Research Article

N_{nc} Z^*-open sets in N_{nc} Topological Spaces

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January 10, 2021

Article History: Received: 11 January 2021; Accepted: 27 February 2021; Published online: 5 April 2021

Abstract: The aim of this paper is to introduce and study the notion of $N_{nc}Z^*o$-sets in N_{nc} topological space. Some characterizations of these notions are presented.

Keywords and phrases: $N_{nc}o$-sets, $N_{nc}Z^*o$-sets, $N_{nc}Z^*c$-sets. AMS (2000) subject classification: 54D10, 54C05, 54C08.

1 Introduction

Smarandache's neutrosophic framework have wide scope of constant applications for the fields of Computer Science, Information Systems, Applied Mathematics, Artificial Intelligence, Mechanics, dynamic, Medicine, Electrical & Electronic, and Management Science and so forth [1, 2, 3, 4, 19, 20]. Topology is an classical subject, as a generalization topological spaces numerous kinds of topological spaces presented throughout the year. Smarandache [13] characterized the Neutrosophic set on three segment Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Neutrosophic topological spaces (*nts*'s) presented by Salama and Alblowi [10]. Lellies Thivagar et al. [8] was given the geometric existence of N topology, which is a non-empty set equipped with N arbitrary topologies. Lellis Thivagar et al. [9] introduced the notion of N_n-open (closed) sets in N neutrosophic crisp topological spaces. Al-Hamido et al. [5] investigate the chance of extending the idea of neutrosophic crisp topological spaces into N-neutrosophic crisp topological spaces and examine a portion of their essential properties. In 2008, Ekici [6] introduced the notion of e-open sets in topology. In 2020, Vadivel and John Sundar [16] introduced N-neutrosophic δ-open, N-neutrosophic δ-semiopen and Nneutrosophic δ-preopen sets are introduced. The purpose of this paper is to introduce and study the notion of $N_{nc}Z^*o$-sets. Also, some characterizations of these notions are presented.

2 Preliminaries

Salama and Smarandache [12] presented the idea of a neutrosophic crisp set in a set X and defined the inclusion between two neutrosophic crisp sets, the intersection (union) of two neutrosophic crisp sets, the complement of a neutrosophic crisp set, neutrosophic crisp empty (resp., whole) set as more then two types. And they studied some properties related to nutrosophic crisp set operations. However, by selecting only one type, we define the inclusion, the intersection (union), and neutrosophic crisp empty (resp., whole) set again and discover a few properties.

Definition 2.1 Let X be a non-empty set. Then H is called a neutrosophic crisp set (in short, *ncs*) in X if H has the form
$H = (H_1,H_2,H_3)$, where H_1,H_2, and H_3 are subsets of X,

The neutrosophic crisp empty (resp., whole) set, denoted by ϕ_n (resp., X_n) is an *ncs* in X defined by $\phi_n = (\phi,\phi,X)$ (resp. $X_n = (X,X,\phi)$). We will denote the set of all *ncs*'s in X as $ncS(X)$.

In particular, Salama and Smarandache [11] classified a neutrosophic crisp set as the followings.

A neutrosophic crisp set $H = (H_1,H_2,H_3)$ in X is called a neutrosophic crisp set of Type 1 (resp. 2 & 3) (in short, *ncs*-Type 1 (resp. 2 & 3)), if it satisfies $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ (resp. $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ and $H_1 \cup H_2 \cup H_3 = X$ & $H_1 \cap H_2 \cap H_3 = \phi$ and $H_1 \cup H_2 \cup H_3 = X$). $ncS_1(X)$ ($ncS_2(X)$ and $ncS_3(X)$) means set of all *ncs* Type 1 (resp. 2 and 3).

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Definition 2.2 Let $H = (H_1,H_2,H_3),M = (M_1,M_2,M_3) \in ncS(X)$. Then H is said to be contained in (resp. equal to) M, denoted by $H \subseteq M$ (resp. $H = M$), if $H_1 \subseteq M_1,H_2 \subseteq M_2$ and $H_3 \supseteq M_3$ (resp. $H \subseteq M$ and $M \subseteq H$), $H^c = (H_3, H_2^c, H_1)$,

$H \cap M = (H1 \cap M1, H2 \cap M2, H3 \cup M3)$, $H \cup M = (H1 \cup M1, H2 \cup M2, H3 \cap M3)$. Let $(Aj)j_{\in}J \subseteq ncS(X)$, where $H_j = (H_j1, H_j2, H_j3)$. Then $\cap_{j\in J} H_j$ (simply $\cap H_j$) = $(\cap H_j1, \cap H_j2, \cup H_j3)$; $\cup_{j\in J} H_j$ (simply $\cup H_j$), = $(\cup H_j1, \cup H_j2, \cap H_j3)$.

The following are the quick consequence of Definition 2.2.

