gη - Separation Axioms In Topological Spaces

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ABSTRACT: In this paper a new class of separation axioms in topological spaces is used in gη-closed sets. The concept of gη-T_k spaces for k = 0, 1, 2, gη-D_k spaces for k = 0, 1, 2 and gη-R_k spaces for k = 0, 1 and some of their properties are investigated.

Keywords gη-closed sets, gη-open sets, gη-T_k spaces for k = 0, 1, 2 gη-D_k spaces for k = 0, 1, 2 and gη-R_k spaces for k = 0, 1.

1. INTRODUCTION

In recent years a number of generalizations of open sets have been developed by many mathematicians. In 1963, Levine [5] introduced the notion of semi-open sets in topological spaces. In 1984, Andrijevic [1] introduced some properties of the topology of α-sets. In 2016, Sayed and Mansour introduced [6] new near open set in Topological Spaces. The aim of this paper is to introduce new types of separation axioms [2, 3, 4, 7] via gη-open sets, and investigate the relations among these concepts.

2. PRELIMINARIES

Definition : 2.1

A subset A of topological space (X, τ) is called
(i) η-open set if $A \subseteq int(cl(int(A))) \cup cl(int(A))$, η-closed set if $cl(int(cl(A))) \cap int(cl(A)) \subseteq A$.
(ii) gη-closed set if $\eta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition : 2.2

A topological space (X, τ) is said to be
(i) gη-T_0 if for each pair of distinct points x, y in X, there exists a gη-open set U such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.
(ii) gη-T_1 if for each pair of distinct points x, y in X, there exist an two gη-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$.
(iii) gη-T_2 if for each pair of distinct points x, y in X, there exist an two disjoint gη-open sets U and V containing x and y respectively.

Example :2.3

(i) Let X = {a, b, c} with the topology τ = {X, φ, {b, c}}. Here gη-open sets are {X, φ, {b}, {c}, {b, c}}. Since for the distinct points {b} and {c}, there exist a gη-open set U = {b} such that $b \in U$ and $c \notin U$ or U = {c} such that $b \notin U$ and $c \in U$. Therefore X is gη-T_0 space.
(ii) Let X = {a, b, c} with the topology τ = {X, φ, {a}}. Here gη-open sets are {X, φ, {a}, {b}, {c}, {a, b}, {a, c}}. For the distinct points {a} and {c} there exist an two gη-open sets U = {a} and V = {c} such that $a \in U$ but $c \notin U$ and $a \notin V$ but $c \in V$. In a similar manner for any two distinct points gη-open sets may be found out. Therefore X is gη-T_1 space.
(iii) Let X = {a, b, c} with the topology τ = {X, φ, {a}}. Here gη-open sets are {X, φ, {a}, {b}, {c}, {a, b}, {a, c}}. Since for the distinct points {a} and {c} there exist an two disjoint gη-open sets U = {a} and V = {c} containing {a} and {c} satisfying gη-T_2 conditions. And this is true for other pairs of distinct points. Therefore X is gη-T_2 space.

Remark : 2.4 Let (X, τ) be a topological space, then the following statements are true:
(i) Every gη-T_2 space is gη-T_1.
(ii) Every gη-T_1 space is gη-T_0 .

Theorem :2.5 A topological space (X, τ) is gη-T_0 if and only if for each pair of distinct points x, y of X, gη-cl({x}) ≠ gη-cl({y}).
Proof:
Necessity: Let (X, τ) be a gη-T_0 space and x, y be any two distinct points of X. There exists a gη-open set U containing x or y, say x but not y. Then X – U is a gη-closed set which does not contain x but contains y. Since gη-cl({y}) is the smallest gη-closed set containing y, gη-cl({y}) ⊆ X – U and therefore x ∉ gη-cl({y}). Consequently gη-cl({x}) ≠ gη-cl({y}).
Sufficiency: Suppose that x, y ∈ X, x ≠ y and gη-cl({x}) ≠ gη-cl({y}). Let z be a point of X such that z ∈ gη-cl({x}) but z ∉ gη-cl({y}). We claim that x ∉ gη-cl({y}). For if x ∈ gη-cl({y}) then gη-cl({x}) ⊆ gη-cl({y}). This contradicts the fact that z ∉ gη-cl({y}). Consequently x belongs to the gη-open set X – gη-cl({y}) to which y does not belong to. Hence (X, τ) is a gη-T_0 space.

