Z-open sets in a Neutrosophic Topological Spaces

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Abstract-In this paper, introduce a neutrosophic open sets in neutrosophic topological spaces. Also, discuss about near open sets, their properties and examplesZ-open set which is a union of neutrosophic P-open sets and neutrosophic δof a neutrosophic S Z-open set. Moreover, we investigate some of their basic properties and examples of neutrosophic Z-interior and Z-closure in a neutrosophic topological spaces.

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1 Introduction


2 Preliminaries

The needful basic definitions & properties of neutrosophic topological spaces are discussed in this section.

Definition 2.1 [9] Let X be a non-empty set. A neutrosophic set (briefly, Ns) L is an object having the form L = \{ (y,μ_L(y),σ_L(y),ν_L(y)) : y ∈ X \} where \( μ_L → [0,1] \) denote the degree of membership function, \( σ_L → [0,1] \) denote the degree of indeterminacy function and \( ν_L → [0,1] \) denote the degree of non-membership function respectively of each element \( y ∈ X \) to the set \( L \) and \( 0 ≤ μ_L(y) + σ_L(y) + ν_L(y) ≤ 3 \) for each \( y ∈ X \).

Remark 2.1 [9] A Ns \( L = \{ (y,μ_L(y),σ_L(y),ν_L(y)) : y ∈ X \} \) can be identified to an ordered triple \( ⟨ y,μ_L(y),σ_L(y),ν_L(y)⟩ \) in \([0,1]\) on \( X \).

Definition 2.2 [9] Let X be a non-empty set & the Ns’s \( L \) & \( M \) in the form \( L = \{ (y,μ_L(y),σ_L(y),ν_L(y)) : y ∈ X \} \), \( M = \{ (y,μ_M(y),σ_M(y),ν_M(y)) : y ∈ X \} \), then

(i) \( 0_N = (y,0,0,1) \) and \( 1_N = (y,1,1,0) \),
(ii) \( L ⊆ M \) iff \( μ_L(y) ≤ μ_M(y), σ_L(y) ≤ σ_M(y) & ν_L(y) ≥ ν_M(y) : y ∈ X \),
(iii) \( L = M \) iff \( L ⊆ M \) and \( M ⊆ L \),
(iv) \( 1_N - L = \{ (y,ν_L(y),1 - σ_L(y),μ_L(y)) : y ∈ X \} = L^C \),
(v) \( L ∪ M = \{ (y,max(μ_L(y),μ_M(y)),max(σ_L(y),σ_M(y)),min(ν_L(y),ν_M(y))) : y ∈ X \} \),
(vi) \( L ∩ M = \{ (y,min(μ_L(y),μ_M(y)),min(σ_L(y),σ_M(y)),max(ν_L(y),ν_M(y))) : y ∈ X \} \).

Definition 2.3 [8] A neutrosophic topology (briefly, \( N τ \)) on a non-empty set \( X \) is a family \( τ_N \) of neutrosophic subsets of \( X \) satisfying

(i) \( 0_N, 1_N ∈ τ_N \),
(ii) \( L_1 ∩ L_2 ∈ τ_N \) for any \( L_1,L_2 ∈ τ_N \),
(iii) \( ∪ L_α ∈ τ_N ∩ A ∈ A ⊆ τ_N \).
Then \((X, \tau_0)\) is called a neutrosophic topological space (briefly, \(Nts\)) in \(X\). The \(\tau_0\) elements are called neutrosophic open sets (briefly, \(Nos\)) in \(X\). A \(N\) \(C\) is called a neutrosophic closed sets (briefly, \(Ncs\)) iff its complement \(C\) is \(Nos\).

**Definition 2.4** [8] Let \((X, \tau_0)\) be \(Nts\) on \(X\) and \(L\) be an \(N\) \(S\) on \(X\), then the neutrosophic interior of \(L\) (briefly, \(Nint(L)\)) and the neutrosophic closure of \(L\) (briefly, \(Ncl(L)\)) are defined as

\[
Nint(L) = \bigcup \{I : I \subseteq L & I is a Nos in X\} \quad Ncl(L) = \bigcap \{I : L \subseteq I & I is a Ncs in X\}.
\]

**Definition 2.5** [1] Let \((X, \tau_0)\) be \(Nts\) on \(X\) and \(L\) be an \(N\) \(S\) on \(X\). Then \(L\) is said to be a neutrosophic regular (resp. pre, semi, \(\alpha \& \beta\)) open set (briefly, \(Nros\) (resp. \(NPos, NSos, Naos \& \Nbetaos\)) if \(L = Nint(Ncl(L))\) (resp. \(L \subseteq Nint(Ncl(L)), L \subseteq Ncl(Nint(L)), L \subseteq Nint(Ncl(Nint(L))) \& L \subseteq Ncl(Nint(Ncl(L)))\)).