Proposition 2.1 [7] Let $L,M,O \in ncS(X)$. Then

(i) $\phi_n \subseteq L \subseteq X_n$,
(ii) if $L \subseteq M$ and $M \subseteq O$, then $L \subseteq O$,
(iii) $L \cap M \subseteq L$ and $L \cap M \subseteq M$,
(iv) $L \subseteq L \cup M$ and $M \subseteq L \cup M$,
(v) $L \subseteq M$ iff $L \cap M = L$,
(vi) $L \subseteq M$ iff $L \cup M = M$.

Likewise the following are the quick consequence of Definition 2.2.

Proposition 2.2 [7] Let $L,M,O \in ncS(X)$. Then

(i) $L \cup L = L$, $L \cap L = L$ (Idempotent laws),
(ii) $L \cup M = M \cup L$, $L \cap M = M \cap L$ (Commutative laws),
(iii) (Associative laws) : $L \cup (M \cup O) = (L \cup M) \cup O$, $L \cap (M \cap O) = (L \cap M) \cap O$,
(iv) (Distributive laws:) $L \cup (M \cap O) = (L \cup M) \cap (L \cup O)$, $L \cap (M \cup O) = (L \cap M) \cup (L \cap O)$,
(v) (Absorption laws) : $L \cup (L \cap M) = L$, $L \cap (L \cup M) = L$,
(vi) (DeMorgan's laws) : $(L \cup M)^c = L^c \cap M^c$, $(L \cap M)^c = L^c \cup M^c$,
(vii) $(L^c)^c = L$,
(viii) (a) $L \cup \phi_n = L$, $L \cap \phi_n = \phi_n$,
(b) $L \cup X_n = X_n$, $L \cap X_n = L$,
(c) $X_n{}^c = \phi$, $\phi^c{}_n = X_n$,
(d) in general, $L \cup L^c = \square\ X_n$, $L \cap L^c = \square\ \phi_n$.

Proposition 2.3 [7] Let $L \in ncS(X)$ and let $(L_j)_j \in J \subseteq ncS(X)$. Then

(i) $(\bigcap L_j)^c = \bigcup L_j^c$, $(\bigcup L_j)^c = \bigcap Lc_j$,
(ii) $L \cap (\cup L_j) = \cup(L \cap L_j)$, $L \cup (\cap L_j) = \cap(L \cup L_j)$.

Definition 2.3 [11] A neutrosophic crisp topology (briefly, *ncts*) on a non-empty set X is a family τ of *nc* subsets of X satisfying the following axioms

(i) $\phi_n, X_n \in \tau$.
(ii) $H_1 \cap H_2 \in \tau\ \forall\ H_1\ \&\ H_2 \in \tau$.
(iii) $\cup_a H_a \in \tau$, for any $\{H_a : a \in J\} \subseteq \tau$.

Then (X,τ) is a neutrosophic crisp topological space (briefly, *ncts*) in X. The τ elements are called neutrosophic crisp open sets (briefly, *ncos*) in X. A *ncs* C is closed set (briefly, *nccs*) iff its complement C^c is *ncos*.

Definition 2.4 [5] Let X be a non-empty set. Then ${}_{nc}\tau_1, {}_{nc}\tau_2, \cdots, {}_{nc}\tau_N$ are N-arbitrary crisp topologies defined on X and the collection $N_{nc}\tau = \{A \subseteq X : A = (\bigcup_{j=1}^{N} H_j) \cup (\bigcap_{j=1}^{N} L_j)$, $H_j, L_j \in nc\tau_j\}$ is called N neutrosophic crisp (briefly, N_{nc})-topology on X if the axioms are satisfied:

(i) $\phi_n, X_n \in N_{nc}\tau$.
(ii) $\bigcup_{j=1}^{\infty} A_j \in N_{nc}\tau\ \forall\ \{A_j\}_{j=1}^{\infty} \in N_{nc}\tau$.
(iii) $\cap_{j=1}^{n} A_j \in N_{nc}\tau\ \forall\ \{A_j\}_{j=1}^{n} \in N_{nc}\tau$.

Then $(X,N_{nc}\tau)$ is called a N_{nc}-topological space (briefly, $N_{nc}ts$) on X. The $N_{nc}\tau$ elements are called N_{nc}-open sets ($N_{nc}os$) on X and its complement is called N_{nc}-closed sets ($N_{nc}cs$) on X. The elements of X are known as N_{nc}-sets ($N_{nc}s$) on X.