Theorem : 2.6 In a topological space (X, τ), if the singletons are gη-closed then X is gη-T_1 space and the converse is true if gη-O(X, τ) is closed under arbitrary union.
Proof: Let {z} is gη-closed for every z ∈ X. Let x, y ∈ X with x ≠ y. Now x ≠ y implies y ∈ X –{x}. Hence X – {x} is a gη-open set that contains y but not x. Similarly X – {y} is a gη-open set containing x but not y. Therefore X is a gη-T_1 space.
Conversely, let (X, τ) be gη-T_1 and x be any point of X. Choose y ∈ X – {x} then x ≠ yand so there exists a gη-open set U such that y ∈ U but x ∉ U. Consequently y ∈ U ⊆ X – {x}, that is X – {x} = ∪ {U_y : y ∈ X – {x}} which is gη-open. Hence {x} is gη-closed.

Theorem : 2.7 Let (X, τ) be a topological space, then the following statements are true:
(i) X is gη-T_2.
(ii) Let x ∈ X. For each y ≠ x, there exists a gη-open set U containing x such that y ∉ gη-cl({U}).
(iii) For each x ∈ X, ∩{ gη-cl({U}) : U ∈ gη-O(X, τ) and x ∈ U } = {x}.
Proof:(i) ⇒ (ii) Let x ∈ X, and for any y ∈ X such that x ≠ y, there exist disjoint gη-open sets U and V containing x and y respectively, since X is gη-T_2 . So U ⊆ X – V. Therefore, gη-cl({U}) ⊆ X – V. So y ∉ gη-cl({U}).
(ii) ⇒ (iii) If possible for some y ≠ x, y ∈ ∩{ gη-cl({U }) : U ∈ gη-O(X, τ) and x ∈ U }. This implies y ∈ gη-cl({U}) for every gη-open set U containing x, which contradicts (ii). Hence ∩{ gη-cl({U }) : U ∈ gη-O(X, τ) and x ∈ U} = {x}.
(iii) ⇒ (i) Let x, y ∈ X and x ≠ y. Then there exists a gη-open set U containing x such that y ∉ gη-cl({U}). Let V = X – gη-cl({U}), then y ∈ V and x ∈ U and also U ∩ V = φ. Therefore X is gη-T_2.

Definition : 2.8 A subset A of a topological space X is called a gη-difference set (briefly gη-D set) if there exists U, V ∈ gη-O(X, τ)such that U ≠ X and A = U – V.

Theorem : 2.9 Every proper gη-open set is a gη-D set.
Proof: Let U be a gη-open set different from X. Take V = φ. Then U = U – V is a gη-D set. But, the converse is not true as shown in the following example.

Example :2.10 Let X = {a, b, c} with τ = {X, φ, {a}, {c}, {a, c}}. Here gη-O(X, τ) = {X, φ, {a}, {c}, {a, b}, {a, c}, {b, c}}, then U = {a, b}≠ X and V = {a, c} are gη-open sets in X. Let A = U – V = {a, b} – {a, c}= {b}. Then A = {b} is a gη-D set but it is not gη-open.

Definition : 2.11 A topological space (X, τ) is said to be
(i) gη-D_0 if for any pair of distinct points x and y of X there exists a gη-D set of X containing x but not y or a gη-D set of X containing y but not x.
(ii) gη-D_1 if for any pair of distinct points x and y of X there exists a gη-D set of X containing x but not y and a gη-D set of X containing y but not x.

(iii) gη-D_2 if for any pair of distinct points x and y of X there exists two disjoint gη-D sets of X containing x and y, respectively.

Remark :2.12 For a topological space (X, τ), the following properties hold:

(i) If (X, τ) is gη-T_k , then it is gη-D_k , for k = 0, 1, 2.
(ii) If (X, τ) is gη-D_k , then it is gη-D_{k-1} for k = 1, 2.
(iii)

Theorem :2.13 A topological space (X, τ) is gη-D_0 if and only if it is gη-T_0 .

