*Research Article*

# $Z$-open sets in a Neutrosophic Topological Spaces

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**Abstract-**In this paper, introduce a neutrosophicopen sets in neutrosophic topological spaces. Also, discuss about near open sets, their properties and examples$Z$-open set which is a union of neutrosophic P-open sets and neutrosophic $\delta$of a neutrosophicS $Z$-open set. Moreover, we investigate some of their basic properties and examples of neutrosophic $Z$-interior and $Z$-closure in a neutrosophic topological spaces.

**Keywords and phrases:** neutrosophic $Z$-open sets, neutrosophic $Z$-closed sets, $NZint$ ($K$) and $NC$ ($K$). **AMS (2000) Subject classification:** 03E72, 54A10, 54A40

## 1 Introduction

In mathematics, concept of fuzzy set between the intervals was first introduced by Zadeh [16] in discipline of logic and set theory. The general topology has been framework with fuzzy set was undertaken by Chang [4] as fuzzy topological space. In 1983, Atanassov [2] initiated intuitionistic fuzzy set which contains a membership and non-membership values. Coker [5] created intuitionistic fuzzy set in a topology entitled as intuitionistic fuzzy topological spaces. The concepts of neutrosophy and neutrosophic set was introduced Smarandache [11, 12] at the beginning of $20^{th}$ century. Salama and Alblowi [8] in 2012, originated neutrosophic set in a neutrosophic topological space. Saha [13] defined $\delta$-open sets in fuzzy topological spaces. In 2008, Ekici [6] introduced the notion of $e$-open sets in a general topology. In 2014, Seenivasan et. al. [10] introduced fuzzy $e$-open sets in a topological space along with fuzzy $e$-continuity. Vadivel et al. [3] studied fuzzy $e$-open sets in intuitionistic fuzzy topological space. Vadivel et al. [14] introduced $e$-open sets in a neutrosophic topological space. From 2011, El-Maghrabi and Mubarki [7] introduced and studied some properties of Z-open sets and maps in topological spaces. In this paper, we develop the concept of neutrosophic $Z$-open sets in a neutrosophic topological spaces and also specialized some of their basic properties with examples. Also, we discuss about neutrosophic $Z$-interior and $Z$-closure in neutrosophic topological spaces.

## 2 Preliminaries

The needful basic definitions & properties of neutrosophic topological spaces are discussed in this section.

**Definition 2.1** [9] Let $X$ be a non-empty set. A neutrosophic set (briefly, $Ns$) $L$ is an object having the form $L = \{\langle y,\mu_L(y),\sigma_L(y),\nu_L(y)\rangle : y \in X\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\sigma_L \to [0,1]$ denote the degree of indeterminacy function and $\nu_L \to [0,1]$ denote the degree of non-membership function respectively of each element $y \in X$ to the set $L$ and $0 \le \mu_L(y) + \sigma_L(y) + \nu_L(y) \le 3$ for each $y \in X$.

**Remark 2.1** [9] A $Ns$ $L = \{\langle y,\mu_L(y),\sigma_L(y),\nu_L(y)\rangle : y \in X\}$ can be identified to an ordered triple $\langle y,\mu_L(y),\sigma_L(y),\nu_L(y)\rangle$ in [0,1] on $X$.

**Definition 2.2** [9] Let $X$ be a non-empty set & the $Ns$'s $L$ & $M$ in the form $L = \{\langle y,\mu_L(y),\sigma_L(y),\nu_L(y)\rangle : y \in X\}$, $M = \{\langle y,\mu_M(y),\sigma_M(y),\nu_M(y)\rangle : y \in X\}$, then

(i) $0_N = \langle y,0,0,1\rangle$ and $1_N = \langle y,1,1,0\rangle$,

(ii) $L \subseteq M$ iff $\mu_L(y) \le \mu_M(y)$, $\sigma_L(y) \le \sigma_M(y)$ & $\nu_L(y) \ge \nu_M(y) : y \in X$,

(iii) $L = M$ iff $L \subseteq M$ and $M \subseteq L$,

(iv) $1_N - L = \{\langle y,\nu_L(y),1-\sigma_L(y),\mu_L(y)\rangle : y \in X\} = L^c$,

(v) $L \cup M = \{\langle y,\max(\mu_L(y),\mu_M(y)),\max(\sigma_L(y),\sigma_M(y)),\min(\nu_L(y),\nu_M(y))\rangle : y \in X\}$,

(vi) $L \cap M = \{\langle y,\min(\mu_L(y),\mu_M(y)),\min(\sigma_L(y),\sigma_M(y)),\max(\nu_L(y),\nu_M(y))\rangle : y \in X\}$.

