

## Inventory Model with Demand Dependant on Unit Cost- Input Parameters as Triangular Fuzzy Numbers

R.Kasthuri<sup>a</sup>, P.Vasanthi<sup>b</sup> and S.Ranganayaki<sup>c</sup>

<sup>a,b,c</sup>

Assistant Professor, Department of Mathematics, Sri Ramakrishna Engineering College, Coimbatore.

**Article History:** Received: 11 January 2021; Accepted: 27 February 2021; Published online: 5 April 2021

**Abstract:** This paper considers an inventory model in which the shortages are backlogged and the demand is dependent on unit cost. An optimum value for average total cost is calculated by considering various input costs, lot size and maximum inventory under fuzzy environment. The process of defuzzification is done by using the signed distance method. Numerical example and sensitivity analysis is given for calculating both crisp and fuzzy values of the total cost.

**Keywords:** Inventory model-Triangular fuzzy numbers-Defuzzification-Crisp and fuzzy values.

### 1. Introduction

The incorporation of fuzzy set theory in decision making has drawn much attention in decision making. The coupling of fuzziness with inventory has many real applications and are essential to improve the decision making problems.

The main aim of the production process is to maximize the profit with the minimum available stock and costs. The inventory costs consists of four costs namely purchase cost, ordering cost, inventory carrying cost and shortage cost.

The effect of different costs results in establishing a minimum total cost. The development of an effective inventory model is based on a proper balance between the carrying cost and ordering cost which will determine the lot-size with minimum total annual cost.

Park & Vujosevic proposed the inventory model in fuzzy sense with cost parameters as fuzzy numbers. Vujosevic et al developed an inventory model using triangular and trapezoidal fuzzy numbers and applied centroid method for defuzzification to estimate the total cost.

But later it was found that the signed distance method is more precise when related to centroid method for defuzzification. Petronic.Retal calculated the total cost with backorder by using trapezoidal fuzzy numbers. Harish Nagar & Priyankasurana developed an inventory model for deteriorating items with varying demand and used pentagonal fuzzy numbers. K.Syed&L.A.Aziz proposed an inventory model without backorders using the method of signed distance for defuzzification. Urgeletti developed EOQ model using triangular fuzzy numbers. D.Dutta and Pavankumar proposed inventory model in which the demand and various costs are taken under fuzzy environment. Jeuhan Chiang et al applied signed distance method for defuzzification of an inventory model with backorders. P.K.DeApurvaRawat prepared EOQ model without shortages using fuzzy numbers and computed the annual inventory cost.

In this research work the decision variables are computed for an inventory model in which the demand varies with respect to unit cost in both crisp and fuzzy environment. Signed distance method is employed for defuzzification with numerical example.

### 2. Definition and Preliminaries

#### Definition: 2.1 Fuzzy point

Let  $\tilde{A}$  be a fuzzy set on the real line  $R$ . Then the membership function of  $\tilde{A}$  is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & x = a \\ 0, & x \neq a \end{cases}$$

#### Definition:2.2 [Triangular fuzzy number]

Let  $\tilde{A} = (u, v, w)$ ,  $u < v < w$  be a fuzzy set on  $R = (-\infty, \infty)$ . Then the membership function of the triangular fuzzy number is

$$\mu_{\tilde{A}}(x) = \left\{ \begin{array}{l} \frac{x-u}{v-u}, u \leq x \leq v \\ \frac{w-x}{w-v}, v \leq x \leq w \\ 0, otherwise \end{array} \right.$$

**Definition:2.3 [Signed distance for triangular fuzzy number]**

Let B=(p,q,r) be a triangular fuzzy number and the signed distance of B

measured from  $\tilde{O}$  is defined as 
$$d(B, \tilde{O}) = \frac{1}{4}[p+ 2q + r]$$

**3. Model Description**

**3.1. Notations**

We use the following notations and related parameters for crisp as well as fuzzy inventory model with backorders.

- AC-Annual optimum cost
- B- Each time ordering lot size
- IL-Inventory level
- SC-Each time ordering cost
- R-Demand per cycle.
- p-unit cost
- HC-Annual holding cost per item
- PC-Penalty cost per unit of time

**3.2. Assumptions**

The following assumptions are considered:

- \*Shortage cost is allowed
- \*Time plan is fixed
- \*Demand R is considered as a function of unit cost

(i.e)  $R = Ap^{-\beta}$  where  $A > 0, 0 < \beta < 1$  are constant.

**4. Karush-Kuhn Tucker Method**

Karush-Kuhn Tucker conditions also known as Kuhn-Tucker conditions is a derivative test for estimating the optimum solution of a non-linear programming problem. In this paper the Kuhn-Tucker conditions approach is used to optimize the imprecise inventory annual cost function by taking the costs like carrying cost, order cost and penalty cost as input parameters and the lot-size, inventory level and unit price as decision variables.