Proof: Suppose that X is gη-D_0. Then for each distinct pair x, y ∈ X, at least one of x, y say x, belongs to a gη-D set P but y ∉ P. As P is gη-D set, P can be written as P = $U_1 – U_2$ where $U_1 \neq X$ and U_1, U_2 ∈ gη-O(X, τ). Then x ∈ U_1, and for y ∉ P we have two cases: (i) y ∉ U_1, (ii) y ∈ U_1 and y ∈ U_2. In case (i), x ∈ U_1 but y ∉ U_1. In case (ii), y ∈ U_2 but $x \notin U_2$ .Thus in both the cases, we obtain that X is gη-T_0.

Conversely, if X is gη-T_0, by Remark 2.12 (i) X is gη-D_0.

Theorem : 2.14 Suppose gη-O(X, τ) is closed under arbitrary union, then X is gη-D_1 if and only if it is gη-D_2.

Proof:

Necessity: Let x, y ∈ X and x ≠ y. Then there exist a gη-D sets P_1, P_2 in X such that x ∈ P_1, y ∉ P_1 and y ∈ P_2, x ∉ P_2. Let $P_1 = U_1 – U_2$ and $P_2 = U_3 – U_4$, Where U_1, U_2 , U_3 and U_4 are gη-open sets in X. From x ∉ P_2, the following two cases arise: Case (i): x ∉ U_3. Case (ii): x ∈ U_3 and x ∈ U_4.

Case (i): x ∉ U_3. By y ∉ P_1we have two sub cases:

(a) y ∉ U_1 . Since x ∈ $U_1 – U_2$, it follows that x ∈ $U_1 – (U_2 \cup U_3)$, and since y ∈ $U_3 – U_4$ we have y ∈ $U_3 – (U_1 \cup U_4)$, and $(U_1 – (U_2 \cup U_3)) \cap (U_3 – (U_1 \cup U_4)) = φ$.
(b) y ∈ $U_1$and y ∈ U_2 . We have x ∈ $U_1 – U_2$, and y ∈ U_2, and $(U_1 – U_2) \cap U_2 = φ$.

Case (ii): x ∈ U_3 and x ∈ U_4. We have y ∈ $U_3 – U_4$, and x ∈ U_4. Hence $(U_3 – U_4) \cap U_4 = φ$. Thus both case (i) and in case (ii), X is gη-D_2.

Sufficiency: Follows from Remark 2.12(ii).

Corollary :2.15 If a topological space (X, τ) is gη-D_1, then it is gη-T_0.

Proof: Follows from 2.12 (ii) and Theorem 2.13.

Definition :2.16 A point x ∈ X which has only X as the gη-neighbourhood is called a gη-neat point.

Proposition:2.17 For a gη-T_0 topological space (X, τ) which has at least two elements, the following are equivalent:

(i) (X, τ) is gη-D_1 space.
(ii) (X, τ) has no gη-neat point.

Proof:(i) ⇒ (ii): Since (X, τ) is a gη-D_1 space, each point x of X is contained in a gη-D set A = U – V and thus in U. By definition U ≠ X. This implies that x is not a gη-neat point. Therefore (X, τ) has no gη-neat point.

(ii) ⇒ (i): Let X be a gη-T_0, then for each distinct pair of points x, y ∈ X, atleast one of them, x (say) has a gη-neighbourhood U containing x and not y. Thus U which is different from X is a gη-D set. If X has no gη-neat point, then y is not gη-neat point. This means that there exists a gη-neighbourhood V of y such that V ≠ X. Thus y ∈ V – U but not x and V – U is a gη-D set. Hence X is gη-D_1.

Definition :2.18 A topological space (X, τ) is said to be gη-symmetric if for any pair of distinct points x and y in X, x ∈ gη-cl({y}) implies y ∈ gη-cl({x}).

Theorem :2.19 If (X, τ) is a topological space, then the following are equivalent:

(i) (X, τ) is a gη-symmetric space.
(ii) {x} is gη-closed, for each x ∈ X.