**Definition 2.3** [8] A neutrosophic topology (briefly, $Nt$) on a non-empty set $X$ is a family $\tau_N$ of neutrosophic subsets of $X$ satisfying

(i) $0_N, 1_N \in \tau_N$.

(ii) $L_1 \cap L_2 \in \tau_N$ for any $L_1,L_2 \in \tau_N$.

(iii) $\cup L_a \in \tau_N$, $\forall L_a : a \in A \subseteq \tau_N$.

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Then $(X,\tau_N)$ is called a neutrosophic topological space (briefly, *Nts*) in *X*. The $\tau_N$ elements are called neutrosophic open sets (briefly, *Nos*) in *X*. A *Ns C* is called a neutrosophic closed sets (briefly, *Ncs*) iff its complement $C^c$ is *Nos*.

**Definition 2.4** [8] Let $(X,\tau_N)$ be *Nts* on *X* and *L* be an *Ns* on *X*, then the neutrosophic interior of *L* (briefly, *Nint*(*L*)) and the neutrosophic closure of *L* (briefly, *Ncl*(*L*)) are defined as

$$Nint(L) = \cup\{I : I \subseteq L \ \& \ I \text{ is a } Nos \text{ in } X\} \quad Ncl(L) = \cap\{I : L \subseteq I \ \& \ I \text{ is a } Ncs \text{ in } X\}.$$

**Definition 2.5** [1] Let $(X,\tau_N)$ be *Nts* on *X* and *L* be an *Ns* on *X*. Then *L* is said to be a neutrosophic regular (resp. pre, semi, $\alpha$ & $\beta$) open set (briefly, *Nros* (resp. *NPos*, *NSos*, *Nαos* & *Nβos*)) if $L = Nint(Ncl(L))$ (resp. $L \subseteq Nint(Ncl(L))$, $L \subseteq Ncl(Nint(L))$, $L \subseteq Nint(Ncl(Nint(L)))$ & $L \subseteq Ncl(Nint(Ncl(L)))$).

The complement of an *NPos* (resp. *NSos*, *Nαos*, *Nros* & *Nβos*) is called a neutrosophic pre (resp. semi, $\alpha$, regular & $\beta$) closed set (briefly, *NPcs* (resp. *NScs*, *Nαcs*, *Nrcs* & *Nβcs*)) in *X*.

The family of all *NPos* (resp. *NPcs*, *NSos*, *NScs*, *Nαos*, *Nαcs*, *Nβos* & *Nβcs*) of *X* is denoted by *NPOS*(*X*) (resp. *NPCS*(*X*), *NSOS*(*X*), *NSCS*(*X*), *NαOS*(*X*), *NαCS*(*X*), *NβOS*(*X*) & *NβCS*(*X*)).

**Definition 2.6** [14] A set *L* is said to be a neutrosophic

(i) $\delta$ interior of *L* (briefly, *Nδint*(*L*)) is defined by $N\delta int(L) =\cap \cup\{I :\subseteq I \subseteq L \ \& \ I$ is a *Nros* in $X\}\}$.

(ii) $\delta$ closure of *L* (briefly, *Nδcl*(*L*)) is defined by $N\delta cl(L) = \{A : L \quad A \ \& \ A$ is a *Nrcs* in *X* .

**Definition 2.7** [14] A set *L* is said to be a neutrosophic

1. $\delta$-open set (briefly, *Nδos*) if $L = N\delta int(L)$.
2. $\delta$-semi open set (briefly, *NδSos*) if $L \subseteq Ncl(N\delta int(L))$.

The complement of an *Nδos* (resp. *NδSos* ) is called a neutrosophic $\delta$ (resp. $\delta$-semi) closed set (briefly, *Nδcs* (resp. *NδScs* )) in *X*.