**5. Crisp Inventory model**

The crisp total cost function of the proposed model is given by

$$AC = pR + \frac{SC.R}{B} + \frac{IL^2 H}{2B} + \frac{(B-IL)^2 PC}{2B}$$

Let  $R = Ap^{-\beta}$ , where  $A > 0$  and  $0 < \beta < 1$  are constants.

$$AC(p, B, IL) = Ap^{1-\beta} + \frac{SC.Ap^{-\beta}}{B} + \frac{IL^2.HC}{2B} + \frac{(B-IL)^2 PC}{2B} \dots \dots \dots \rightarrow (1)$$

where p, B and IL are the decision variables.

The crisp optimal solutions are

$$Optimal \ unit \ cost \ p = \left[ \frac{PC.HC.SC\beta^2}{2A(HC + PC)(1 - \beta)^2} \right]^{\frac{1}{2-\beta}}$$

$$Optimal \ lot \ size \ B = \frac{SC.\beta}{p(1 - \beta)}$$

$$Optimal \ Inventory \ level \ IL = \frac{B.PC}{HC + PC}$$

**6. Fuzzy Inventory model**

The annual cost function under fuzzy environment with imprecise holding cost, order cost and shortage cost is given by

$$\tilde{A}C = A\tilde{p}^{1-\beta} + \frac{\tilde{S}C.A\tilde{p}^{-\beta}}{\tilde{B}} + \frac{\tilde{H}C.\tilde{I}L^2}{2\tilde{B}} + \frac{\tilde{P}C(\tilde{B}-\tilde{I}L)^2}{2\tilde{B}} \dots \dots \dots > (2)$$

The input parameters HC,PC ,SC and also the decision quantities IL,B and p are considered as triangular fuzzy numbers which can be taken as

$$SC = (SC - \delta_1, SC, SC + \delta_2), SC > \delta_1$$

$$HC = (HC - \delta_3, HC, HC + \delta_4), HC > \delta_3$$

$$PC = (PC - \delta_5, PC, PC + \delta_6), PC > \delta_5$$

Also  $\tilde{I}L = (IL - \delta_7, IL, IL + \delta_8), IL > \delta_7$

$$B = (B - \delta_9, B, B + \delta_{10}), B > \delta_9$$

$$\tilde{p} = (p - \delta_{11}, p, p + \delta_{12}), p > \delta_{11}$$

$$\tilde{A}C = (C_1, C_2, C_3)$$

The Signed distance method for defuzzification is given by

$$d(\tilde{A}C) = \frac{1}{4}[C_1 + 2C_2 + C_3]$$

By applying defuzzification, we get

$$d(\tilde{A}C) = \frac{1}{4} \left[ \begin{aligned} & \frac{A(p - \delta_{11})(p + \delta_{12})^{-\beta}}{B + \delta_{10}} \\ & + \frac{A(SC - \delta_1)(p + \delta_{12})^{-\beta}}{B + \delta_{10}} \\ & + \frac{(HC - \delta_3 + PC - \delta_5)(IL - \delta_7)^2}{2(B + \delta_{10})} \\ & + \frac{(B - \delta_9)(PC - \delta_5)}{2} \\ & - (IL + \delta_8)(PC + \delta_6) \end{aligned} \right]$$

$$+ \frac{1}{2} \left[ \begin{aligned} & Ap^{1-\beta} + \frac{ASCp^{-\beta}}{B} + \frac{(HC + PC)IL^2}{2B} + \frac{B.PC}{2} - IL.PC \end{aligned} \right]$$

$$+ \frac{1}{4} \left[ \begin{aligned} & \frac{A(p + \delta_{12})(p - \delta_{11})^{-\beta}}{B - \delta_9} + \frac{A(SC + \delta_2)(p - \delta_{11})^{-\beta}}{B - \delta_9} \\ & + \frac{(HC + \delta_4 + PC + \delta_6)(IL + \delta_8)^2}{2(B - \delta_9)} \\ & + \frac{(B + \delta_{10})(PC + \delta_6)}{2} \\ & - (IL - \delta_7)(PC - \delta_5) \end{aligned} \right]$$

On simplification we get

$$p^* = \left[ \frac{\beta^2 (HC - \delta_3 + 2HC + HC + \delta_4)(SC + \delta_2 + 2SC + SC - \delta_1)}{32A(1 - \beta)^2} \right]^{\frac{1}{(2-\beta)}}$$

$$B^* = \left[ \frac{\beta(SC + \delta_2 + 2SC + SC - \delta_1)}{4p(1 - \beta)} \right]$$

$$IL^* = \left[ \frac{\beta(SC + \delta_2 + 2SC + SC - \delta_1) (PC - \delta_5 + 2PC + PC + \delta_6)}{4p(1 - \beta)(HC - \delta_3 + PC - \delta_5 + 2(HC + PC) + HC + \delta_4 + PC + \delta_6)} \right]$$