Proof:(i) ⇒ (ii): Let (X, τ) be a gη-symmetric space. Assume that {x} ⊆ U ∈ gη-O(X, τ), but gη-cl({x}) ⊄ U. Then gη-cl({x}) ∩ (X – U) ≠ φ. Now, we take y ∈ gη-cl({x}) ∩ (X – U), then by hypothesis x ∈ gη-cl({y}) ⊆ X – U that is, x ∉ U, which is a contradiction. Therefore {x} is gη-closed, for each x ∈ X.

(iii) ⇒ (i) Assume that x ∈ gη-cl({y}), but y ∉ gη-cl({x}). Then {y} ⊆ X – gη-cl({x}) and hence gη-cl({y}) ⊆ X – gη-cl({x}). Therefore x ∈ X – gη-cl({x}), which is a contradiction and hence y ∈ gη-cl({x}).
(iv)

Corollary :2.20 Let gη-O(X, τ)be closed under arbitrary union. If the topological space (X, τ) is a gη-T_1 space, then it is gη-symmetric.

Proof: In a gη-T_1 space, every singleton set is gη-closed by theorem 2.6 therefore, by theorem 2.19, (X, τ) is gη-symmetric.

Corollary :2.21 If a topological space (X, τ) is gη-symmetric and gη-T_0, then (X, τ) is gη-T_1 space.
Proof: Let x ≠ y and as (X, τ) is gη-T_0, we may assume that x ∈ U ⊆ X – {y}for some U ∈ gη-O(X, τ). Then x ∉ gη-cl({y}) and hence y ∉ gη-cl({x}). There exists a gη-open set V such that y ∈ V ⊆ X – {x} and thus (X, τ) is a gη-T_1 space.

Corollary : 2.22 For a gη-symmetric space (X, τ), the following are equivalent:
(i) (X, τ) is gη-T_0 space.
(ii) (X, τ) is gη-D_1 space.
(iii) (X, τ) is gη-T_1 space.
Proof: (i) ⇒ (iii): Follows from Corollary 2.21.
(iv) ⇒ (ii) ⇒ (i): Follows from Remark 2.12 and Corollary 2.15.

Definition :2.23 A topological space (X, τ) is said to be gη-R_0 if U is a gη-open set and x ∈ U then gη-cl({x}) ⊆ U.
Theorem : 2.24 For a topological space (X, τ) the following properties are equivalent:
(i) (X, τ) is gη-R_0 space.
(ii) For any P ∈ gη-C (X, τ), x ∉ P implies P ⊆ U and x ∉ U for some U∈ gη-O (X, τ).
(iii) For any P ∈ gη-C (X, τ), x ∉ P implies P ∩ gη-cl({x}) = φ.
(iv) For any two distinct points x and y of X, either gη-cl({x}) = gη-cl({y}) or gη-cl({x}) ∩ gη-cl({y}) = φ.
Proof: (i) ⇒ (ii) Let P ∈ gη-C(X, τ) and x ∉ P. Then by (1), gη-cl({x}) ⊆ X – P. Set U = X – gη-cl({x}), then U is a gη-open set such that P ⊆ U and x ∉ U.
(ii) ⇒ (iii) Let P ∈ gη-C(X, τ) and x ∉ P. There exists U ∈ gη-O(X, τ) such that P ⊆ U and x ∉ U. Since U ∈ gη-O (X, τ), U ∩ gη-cl({x}) = φ and P ∩ gη-cl({x}) = φ.
(iii) ⇒ (iv) Suppose that gη-cl({x}) ≠ gη-cl({y}) for two distinct points x, y ∈ X. There exists z ∈ gη-cl({x}) such that z ∉ gη-cl({y}) [or z ∈ gη-cl({y}) such that z ∉ gη-cl({x})]. There exists V ∈ gη-O(X, τ) such that y ∉ V and z ∈ V, hence x ∈ V. Therefore, we have x ∉ gη-cl({y}). By (iii), we obtain gη-cl({x}) ∩ gη-cl({y}) = φ.
(iv) ⇒ (i) Let V ∈ gη-O(X, τ) and x ∈ V. For each y ∉ V, x ≠ y and x ∉ gη-cl({y}). This shows that gη-cl({x}) ≠ gη-cl({y}). By (iv), gη-cl({x}) ∩ gη-cl({y}) = φ for each y ∈ X – V and hence gη-cl({x}) ∩ [∪ gη-cl({y}) : y ∈ X – V] = φ. On the other hand, since V ∈ gη-O(X, τ) and y ∈ X – V, we have gη-cl({y}) ⊆ X – V and hence X – V = ∪ {gη-cl({y}) : y ∈ X – V}. Therefore, we obtain (X – V) ∩ gη-cl({x}) = φ and gη-cl({x}) ⊆ V. This shows that (X, τ) is a gη-R_0 space.