Definition 2.5 [5] Let $(X,N_{nc}\tau)$ be $N_{nc}ts$ on X and H be an $N_{nc}s$ on X, then the N_{nc} interior of H (briefly, $N_{nc}int(H)$) and N_{nc} closure of H (briefly, $N_{nc}cl(H)$) are defined as

(i) $N_{nc}int(H) = \cup\{A : A \subseteq H\ \&\ A$ is a $N_{nc}os$ in $X\}$ & $N_{nc}cl(H) = \cap\{C : H \subseteq C\ \&\ C$ is a $N_{nc}cs$ in $X\}$.
(ii) N_{nc}-regular open [14] set (briefly, $N_{nc}ros$) if $H = N_{nc}int(N_{nc}cl(H))$.
(iii) N_{nc}-pre open set (briefly, $N_{nc}Pos$) if $H \subseteq N_{nc}int(N_{nc}cl(H))$.
(iv) N_{nc}-semi open set (briefly, $N_{nc}Sos$) if $H \subseteq N_{nc}cl(N_{nc}int(H))$.

(v) N_{nc}-α-open set (briefly, $N_{nc}\alpha os$) if $H \subseteq N_{nc}int(N_{nc}cl(N_{nc}int(H)))$.
(vi) N_{nc}-γ-open set[14] (briefly, $N_{nc}\gamma os$) if $H \subseteq N_{nc}cl(N_{nc}int(H)) \cup N_{nc}int(N_{nc}cl(H))$.
(vii) N_{nc}-β-open set [15] (briefly, $N_{nc}\beta os$) if $H \subseteq N_{nc}cl(N_{nc}int(N_{nc}cl(H)))$.

The complement of an $N_{nc}ros$ (resp. N_{nc}Sos, N_{nc}Pos, $N_{nc}\alpha os$, $N_{nc}\beta os$ & $N_{nc}\gamma os$) is called an N_{nc}-regular (resp. N_{nc}-semi, N_{nc}-pre, N_{nc}-α, N_{nc}-β & N_{nc}-γ) closed set (briefly, $N_{nc}rcs$ (resp. N_{nc}Scs, N_{nc}Pcs, $N_{nc}\alpha cs$, $N_{nc}\beta cs$ & $N_{nc}\gamma c$)) in X.

The family of all $N_{nc}ros$ (resp. $N_{nc}rcs$, N_{nc}Pos, N_{nc}Pcs, N_{nc}Sos, N_{nc}Scs, $N_{nc}\alpha os$, $N_{nc}\alpha cs$, $N_{nc}\beta os$, $N_{nc}\beta cs$, $N_{nc}\gamma os$ & $N_{nc}\gamma cs$,) of X is denoted by $N_{nc}ROS(X)$ (resp. $N_{nc}RCS(X)$, N_{nc}P$OS(X)$, N_{nc}P$CS(X)$, N_{nc}S$OS(X)$, N_{nc}S$CS(X)$, $N_{nc}\alpha OS(X)$, $N_{nc}\alpha CS(X)$, $N_{nc}\beta OS(X)$, $N_{nc}\beta CS(X)$, $N_{nc}\gamma OS(X)$ & $N_{nc}\gamma CS(X)$).

Definition 2.6 [16] A set H is said to be a
(i) $N_{nc}\delta$ interior of H (briefly, $N_{nc}\delta int(H)$) is defined by $N_{nc}\delta int(H) = \cup\{A : A \subseteq H \text{ \& } A \text{ is a } N_{nc}ros\}$.
(ii) $N_{nc}\delta$ closure of H (briefly, $N_{nc}\delta cl(H)$) is defined by $N_{nc}\delta cl(H) = \cup\{x \in X : N_{nc}int(N_{nc}cl(L)) \cap H \neq \phi, x \in L \text{ \& } L \text{ is a } N_{nc}os\}$.

Definition 2.7 [16] A set H is said to be a
(i) $N_{nc}\delta$-open set (briefly, $N_{nc}\delta os$) if $H = N_{nc}\delta int(H)$.
(ii) $N_{nc}\delta$-pre open set (briefly, $N_{nc}\delta$Pos) if $H \subseteq N_{nc}int(N_{nc}\delta cl(H))$.
(iii) $N_{nc}\delta$-semi open set (briefly, $N_{nc}\delta$Sos) if $H \subseteq N_{nc}cl(N_{nc}\delta int(H))$.
(iv) $N_{nc}a$ open set (briefly, $N_{nc}aos$) if $H \subseteq N_{nc}int(N_{nc}cl(N_{nc}\delta int(H)))$.
(v) $N_{nc}\delta\beta$-open set or $N_{nc}e^*$-open set (briefly, $N_{nc}\delta\beta os$ or $N_{nc}e^*os$) if $H \subseteq N_{nc}cl(N_{nc}int(N_{nc}\delta cl(H)))$.