The family of all *NδSos* (resp. *NδScs*) of *X* is denoted by *NδSOS*(*X*) (resp. *NδSCS*(*X*)).

**Definition 2.8** [14] A set *K* is said to be a neutrosophic

(i) *e*-open set (briefly, *Neos*) if $K \subseteq Ncl(N\delta int(K)) \cup Nint(N\delta cl(K))$.

(ii) *e*-closed set (briefly, *Necs*) if $K \supseteq Ncl(N\delta int(K)) \cap Nint(N\delta cl(K))$.

The complement of a *Neos* is called a *Necs*.

The family of all *Neos* (resp. *Necs*) of *X* is denoted by *NeOS*(*X*) (resp. *NeCS*(*X*)).

## 3 Neutrosophic *Z*-open sets in *Nts*

Throughout the sections 3 & 4, let $(X,\tau_N)$ be any *Nts*. Let *K* and *M* be a *Ns*'s in *Nts*.

**Definition 3.1** A set *K* is said to be a neutrosophic

(i) *Z*-open set (briefly, *NZos*) if $K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K))$.

(ii) *Z*-closed set (briefly, *NZcs*) if $K \supseteq Ncl(N\delta int(K)) \cap Nint(N\delta cl(K))$.

The complement of a *NZos* is called a *NZcs*.

The family of all *NZos* (resp. *NZcs*) of *X* is denoted by *NZOS*(*X*) (resp. *NZCS*(*X*)).

**Definition 3.2** A set *K* is said to be a neutrosophic

(i) *Z* interior of *K* (briefly, *NZint*(*K*)) is defined by $NZint(K) =\cap \cup\{A :\subseteq A \subseteq K \ \& \ A$ is a *NZos* in$\}X\}$.

(ii) *Z* closure of *K* (briefly, *NZcl*(*K*)) is defined by $NZcl(K) = \{A : K \quad A \ \& \ A$ is a *NZcs* in *X* .

**Proposition 3.1** The statements are hold but the converse does not true.

(i) Every *Nδos* (resp. *Nδcs*) is a *Nos* (resp. *Ncs*).
(ii) Every *Nos* (resp. *Ncs*) is a *NδSos* (resp. *NδScs*).
(iii) Every *Nos* (resp. *Ncs*) is a *NPos* (resp. *NPcs*).
(iv) Every *NδSos* (resp. *NδScs*) is a *NZos* (resp. *NZcs*).
(v) Every *NPos* (resp. *NPcs*) is a *NZos* (resp. *NZcs*).
(vi) Every *NZos* (resp. *NZcs*) is a *Neos* (resp. *Necs*).

**Proof.** The proof of (i), (ii) & (iii) are studied in [14, 15].

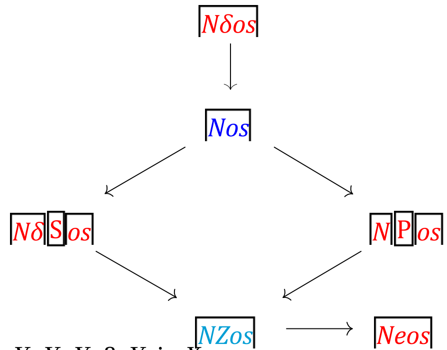
(iv) *K* is a *NδSos*, then $K \subseteq Ncl(N\delta int(K)) \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K))$. $\therefore$ *K* is a *NZos*.

(v) *K* is a *NPos*, then $K \subseteq Nint(Ncl(K)) \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K))$. $\therefore$ *K* is a *NZos*.

(vi) *K* is a *NZos* then $K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K))$. So $K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)) \subseteq Ncl(N\delta int(K)) \cup Nint(N\delta cl(K))$. $\therefore$ *K* is a *Neos*.