Theorem : 2.25 If a topological space (X, τ) is gη-T_0 space and a gη-R_0 space then it is gη-T_1 space.
Proof: Let x and y be any two distinct points of X. Since X is gη-T_0, there exists a gη-open set U such that x ∈ U and y ∉ U. As x ∈ U, gη-cl({x}) ⊆ U as X is gη-R_0 space. Since y ∉ U, so y ∉ gη-cl({x}). Hence y ∈ V = X – gη-cl({x}) and it is clear that x ∉ V. Hence it follows that there exist gη-open sets U and V containing x and y respectively, such that y ∉ U and x ∉ V. This implies that X is gη-T_1 space.

Theorem : 2.26 For a topological space (X, τ) the following properties are equivalent:
(i) (X, τ) is gη-R_0 space.
(ii) x ∈ gη-cl({y}) if and only if y ∈ gη-cl({x}), for any two points x and y in X.
Proof: (i) ⇒ (ii) Assume that X is gη-R_0. Let x ∈ gη-cl({y}) and V be any gη-open set such that y ∈ V. Now by hypothesis, x ∈ V. Therefore, every gη-open set which contain y contains x. Hence y ∈ gη-cl({x}).
(ii) ⇒ (i) Let U be a gη-open set and x ∈ U. If y∉ U, then x ∉ gη-cl({y}) and hence y ∉ gη-cl({x}). This implies that gη-cl({x}) ⊆ U. Hence (X, τ) is gη-R_0 space.

Remark :2.27 From Definition 2.18 and Theorem 2.26 the notion of gη-symmetric and gη-R_0 are equivalent.
Theorem : 2.28 A topological space (X, τ) is gη-R_0 space if and only if for any two points x and y in X, gη-cl({x}) ≠ gη-cl({y}) implies gη-cl({x}) ∩ gη-cl({y}) = φ.
Proof:
Necessity: Suppose that (X, τ) is gη-R_0 and x and y ∈ X such that gη-cl({x}) ≠ gη-cl({y}). Then, there exists z ∈ gη-cl({x}) such that z ∉ gη-cl({y}) [or z ∈ gη-cl({y}) such that z ∉ gη-cl({x})]. There exists V ∈ gη-O (X, τ) such that y ∉ V and z ∈ V, hence x ∈ V. Therefore, we have x ∉ gη-cl({y}). Thus x ∈ [X – gη-cl({y})] ∈ gη-O (X, τ), which implies gη-cl({x}) ⊆ [X – gη-cl({y})] and gη-cl({x}) ∩ gη-cl({y}) = φ.

Sufficiency: Let V ∈ gη-O (X, τ) and let x ∈ V. To show that gη-cl({x}) ⊆ V. Let y ∉ V, that is y ∈ X–V. Then x ≠ y and x ∉ gη-cl({y}). This shows that gη-cl({x}) ≠ gη-cl({y}). By assumption, gη-cl({x}) ∩ gη-cl({y}) = φ. Hence y ∉ gη-cl({x})and therefore gη-cl({x}) ⊆ V. Hence (X, τ) is gη-R_0 space.

Definition:2.29 A topological space (X, τ) is said to be gη-R_1 if for x, y in X with gη-cl({x}) ≠ gη-cl({y}), there exist disjoint gη-open sets U and V such that gη-cl({x})⊆ U and gη-cl({y}) ⊆ V.

Theorem : 2.30 A topological space (X, τ) is gη-R_1 space if it is gη-T_2 space.
Proof: Let x and y be any two points X such that gη-cl({x}) ≠ gη-cl({y}). By remark 2.4 (i), every gη-T_2 space is gη-T_1 space. Therefore, by theorem 2.6, gη-cl({x}) = {x}, gη-cl({y}) = {y} and hence {x} ≠ {y}. Since (X, τ) is gη-T_2, there exist a disjoint gη-open sets U and V such that gη-cl({x}) = {x} ⊆ U and gη-cl({y}) = {y} ⊆ V. Therefore (X, τ) is gη-R_1 space.