The complement of an $N_{nc}\delta os$ (resp. $N_{nc}\delta$Pos, $N_{nc}\delta$Sos, $N_{nc}aos$ & $N_{nc}e^*os$) is called an $N_{nc}\delta$ (resp. $N_{nc}\delta$-pre, $N_{nc}\delta$-semi, $N_{nc}a$) & $N_{nc}e^*$ closed set (briefly, $N_{nc}\delta cs$ (resp. $N_{nc}\delta$Pcs, $N_{nc}\delta$Scs, $N_{nc}\delta\ acs$ & $N_{nc}e^*cs$)) in Y .

The family of all $N_{nc}\delta os$ (resp. $N_{nc}\delta cs$, $N_{nc}\delta$Pos, $N_{nc}\delta$Pcs, $N_{nc}\delta$Sos, $N_{nc}\delta$Scs, $N_{nc}aos$, $N_{nc}\ acs$, $N_{nc}e^*os$ & $N_{nc}e^*cs$) of X is denoted by $N_{nc}\delta OS(X)$ (resp. $N_{nc}\delta CS(X)$, $N_{nc}\delta$P$OS(X)$, $N_{nc}\delta$P$CS(X)$, $N_{nc}\delta$S$OS(X)$, $N_{nc}\delta$S$CS(X)$, $N_{nc}aOS(X)$, $N_{nc}aCS(X)$, $N_{nc}e^*OS(X)$ & $N_{nc}e^*CS(X)$).

Definition 2.8 [17] Let H be an $N_{nc}s$ on a $N_{nc}ts$ X. Then H is said to be a
(i) $N_{nc}e$-open (briefly, $N_{nc}eo$) set if $H \subseteq N_{nc}cl(N_{nc}\delta int(H)) \cup N_{nc}int(N_{nc}\delta cl(H))$.
(ii) $N_{nc}e$-closed (briefly, $N_{nc}ec$) set if $N_{nc}cl(N_{nc}\delta int(H)) \cap N_{nc}int(N_{nc}\delta cl(H)) \subseteq H$.

The complement of an $N_{nc}eo$ set is called an $N_{nc}e$ closed (briefly. $N_{nc}ec$) set in X. The family of all $N_{nc}eo$ (resp. $N_{nc}ec$) set of X is denoted by $N_{nc}eOS(X)$ (resp. $N_{nc}eCS(X)$). The N_{nc} e-interior of H (briefly, $N_{nc}eint(H)$) and N_{nc} e-closure of H (briefly, $N_{nc}ecl(H)$) are defined as $N_{nc}eint(H) = \cup\{G : G \subseteq H \text{ and } G \text{ is a } N_{nc}eo \text{ set in } X\}$ & $N_{nc}ecl(H) = \cap\{F : H \subseteq F \text{ and } F \text{ is a } N_{nc}ec \text{ set in } X\}$.

Lemma 2.1 [16] Let A, B be two subsets of $(X,N_{nc}\tau)$. Then:
(i) A is $N_{nc}\delta$-open iff $A = N_{nc}int_\delta(A)$,
(ii) $X\backslash(N_{nc}int_\delta(A)) = N_{nc}cl_\delta(X\backslash A)$ and $N_{nc}int_\delta(X\backslash A) = X\backslash(N_{nc}cl_\delta(A))$,
(iii) $N_{nc}cl(A) \subseteq N_{nc}cl_\delta(A)$(resp. $N_{nc}int_\delta(A) \subseteq N_{nc}int(A)$), for any subset A of X,
(iv) $N_{nc}cl_\delta(A \cup B) = N_{nc}cl_\delta(A) \cup N_{nc}cl_\delta(B)$,$N_{nc}int_\delta(A \cap B) = N_{nc}int_\delta(A) \cap N_{nc}int_\delta(B)$.

Proposition 2.4 Let A be a subset of a space $(X,N_{nc}\tau)$. Then:
(i) $N_{nc}scl(A) = A \cup N_{nc}int(N_{nc}cl(A))$,$(N_{nc}sint(A) = A \cap N_{nc}cl(N_{nc}int(A))$
(ii) $N_{nc}pcl(A) = A \cup N_{nc}cl(N_{nc}int(A))$,$N_{nc}pint(A) = A \cap N_{nc}int(N_{nc}cl(A))$ (iii) $N_{nc}scl_\delta(X\backslash A) = X\backslash\delta - (N_{nc}sint(A)$,$N_{nc}scl_\delta(A \cup B) \subseteq N_{nc}scl_\delta(A) \cup N_{nc}scl_\delta(B)$
(iv) $N_{nc}pcl_\delta(X\backslash A) = X\backslash N_{nc}pint_\delta(A)$,$N_{nc}pcl_\delta(A \cup B) \subseteq N_{nc}pcl_\delta(A) \cup N_{nc}pcl_\delta(B)$.

