 Let $T_u(X)$ be a trigonometric topological space and $y \in T_u(X)$. If $N \in$ t-N(y), then there exists $M \in$ t-N(y) such that $M \subseteq N$ and $M \in$ t-N(x) for every $x \in M$.

Proof: Assume that $N \in$ t-N(y). Then there exists a t-open set M such that $y \in M \subseteq N$. Since M is t-open, we have M is a t-nbd of each of its points. Therefore, $M \in$ t-N(y) and $M \in$ t-N(x) for every $x \in M$.

4. Conclusion:

In this paper we have introduced t-neighbourhoods in Trigonometric Topological Spaces and studied some of their basic properties.

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