**7. Numerical Calculation**

**7.1. Crisp annual cost function**

Let  $\beta=0.86$  and  $A=100$ . The estimated cost parameters under crisp sense are given as follows:

SC	HC	PC
131.25	0.23	4.65
137.5	0.25	4.75
146.25	0.25	4.78
152.5	0.27	5.23
120	0.22	4.25

By using the above set of crisp values, the corresponding values of decision variables are estimated. Estimated values of  $p, B \& IL$  along with total annual cost are tabulated as follows:

p	B	IL	AC
4.41	182.82	174.20	163.16
4.92	171.68	163.10	165.90
5.20	172.77	164.17	167.00
5.78	162.07	154.11	169.46
3.91	188.53	179.25	158.75

**7.2. Triangular Fuzzy model:**

Let  $\beta=0.86$ , and  $A=100$ . The triangular fuzzy numbers of the different costs say SC, HC & PC are considered as follows:

SC	HC	PC
(30,150,195)	(.06,.25,.36)	(1,5,7.6)
(45,150,205)	(.11,.25,.38)	(1.5,5,7.5)
(60,150,225)	(.09,.25,.39)	(2.6,5,6.5)
(75,150,235)	(.10,.25,.48)	(2.4,5,8.5)
(15,150,165)	(.05,.25,.34)	(.5,5,6.5)

By using the above set of fuzzy cost values, the unit price, batch size and the inventory level are calculated and there by the corresponding fuzzy inventory annual cost is computed.

The estimated values are distributed in the following table:

$\tilde{p}$	$\tilde{B}$	$\tilde{IL}$	$\tilde{AC}$
4.41	182.82	174.21	163.17
4.89	172.72	164.17	165.52
5.12	175.46	166.90	165.90
5.78	162.07	154.11	169.46
3.95	186.62	177.33	160.67

**8. Conclusion:**

In this paper, an EOQ model with shortages and a demand dependent on unit cost is considered with the various costs like the order cost, holding cost and the penalty cost are taken as triangular fuzzy numbers. The signed distance method is used to defuzzify the annual cost function and hence the crisp and fuzzy values are computed. The calculated values from the table shows that the total cost values remains same if we use the signed distance method to defuzzify the triangular fuzzy numbers. Sensitivity analysis is done to illustrate the specified model.

**9. Future work:**

In future, the work can be extended by using Trapezoidal and Pentagonal Fuzzy number to fuzzify the parameters and the result can be compared with this triangular fuzzy parameters. Also the Graded mean integration method can be applied for defuzzification and the accurate result can be concluded.

**References:**

1. Park K S, 'Fuzzy set theoretic interpretation of economic order quantity', *IEEE Transactions on systems man and cybernetics*, 17,1082-1084(1987).
2. Vujosrvicetal, 'EOQ formula when inventory cost is fuzzy,' *International journal of production Economic*, 45,499-504(1996).
3. Zadeh,L.A, 'Fuzzy sets',*Information& Control*, 8, 338-353(1965).
4. Syed J.K & Aziz L.A,'Fuzzy Inventory model without shortages using signed distance method,'*Applied Mathematics and Information Sciences*,1(2),203-209(2007).
5. UrgelettiTinarelli.G,'Inventory control Models and Problems', *European Journal of Operations Research*,Vol:14,1-12(1983).
6. Dutta.D&Pavankumar,'Fuzzy Inventory Model without shortages using trapezoidal fuzzy numbers with sensitivity analysis', *IOSR-Journal of mathematics*,Vol:4(3),32-37(2012).
7. Jershan Chiang-Shing Yao and Lee.H.M,'Fuzzy inventory with backorder defuzzification by signed distance method', *Journal of Information Science and Engineering*,Vol:21,673-679(2005).
8. De.ApurvaRawat.P.K,'A Fuzzy Inventory model without shortage using triangular fuzzy number',*Fuzzy Inf.Eng*,Vol:1,59-68(2011).
9. Kasthuri, R ,Vasanthi, P &Ranganayaki, S, 2020 'Optimization of Inventory Model-Cost parameters, Inventory and Lot-size as Fuzzy Numbers', *International Journal of Engineering and Advanced Technology*, vol:10(1), pp.365-368.
10. Vasanthi, P, Ranganayaki, S &Kasthuri, R, 2019 'Fuzzy EoQ model with Shortages using Kuhn-Tucker conditions', *International Journal of Engineering and Advanced Technology*, vol.8, issue.6, pp. 822-827.