Lemma 2.2 [17] Let H be an $N_{nc}s$ on a $N_{nc}ts$ X. Then the following are hold.
(i) $N_{nc}\delta$P$cl(H) = H \cup N_{nc}cl(N_{nc}\delta int(H))$ and $N_{nc}\delta$P$int(H) = H \cap N_{nc}int(N_{nc}\delta cl(H))$,
(ii) $N_{nc}\delta$S$int(H) = H \cap N_{nc}cl(N_{nc}\delta int(H))$ and $N_{nc}\delta$S$cl(H) = H \cup N_{nc}int(N_{nc}\delta cl(H))$,
(iii) $N_{nc}cl(N_{nc}\delta int(H)) = N_{nc}\delta cl(N_{nc}\delta int(H))$, (iv) $N_{nc}int(N_{nc}\delta cl(H)) = N_{nc}\delta int(N_{nc}\delta cl(H))$.

3 $N_{nc}Z^*$-open sets and $N_{nc}Z^*$-closed sets

Definition 3.1 Let $(X,N_{nc}\tau)$ be a $N_{nc}ts$. Let A be an $N_{nc}s$ in $(X,N_{nc}\tau)$. Then A is said to be a
(i) $N_{nc}Z^*$-open (briefly, $N_{nc}Z^*o$) if $A \subseteq N_{nc}cl(N_{nc}int(A)) \cup N_{nc}int(N_{nc}cl_\delta(A))$,
(ii) $N_{nc}Z^*$-closed (briefly, $N_{nc}Z^*c$) if $N_{nc}int(N_{nc}cl(A)) \cap N_{nc}cl(N_{nc}int_\delta(A)) \subseteq A$.

The family of all $N_{nc}Z^*o$ (resp. $N_{nc}Z^*c$) subsets of a space $(X,N_{nc}\tau)$ will be as always denoted by $N_{nc}Z^*OS(X)$ (resp. $N_{nc}Z^*CS(X)$).

Remark 3.1 The following holds for a space $(X,N_{nc}\tau)$.
(i) Every $N_{nc}\gamma o$ (resp. $N_{nc}eo$) set is $N_{nc}Z^*o$,
(ii) Every $N_{nc}Z^*o$ set is $N_{nc}e^*o$.

But not conversely.

Remark 3.2 The following diagram holds for a N_{nc} set of a $N_{nc}ts$ X:

$$\begin{tikzcd}
N_{nc}o \arrow[r] & N_{nc}\alpha o \arrow[r] & N_{nc}\mathrm{S}o \arrow[r] & N_{nc}\gamma o \arrow[r] \arrow[d] & N_{nc}\beta o \arrow[d] \\
 & & & N_{nc}Z^{*}o \arrow[r] & N_{nc}e^{*}o \\
N_{nc}\delta o \arrow[r] \arrow[uu] & N_{nc}ao \arrow[r] \arrow[uu] & N_{nc}\delta\mathrm{P}o \arrow[r] & N_{nc}eo \arrow[u] &
\end{tikzcd}$$

Example 3.1 Let $X = \{a,b,c,d\}$, ${}_{nc}\tau_1 = \{\phi_N,X_N,A,B,C,D\}$, ${}_{nc}\tau_2 = \{\phi_N,X_N,E,F\}$. $A = \langle\{a\},\{\phi\},\{b,c,d\}\rangle$, $B = \langle\{c\},\{\phi\},\{a,b,d\}\rangle$, $C = \langle\{a,c\},\{\phi\},\{b,d\}\rangle$, $D = \langle\{a,c,d\},\{\phi\},\{b\}\rangle$, $E = \langle\{a,b\},\{\phi\},\{c,d\}\rangle$, $F = \langle\{a,b,c\},\{\phi\},\{d\}\rangle$, then we have $2_{nc}\tau = \{\phi_N,X_N,A,B,C,D,E,F\}$. The set

(i) $\langle\{b,c\},\{\phi\},\{a,d\}\rangle$ is a $2_{nc}Z^*os$ but not $2_{nc}\gamma os$.

(ii) $\langle\{a,d\},\{\phi\},\{b,c\}\rangle$ is a $2_{nc}Z^*os$ but not $2_{nc}eos$.

(iii) $\langle\{b,d\},\{\phi\},\{a,c\}\rangle$ is a $2_{nc}e^*os$ but not $2_{nc}Z^*os$.

Proposition 3.1 Let $(X,N_{nc}\tau)$ be a $N_{nc}ts$. Then the $N_{nc}\delta$-closure of a $N_{nc}Z^*o$ set of $(X,N_{nc}\tau)$ is $N_{nc}\delta\mathrm{S}o$.