It is also true for their respective closed sets. Ξ

**Remark 3.1** The diagram shows *NZos*'s in *fnts*.




**Example 3.1** Let $Y = \{a,b,c\}$ and define $Ns$'s $Y_1,Y_2,Y_3$ & $Y_4$ in $X$ are

$$Y_1 = \langle Y, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.3}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.7}, \frac{\nu_c}{0.6})\rangle,$$
$$Y_2 = \langle Y, (\frac{\mu_a}{0.1}, \frac{\mu_b}{0.1}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.9}, \frac{\nu_b}{0.9}, \frac{\nu_c}{0.6})\rangle$$
$$Y_3 = \langle Y, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.2}, \frac{\mu_c}{0.3}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.8}, \frac{\nu_c}{0.7})\rangle,$$
$$Y_4 = \langle Y, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6})\rangle$$
$$Y_5 = \langle Y, (\frac{\mu_a}{0.8}, \frac{\mu_b}{0.7}, \frac{\mu_c}{0.8}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.2}, \frac{\nu_b}{0.3}, \frac{\nu_c}{0.2})\rangle,$$

Then we have $\tau_N = \{0_N,Y_1,Y_2,1_N\}$ is a $Nts$ in $X$, then

(i) $Y_3$ is a $N$P$os$ but not $Nos$.
(ii) $Y_4$ is a $NZos$ but not $N$P$os$.
(iii) $Y_5$ is a $Neos$ but not $NZos$.

**Example 3.2** Let $Y = \{a,b,c\}$ and define $Ns$'s $Y_1,Y_2$ & $Y_3$ in $X$ are

$$Y_1 = \langle Y, (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.6}, \frac{\mu_c}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.6}, \frac{\nu_b}{0.4}, \frac{\nu_c}{0.5})\rangle$$
$$Y_2 = \langle Y, (\frac{\mu_a}{0.6}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.4}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6})\rangle,$$
$$Y_3 = \langle Y, (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}, \frac{\mu_c}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.6}, \frac{\nu_b}{0.5}, \frac{\nu_c}{0.5})\rangle,$$

Then we have $\tau_N = \{0_N,Y_1,Y_2,Y_1\cup Y_2,Y_1\cap Y_2,1_N\}$ is a $Nts$ in $X$, then $Y_3$ is a $NZos$ but not ${}^{N\delta}$S$os$.

The other implications are shown in [14].

**Theorem 3.1** Let $(X,\tau_N)$ be a $Nts$. Then if $M \in N\delta OS(X)$ and $M \in NZOS(X)$, then $H \cap M$ is $NZo$.

**Proof.** Suppose that $H \in N\delta OS(X)$. Then $H = Nint_\delta(H)$. Since $M \in NZOS(X)$, then $M \subseteq Ncl(Nint_\delta(M)) \cup Nint(Ncl(M))$ and hence

$$\begin{aligned} H \cap M &\subseteq Nint_\delta(H) \cap (Ncl(Nint_\delta(M)) \cup Nint(Ncl(M))) \\ &= (Nint_\delta(H) \cap Ncl(Nint_\delta(M))) \cup (Nint_\delta(H) \cap Nint(Ncl(M))) \\ &\subseteq Ncl(Nint_\delta(H) \cap (Nint_\delta(M))) \cup Nint(Nint(H) \cap Ncl(M)) \subseteq Ncl(Nint_\delta(H \cap M)) \cup Nint(Ncl(H \cap M)). \end{aligned}$$

Thus $H \cap M \subseteq Ncl(Nint_\delta(H \cap M)) \cup Nint(Ncl(H \cap M))$. Therefore, $H \cap M$ is $NZo$.

**Proposition 3.2** Let $(X,\tau_N)$ be a $Nts$. Then the closure of a $NZo$ set of $X$ is $N$S$o$. ■

**Proof.** Let $H \in NZOS(X)$. Then

$$\begin{aligned} Ncl(H) &\subseteq Ncl(Ncl(Nint_\delta(H)) \cup Nint(Ncl(H))) \\ &\subseteq Ncl(Nint_\delta(H)) \cup Ncl(Nint(Ncl(H))) = Ncl(Nint(Ncl(H))). \end{aligned}$$

Therefore, $Ncl(H)$ is $N$S$o$.

**Theorem 3.2** The statements are true. ■

(i) $N$P$cl(K) \supseteq K \cup Ncl(Nint(K))$.
(ii) $N$P$int(K) \subseteq K \cap Nint(Ncl(K))$.
(iii) $N\delta$S$cl(K) \supseteq K \cup Nint(N\delta cl(K))$.
(iv) $N\delta$S$int(K) \subseteq K \cap Ncl(N\delta int(K))$.