Theorem : 2.31 If a topological space (X, τ) is gη-symmetric, then the following are equivalent:
(i) (X, τ) is gη-T_2 space.
(ii) (X, τ) is gη-R_1 space and gη-T_1 space.
(iii) (X, τ) is gη-R_1 space and gη-T_0 space.
Proof: (i) ⇒ (ii) and (ii) ⇒ (iii) obvious.
(iii) ⇒ (i) Let x, y ∈ X such that x ≠ y. Since (X, τ) is gη-T_0 space. By theorem 2.5, gη-cl({x}) ≠ gη-cl({y}), since X is gη-R_1, there exist disjoint gη-open sets U and V such that gη-cl({x}) ⊆ U and gη-cl({y}) ⊆ V. Therefore, there exist disjoint gη-open set U and V such that x ∈ U and y ∈ V. Hence (X, τ) is gη-T_2 space.

Remark :2.32 For a topological space (X, τ) the following statements are equivalent.
(i) (X, τ) is gη-R_1 space.
(ii) If x, y ∈ X such that gη-cl({x}) ≠ gη-cl({y}), then there exist gη-closed sets P_1 and P_2 such that x ∈ P_1, y ∉ P_1, y ∈ P_2, x ∉ P_2, and X = $P_1 \cup P_2$.
(iii)

Theorem : 2.33 A topological space (X, τ) is gη-R_1 space, then (X, τ) is gη-R_0 space.
Proof: Let U be a gη-open such that x ∈ U. If y ∉ U, then x ∉ gη-cl({y}), therefore gη-cl({x}) ≠ gη-cl({y}). So, there exists a gη-open set V such that gη-cl({y}) ⊆ V and x ∉ V, which implies y∉ gη-cl({x}). Hence gη-cl({x}) ⊆ U. Therefore, (X, τ) is gη-R_0 space.

Theorem:2.34 A topological space (X, τ) is gη-R_1 space if and only if x ∈ X – gη-cl({y}) implies that x and y have disjoint gη-open neighbourhoods.
Proof:
Necessity: Let (X, τ) be a gη-R_1 space. Let x ∈ X – gη-cl({y}). Then gη-cl({x}) ≠ gη-cl({y}), so x and y have disjoint gη-open neighbourhoods.
Sufficiency: First to show that (X, τ) is gη-R_0 space. Let U be a gη-open set and x ∈ U. Suppose that y ∉ U. Then, gη-cl({y}) ∩ U = φ and x ∉ gη-cl({y}). There exist a gη-open sets U_x and U_y such that x ∈ U_x, y ∈ U_y and and $U_x \cap U_y$ = φ. Hence, gη-cl({x}) ⊆ gη-cl({U_x }) and gη-cl({x}) ∩ U_y ⊆ gη-cl({U_x}) ∩ U_y = φ. [For since U_y is gη-open set, U_y^c is gη-closed set. So gη-cl({U_y^c}) = U_y^c. Also since $U_x \cap U_y$ = φ and $U_x \subseteq U_y$. So gη-cl({U_x}) ⊆ gη-cl({U_y^c}). Thus gη-cl({U_x }) ⊆ U_y^c]. Therefore, y∉gη-cl({x}). Consequently, gη-cl({x}) ⊆ U and (X, τ) is gη-R_0 space. Next, (X, τ) is gη-R_1 space. Suppose that gη-cl({x}) ≠ gη-cl({y}). Then, assume that there exists z ∈ gη-cl({x}) such that z ∉ gη-cl({y}). There exist a gη-open sets V_z and V_y such that z ∈ V_z, y ∈ V_y and $V_z \cap V_y$ = φ. Since z ∈ gη-cl({x}), x ∈ V_z . Since (X, τ) is gη-R_0 space, we obtain gη-cl({x}) ⊆ V_z, gη-cl({y}) ⊆ V_y and $V_z \cap V_y$ = φ. Therefore (X, τ) is gη-R_1 space.

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