Proof. Let $A \in N_{nc}Z^*OS(X)$. Then $N_{nc}cl_\delta(A) \subseteq N_{nc}cl_\delta(N_{nc}cl(N_{nc}int(A)) \cup N_{nc}int(N_{nc}cl_\delta(A))) \subseteq N_{nc}cl_\delta(N_{nc}cl(N_{nc}int(A)))\cup N_{nc}cl_\delta(N_{nc}int(N_{nc}cl_\delta(A))) \subseteq N_{nc}cl_\delta(N_{nc}int(A))\cup N_{nc}cl_\delta(N_{nc}int(N_{nc}cl_\delta(A))) = N_{nc}cl_\delta(N_{nc}int(N_{nc}cl_\delta(A))) = N_{nc}cl_\delta(N_{nc}int_\delta(N_{nc}cl_\delta(A))) = N_{nc}cl(N_{nc}\,int_\delta(N_{nc}cl_{delta}(A)))$. Therefore $N_{nc}cl_\delta(A) \in N_{nc}\delta SOS(X)$.

Lemma 3.1 Let $(X,N_{nc}\tau)$ be a $N_{nc}ts$. Then the following statements are hold.

(i) The union of arbitrary $N_{nc}Z^*o$ sets is $N_{nc}Z^*o$,

(ii) The intersection of arbitrary $N_{nc}Z^*c$ sets is $N_{nc}Z^*c$.

Proof. (i) It is clear. ■

Remark 3.3 By the following we show that the intersection of any two $N_{nc}Z^*o$ sets is not $N_{nc}Z^*o$.

Example 3.2 In Example 3.1, the sets $\langle\{a,d\},\{\phi\},\{b,c\}\rangle$ and $\langle\{b,c,d\},\{\phi\},\{a\}\rangle$ are $N_{nc}Z^*o$ sets but the intersection $\langle\{d\},\{\phi\}, \{a,b,c\}\rangle$ is not $N_{nc}Z^*o$ set.

Definition 3.2 Let $(X,N_{nc}\tau)$ be a $N_{nc}ts$. Then:

(i) The union of all $N_{nc}Z^*o$ sets of X contained in A is called the $N_{nc}Z^*$-interior of A and is denoted by $N_{nc}Z^*int(A)$,

(ii) The intersection of all $N_{nc}Z^*c$ sets of X containing A is called the $N_{nc}Z^*$-closure of A and is denoted by $N_{nc}Z^*cl(A)$.

Theorem 3.1 Let A, B be two subsets of a $N_{nc}ts$ $(X,N_{nc}\tau)$. Then the following are hold:

(i) $N_{nc}Z^*cl(X\backslash A) = X\backslash N_{nc}Z^*int(A)$,

(ii) $N_{nc}Z^*int(X\backslash A) = X\backslash N_{nc}Z^*cl(A)$,

(iii) If $A \subseteq B$, then $N_{nc}Z^*cl(A) \subseteq N_{nc}Z^*cl(B)$ and $N_{nc}Z^*int(A) \subseteq N_{nc}Z^*int(B)$,

(iv) $x \in N_{nc}Z^*cl(A)$ iff for each a $N_{nc}Z^*o$ set U contains x, $U \cap A \neq \phi$,

(v) $x \in N_{nc}Z^*int(A)$ iff there exist a $N_{nc}Z^*o$ set W such that $x \in W \subseteq A$,

(vi) A is $N_{nc}Z^*o$ set iff $A = N_{nc}Z^*int(A)$,

(vii) A is $N_{nc}Z^*c$ set iff $A = N_{nc}Z^*cl(A)$,

(viii) $N_{nc}Z^*cl(N_{nc}Z^*cl(A)) = N_{nc}Z^*cl(A)$ and $N_{nc}Z^*int(N_{nc}Z^*int(A)) = N_{nc}Z^*int(A)$,

(ix) $N_{nc}Z^*cl(A) \cup N_{nc}Z^*cl(B) \subseteq N_{nc}Z^*cl(A \cup B)$ and $N_{nc}Z^*int(A) \cup N_{nc}Z^*int(B) \subseteq N_{nc}Z^*int(A \cup B)$, (x) $N_{nc}Z^*int(A \cap B) \subseteq N_{nc}Z^*int(A) \cap N_{nc}Z^*int(B)$ and $N_{nc}Z^*cl(A \cap B) \subseteq N_{nc}Z^*cl(A) \cap N_{nc}Z^*cl(B)$.

Remark 3.4 By the following example we show that the inclusion relation in parts (ix) and (x) of the above theorem cannot be replaced by equality.