**Proof.** (i) Since $N$P$cl(K)$ is $N$P$cs$, we have

$$Ncl(Nint(K)) \subseteq Ncl(Nint(N\text{P}cl(K))) \subseteq N\text{P}cl(K).$$

Thus $K \cup Ncl(Nint(K)) \subseteq N$P$cl(K)$.

The other cases are similar. Ξ

**Theorem 3.3** Let $K$ is a $NZos$ iff $K = NPint(K) \cup N\delta Sint(K)$.

**Proof.** Let $K$ is a $NZos$. Then $K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K))$. By Theorem 3.2, we have

$$NPint(K) \cup N\delta Sint(K) = K \cap (Nint(Ncl(K))) \cup (K \cap Ncl(N\delta int(K))) = K \cap (Nint(Ncl(K))) \cup Ncl(N\delta int(K)) = K.$$

Conversely, if $K = NPint(K) \cup N\delta Sint(K)$ then, by Theorem 3.2

$$K = NPint(K) \cup N\delta Sint(K)$$
$$= (K \cap Nint(Ncl(K))) \cup (K \cap Ncl(N\delta int(K)))$$
$$= K \cap (Nint(Ncl(K)) \cup Ncl(N\delta int(K))) \subseteq Nint(Ncl(K)) \cup Ncl(N\delta int(K))$$

and hence $K$ is a $NZos$. Ξ

**Theorem 3.4** The union (resp. intersection) of any family of $NZOS(X)$ (resp. $NZCS(X)$) is a $NZOS(X)$ (resp. $NZCS(X)$).

**Proof.** Let $\{K_a : a \in \tau_N\}$ be a family of $NZos$'s. For each $a \in \tau_N$, $K_a \subseteq Ncl(N\delta int(K_a)) \cup Nint(Ncl(K_a))$.

$$\bigcup_{a\in\tau_N} K_a \subseteq \bigcup_{a\in\tau_N} Ncl(N\delta int(K_a)) \cup Nint(Ncl(K_a))$$
$$\subseteq Ncl(N\delta int(\cup K_a)) \cup Nint(Ncl(\cup K_a))$$

The other case is similar. Ξ

**Remark 3.2** The intersection of two $NZos$'s need not be $NZos$.

**Example 3.3** Let $Y = \{a,b\}$ and define $Ns$'s $Y_1, Y_2$ & $Y_3$ in $X$ are

$$Y_1 = \langle Y, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.1}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.5}) \rangle$$
$$Y_2 = \langle Y, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.2}) \rangle,$$
$$Y_3 = \langle Y, (\frac{\mu_a}{0.1}, \frac{\mu_b}{0.2}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}), (\frac{\nu_a}{0.1}, \frac{\nu_b}{0.1}) \rangle.,$$

Then we have $\tau_N = \{0_N, Y_1, 1_N\}$ is a $Nts$ in $X$, then $Y_2$ & $Y_3$ are $NZos$ but $Y_2 \cap Y_3$ is not $NZos$.

**Proposition 3.3** Let $K$ is a

(i) $NZos$ and $N\delta int(K) = 0_N$, then $K$ is a $NPos$.
(ii) $NZos$ and $Ncl(K) = 0_N$, then $K$ is a $N\delta Sos$.
(iii) $NZos$ and $N\delta cs$, then $K$ is a $N\delta Sos$.
(iv) $N\delta Sos$ and $Ncs$, then $K$ is a $NZos$.

**Proof.** (i) Let $K$ be a $NZos$, that is

$$K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)) = 0_N \cup Nint(Ncl(K)) = Nint(Ncl(K))$$

Hence $K$ is a $NPos$.

(ii) Let $K$ be a $NZos$, that is

$$K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)) = Ncl(N\delta int(K)) \cup 0_N = Ncl(N\delta int(K))$$

Hence $K$ is a $N\delta Sos$.

(iii) Let $K$ be a $NZos$ and $N\delta cs$, that is

$$K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)) = Ncl(N\delta int(K)) \cup Nint(Ncl(K)) = Ncl(N\delta int(K)).$$

Hence $K$ is a $N\delta Sos$.