Example 3.3 Let $X = \{a,b,c,d,e\}$, ${}_{nc}\tau_1 = \{\phi_N,X_N,A,B,C\}$, ${}_{nc}\tau_2 = \{\phi_N,X_N\}$. $A = \langle\{c\},\{\phi\},\{a,b,d,e\}\rangle$, $B = \langle\{a,b\},\{\phi\},\{c,d,e\}\rangle$, $C = \langle\{a,b,c\},\{\phi\},\{d,e\}\rangle$, then we have $2_{nc}\tau = \{\phi_N,X_N,A,B,C\}$. Then, the sets

(i) $A = \langle\{a,b\},\{\phi\},\{c,d,e\}\rangle$ and $B = \langle\{c,d\},\{\phi\},\{a,b,e\}\rangle$, then $A\cup B = \langle\{a,b,c,d\},\{\phi\},\{e\}\rangle$. $2_{nc}Z^*cl(A) = \langle\{a,b\},\{\phi\}, \{c,d,e\}\rangle$, $2_{nc}Z^*cl(B) = \langle\{c,d\},\{\phi\},\{a,b,e\}\rangle$ and $2_{nc}Z^*cl(A \cup B) = X$. Thus $2_{nc}Z^*cl(A \cup B) \not\subset 2_{nc}Z^*cl(A) \cup 2_{nc}Z^*cl(B)$.

(ii) $C = \langle\{a,c\},\{\phi\},\{b,d,e\}\rangle$ and $D = \langle\{c,d\},\{\phi\},\{a,b,e\}\rangle$, then $C\cap D = \langle\{c\},\{\phi\},\{a,b,d,e\}\rangle$. $2_{nc}Z^*cl(C) = \langle\{a,c,d,e\}, \{\phi\},\{b\}\rangle$, $2_{nc}Z^*cl(D) = \langle\{c,d\},\{\phi\},\{a,b,e\}\rangle$ and $2_{nc}Z^*cl(C \cap D) = \langle\{c\},\{\phi\},\{a,b,d,e\}\rangle$. Thus $2_{nc}Z^*cl(C) \cap 2_{nc}Z^*cl(D) \not\subset 2_{nc}Z^*cl(C \cap D)$.

(iii) $E = \langle\{a,d\},\{\phi\},\{b,c,e\}\rangle$ and $F = \langle\{b,d\},\{\phi\},\{a,c,e\}\rangle$, then $E \cup F = \langle\{a,b,d\},\{\phi\},\{c,e\}\rangle$. $2_{nc}Z^*int(E) = \langle\{a\},\{\phi\},\{b,c,d,e\}\rangle$, $2_{nc}Z^*int(F) = \langle\{b\},\{\phi\},\{a,c,d,e\}\rangle$ and $2_{nc}Z^*int(E \cup F) = \langle\{a,b,d\},\{\phi\},\{c,e\}\rangle$. Thus $2_{nc}Z^*int(E \cup F) \not\subset 2_{nc}Z^*int(E) \cup 2_{nc}Z^*int(F)$.

Theorem 3.2 Let A, B be two N_{nc} sets of a $N_{nc}ts$ $(X,N_{nc}\tau)$. Then the following are hold:

(i) $N_{nc}Z^*cl(N_{nc}cl(A) \cup B) = N_{nc}cl(A) \cup N_{nc}Z^*cl(B)$,

(ii) $N_{nc}Z^*int(N_{nc}int(A) \cap B) = N_{nc}int(A) \cap N_{nc}Z^*int(B)$.

Proof. (i) $N_{nc}Z^*cl(N_{nc}cl(A) \cup B) \supseteq N_{nc}Z^*cl(N_{nc}cl(A)) \cup N_{nc}Z^*cl(B) \supseteq N_{nc}cl(A) \cup N_{nc}Z^*cl(B)$. The other inclusion, $N_{nc}cl(A) \cup B \subseteq N_{nc}cl(A) \cup N_{nc}Z^*cl(B)$ which is $N_{nc}Z^*c$. Hence, $N_{nc}Z^*cl(N_{nc}cl(A) \cup B) \subseteq N_{nc}cl(A) \cup N_{nc}Z^*cl(B)$. Therefore, $N_{nc}Z^*cl(N_{nc}cl(A) \cup B) = N_{nc}cl(A) \cup N_{nc}Z^*cl(B)$.

(ii) It is follows from (i). ■

Theorem 3.3 Let $(X,N_{nc}\tau)$ be a $N_{nc}ts$ and $A \subseteq X$. Then A is a $N_{nc}Z^*o$ set iff $A = (N_{nc}sint(A)) \cup N_{nc}pint_\delta(A)$.

Proof. It is clear. ■

Proposition 3.2 Let $(X,N_{nc}\tau)$ be a $N_{nc}ts$ and $A \subseteq X$. Then A is a $N_{nc}Z^*c$ set iff $A = N_{nc}scl(A) \cap N_{nc}pcl_\delta(A)$.

Proof. It follows from Theorem 3.3. ■

Proposition 3.3 Let A be a N_{nc} set of a $N_{nc}ts$ $(X,N_{nc}\tau)$. Then:

(i) $N_{nc}Z^*cl(A) = N_{nc}scl(A) \cap N_{nc}pcl_\delta(A)$,

(ii) $N_{nc}Z^*int(A) = N_{nc}sint(A) \cup N_{nc}pint_\delta(A)$.