(iv) Let $K$ be a $N\delta Sos$ and $Ncs$, that is

$$K \subseteq Ncl(N\delta int(K)) \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)).$$

Hence $K$ is a $NZos$. Ξ

**Theorem 3.5** Let $K$ be a $NZcs$ (resp. $NZos$) iff $K = NZcl(K)$ (resp. $K = NZint(K)$).

**Proof.** Suppose $K = NZcl(K) = \cap\{A : K \subseteq A \ \&\ A \text{ is a } NZcs\}$. This means $K \in \cap\{A : K \subseteq A \ \&\ A \text{ is a } NZcs\}$ and hence $K$ is $NZcs$.

Conversely, suppose $K$ be a $NZcs$ in $X$. Then, we have $K \in \cap\{A : K \subseteq A \ \&\ A \text{ is a } NZcs\}$. Hence, $K \subseteq A$ implies $K = \cap\{A : K \subseteq A \ \&\ A \text{ is a } NZcs\} = NZcl(K)$.

Similarly for $K = NZint(K)$. Ξ

**Proposition 3.4** Let $K$ and $L$ are in $X$, then

(i) $NZcl(\overline{K}) = \overline{NZint(K)}$, $NZint(\overline{K}) = \overline{NZcl(K)}$.
(ii) $NZcl(K \cup L) \supseteq NZcl(K) \cup NZcl(L)$, $NZcl(K \cap L) \subseteq NZcl(K) \cap NZcl(L)$.
(iii) $NZint(K \cup L) \supseteq NZint(K) \cup NZint(L)$, $NZint(K \cap L) \subseteq NZint(K) \cap NZint(L)$.

**Proof.**

(i) The proof is directly from definition.

(ii) $K \subseteq K \cup L$ or $L \subseteq K \cup L$. Hence $NZcl(K) \subseteq NZcl(K \cup L)$ or $NZcl(L) \subseteq NZcl(K \cup L)$. Therefore, $NZcl(K \cup L) \supseteq NZcl(K) \cup NZcl(L)$. The other one is similar.

(iii) $K \subseteq K \cup L$ or $L \subseteq K \cup L$. Hence $NZint(K) \subseteq NZint(K \cup L)$ or $NZint(L) \subseteq NZint(K \cup L)$. Therefore, $NZint(K \cup L) \supseteq NZint(K) \cup NZint(L)$. The other one is similar.

Ξ

**Remark 3.3** The equality of (ii) in Proposition 3.4 can not be true in the given example.

**Example 3.4** Let $Y = \{a,b,c,d\}$ and define *Ns*'s $Y_1,Y_2,Y_3$ & $Y_4$ in $X$ are

$$Y_1 = \langle Y, (\frac{\mu_a}{1}, \frac{\mu_b}{0}, \frac{\mu_c}{0.2}, \frac{\mu_d}{0}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}, \frac{\sigma_d}{0.5}), (\frac{\nu_a}{0}, \frac{\nu_b}{1}, \frac{\nu_c}{0.7}, \frac{\nu_d}{1})\rangle$$

$$Y_2 = \langle Y, (\frac{\mu_a}{0}, \frac{\mu_b}{1}, \frac{\mu_c}{0}, \frac{\mu_d}{0}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}, \frac{\sigma_d}{0.5}), (\frac{\nu_a}{1}, \frac{\nu_b}{0}, \frac{\nu_c}{1}, \frac{\nu_d}{0.1})\rangle,$$

$$Y_3 = \langle Y, (\frac{\mu_a}{1}, \frac{\mu_b}{0}, \frac{\mu_c}{0}, \frac{\mu_d}{1}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}, \frac{\sigma_d}{0.5}), (\frac{\nu_a}{0}, \frac{\nu_b}{0.2}, \frac{\nu_c}{0}, \frac{\nu_d}{0})\rangle,$$

$$Y_4 = \langle Y, (\frac{\mu_a}{0}, \frac{\mu_b}{0.9}, \frac{\mu_c}{0.3}, \frac{\mu_d}{1}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}, \frac{\sigma_d}{0.5}), (\frac{\nu_a}{1}, \frac{\nu_b}{0}, \frac{\nu_c}{0.2}, \frac{\nu_d}{0})\rangle.$$

Then we have $\tau_N = \{0_N, Y_1, Y_2, Y_1 \cap Y_2, 1_N\}$ is a *Nts* in $X$, then $NZcl(Y_3 \cup Y_4) = NZcl(Y_3) \cup NZcl(Y_4)$.