Lemma 3.2 Let A be a N_{nc} set of a $N_{nc}ts$ $(X,N_{nc}\tau)$. Then the following are hold:

(i) $N_{nc}pcl(N_{nc}pint_\delta(A)) = N_{nc}pint_\delta(A) \cup N_{nc}cl(N_{nc}int(A))$,

(ii) $N_{nc}pint(N_{nc}pcl_\delta(A)) = N_{nc}pcl_\delta(A) \cap N_{nc}int(N_{nc}cl(A))$.

Proof. (i) By Lemma 2.2 and Proposition 2.4, $N_{nc}pcl(N_{nc}pint_\delta(A)) = N_{nc}pint_\delta(A) \cup N_{nc}cl(N_{nc}int(N_{nc}pint_\delta(A))) = N_{nc}pint_\delta(A) \cup N_{nc}cl(N_{nc}int(A \cap N_{nc}cl_\delta(N_{nc}int(A)))) = N_{nc}pint_\delta(A) \cup N_{nc}cl(N_{nc}int(A))$.

(ii) It follows from (i). ■

Proposition 3.4 Let A be a N_{nc} set of a $N_{nc}ts$ $(X,N_{nc}\tau)$. Then:

(i) $N_{nc}Z^*cl(A) = A \cup N_{nc}pint(N_{nc}pcl_\delta(A))$,

(ii) $N_{nc}Z^*int(A) = A \cap N_{nc}pcl(N_{nc}pint_\delta(A))$.

Proof. (i) By Lemma 3.2, $A \cup N_{nc}pint(N_{nc}pcl_\delta(A)) = A \cup (N_{nc}pcl_\delta(A) \cap N_{nc}int(N_{nc}cl(A))) = (A \cup N_{nc}pcl_\delta(A)) \cap (A \cup N_{nc}int(N_{nc}cl(A))) = N_{nc}pcl_\delta(A) \cap N_{nc}scl(A) = N_{nc}Z^*cl(A)$.

(ii) It follows from (i). ■

Theorem 3.4 Let A be a N_{nc} set of a $N_{nc}ts$ $(X,N_{nc}\tau)$. Then the following are equivalent:

(i) A is a $N_{nc}Z^*o$ set,

(ii) $A \subseteq N_{nc}pcl(N_{nc}pint_\delta(A))$,

(iii) there exists $U \in N_{nc}\delta POS(X)$ such that $U \subseteq A \subseteq N_{nc}pcl(U)$,

(iv) $N_{nc}pcl(A) = N_{nc}pcl(N_{nc}pint_\delta(A))$.

Proof. (i) ⇒ (ii). Let A be a $N_{nc}Z^*o$ set. Then, $A = N_{nc}Z^*int(A)$ and by Proposition 3.4, $A = A \cap N_{nc}pcl(N_{nc}pint_\delta(A))$ and hence ,$A \subseteq N_{nc}pcl(N_{nc}pint_\delta(A))$.

(iii) ⇒ (i). Let $A \subseteq N_{nc}pcl(pint_\delta(A))$. Then by Proposition 3.4, $A \subseteq A \cap N_{nc}pcl(N_{nc}pint_\delta(A)) = N_{nc}Z^*int(A)$ and hence $A = N_{nc}Z^*int(A)$. Thus A is $N_{nc}Z^*o$.

(ii) ⇒ (iii). It follows from putting $U = N_{nc}pint_\delta(A)$.

(iii) → (ii). Let there exists $U \in N_{nc}\delta POS(X)$ such that $U \subseteq A \subseteq N_{nc}pcl(U)$. Since $U \subseteq A$, then $N_{nc}pcl(U) \subseteq N_{nc}pcl(N_{nc}pint_\delta(A))$ therefore $A \subseteq N_{nc}pcl(U) \subseteq N_{nc}pcl(N_{nc}pint_\delta(A))$. (iv) ⇔ (i). It is clear. ■

Theorem 3.5 Let A be a N_{nc} set of a $N_{nc}ts$ X. Then the following are equivalent: (i) A is a $N_{nc}Z^*c$ set,

(ii) $N_{nc}pint_\delta(N_{nc}pcl(A)) \subseteq A$,

(iii) there exists $U \in N_{nc}\delta PCS(X)$ such that $N_{nc}pint(U) \subseteq A \subseteq U$,

(iv) $N_{nc}pint(A) = N_{nc}pint(N_{nc}pcl_\delta(A))$.

Proof. It follows from Theorem 3.4. ■

Proposition 3.5 If A is a $N_{nc}Z^*o$ set of a $N_{nc}ts$ $(X,N_{nc}\tau)$ such that $A \subseteq B \subseteq N_{nc}pcl(A)$, then B is $N_{nc}Z^*o$.

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