**Proposition 3.5** Let $K$ be a neutrosophic set in a neutrosophic topological space $X$. Then $Nint(K) \subseteq NZint(K) \subseteq K \subseteq NZcl(K) \subseteq Ncl(K)$.

**Proof.** It follows from the definitions of corresponding operators. Ξ

**Theorem 3.6** Let $K$ and $L$ in $X$, then the *NZint* sets have

(i) $NZcl(0_N) = 0_N$, $NZcl(1_N) = 1_N$.
(ii) $NZcl(K)$ is a *NZcs* in $X$.
(iii) $NZcl(K) \subseteq NZcl(L)$ if $K \subseteq L$.
(iv) $K \subseteq NZcl(K)$.
(v) $K$ is *NZc* set in $X \Leftrightarrow NZcl(K) = K$.
(vi) $NZint(NZint(K)) = NZint(K)$.

**Proof.** The proofs (i) to (iv) and (vi) are directly from definitions of *NZcl* set.

(v) Let $K$ be *NZc* set in $X$. By using Proposition 3.4, $K$ is *NZo* set in $X$. By Proposition 3.4, $NZint(\overline{K}) = \overline{K} \Leftrightarrow$ $NZcl(K) = K \Leftrightarrow NZcl(K) = K$. Ξ

**Theorem 3.7** Let $K$ and $L$ in $X$, then the *NZint* sets have

(i) $NZint(0_N) = 0_N$, $NZint(1_N) = 1_N$.
(ii) $NZint(K)$ is a *NZos* in $X$.
(iii) $NZint(K) \subseteq NZint(L)$ if $K \subseteq L$.
(iv) $NZint(NZint(K)) = NZint(K)$.

**Proof.** The proofs are directly from definitions of *NZint* set.

Ξ

**Proposition 3.6** If $K$ and $L$ is in $X$, then (i) $NZcl(K) \supseteq K \cup NZcl(NZint(K))$.
(ii) $NZint(K) \subseteq K \cap Nint(NZcl(K))$.
(iii) $Nint(NZcl(K)) \supseteq Nint(NZcl(NZint(K)))$.

**Proof.** (i) By Theorem 3.6 $K \subseteq NZcl(K) \rightarrow (1)$. Again using Theorem 3.6, $NZint(K) \subseteq K$. Then $NZcl(NZint(K)) \subseteq NZcl(K) \rightarrow (2)$. By (1) and (2) we have, $K \cup NZcl(NZint(K)) \subseteq NZcl(K)$.

(ii) By Theorem 3.6, $NZint(K) \subseteq K \rightarrow (1)$. Again using Theorem 3.6, $K \subseteq NZint(K)$. Then $NZint(K) \subseteq NZint(NZcl(K) \rightarrow (2)$. By (1) and (2) we have, $NZint(K) \subseteq K \cup NZint(NZcl(K))$.

(iii) By Theorem 3.6, $NZcl(K) \subseteq Ncl(K)$, we get $Nint(NZcl(K)) \subseteq Nint(Ncl(K))$. Hence (iii).

(iv) By (i), $NZcl(K) \supseteq K \cup NZ(NZint(K))$. We have, $Nint(NZcl(K)) \supseteq Nint(K \cup NZcl(NZint(K)))$. Since $Nint(K \cup L) \supseteq Nint(K) \cup Nint(L)$, $Nint(NZcl(K)) \supseteq Nint(K) \cup Nint(NZcl(NZint(K))) \supseteq Nint(NZcl(NZint(K)))$. Ξ

(v)

## 4 Conclusion

We have studied about neutrosophic *Z*-open set and neutrosophic *Z*-closed set and their respective interior and closure operators of neutrosophic topological space in this paper. Also studied some of their fundamental properties along with examples in *Nts*. Also, we have discussed a near open sets of neutrosophic *Z*-open sets in *Nts*. In future, we can be extended to neutrosophic *Z* continuous mappings, neutrosophic *Z*-open mappings and neutrosophic *Z*-closed mappings in *Nts*.

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