The complement of an \(NPos\) (resp. \(NSos, Naos, Nros \& \Nbetaos\)) is called a neutrosophic pre (resp. semi, \(\alpha\), regular \& \(\beta\)) closed set (briefly, \(Npcs\) (resp. \(NScs, Nacs, Nrcs \& \Nbetacs\)) in \(X\).

The family of all \(NPos\) (resp. \(NPcs, NSos, NScs, Naos, Nacs, Nrcs, \& \Nbetaos\)) of \(X\) is denoted by \(NPOS(X)\) (resp. \(NPCS(X), NSOS(X), NSCS(X), NaOS(X), NaCS(X), \& NaCS(X)\)).

**Definition 2.6** [14] A set \(L\) is said to be a neutrosophic

(i) \(\delta\) interior of \(L\) (briefly, \(N\) \(\delta\) (\(int\))\) is defined by \(N\) \(\delta\) (\(int\)) \(L\) = \(\cap \{I : I \subseteq L & I is a Nros in X\}\).

(ii) \(\delta\) closure of \(L\) (briefly, \(N\) \(\delta\) (\(cl\))\) is defined by \(N\) \(\delta\) (\(cl\)) \(L\) = \(\{A : L \subseteq A \& A is a Nrcs in X\}\).

**Definition 2.7** [14] A set \(L\) is said to be a neutrosophic

1. \(\delta\)-open set (briefly, \(N\) \(\delta\) \(os\)) if \(\delta L = N\) \(\delta\) \(int\) \(L\).

2. \(\delta\)-semi open set (briefly, \(N\) \(\delta\) \(so\)) if \(L \subseteq N\) \(\delta\) (\(cl\)) \(L\).

The complement of an \(N\) \(\delta\) \(os\) (resp. \(N\) \(\delta\) \(so\)) is called a neutrosophic \(\delta\) (resp. \(\delta\)-semi) closed set (briefly, \(N\) \(\delta\) \(cs\) (resp. \(N\) \(\delta\) \(sCs\))) in \(X\).

The family of all \(N\) \(\delta\) \(os\) (resp. \(N\) \(\delta\) \(sCs\)) of \(X\) is denoted by \(N\) \(\delta\) \(OS\)(\(X\)) (resp. \(N\) \(\delta\) \(SCS\)(\(X\))).

**Definition 2.8** [14] A set \(K\) is said to be a neutrosophic

(i) \(e\)-open set (briefly, \(Neos\)) if \(\subseteq N\) \(\delta\) (\(cl\)) \(K\).

(ii) \(e\)-closed set (briefly, \(Necs\)) if \(\subseteq N\) \(\delta\) (\(int\)) \(K\).

The complement of a \(Neos\) is called a \(Necs\).

The family of all \(Neos\) (resp. \(Necs\)) of \(X\) is denoted by \(NeOS(X)\) (resp. \(NeCS(X)\)).

3 **Neutrosophic Z-open sets in Nts**

Throughout the sections 3 & 4, let \((X, \tau_0)\) be any \(Nts\). Let \(K\) and \(M\) be a \(N\) \(s\)’s in \(Nts\).

**Definition 3.1** A set \(K\) is said to be a neutrosophic

(i) \(Z\)-open set (briefly, \(NZos\)) if \(\subseteq N\) \(\delta\) (\(cl\)) \(K\).

(ii) \(Z\)-closed set (briefly, \(NZcs\)) if \(\subseteq N\) \(\delta\) (\(int\)) \(K\).

The complement of a \(NZos\) is called a \(NZcs\).

The family of all \(NZos\) (resp. \(NZcs\)) of \(X\) is denoted by \(NZOS(X)\) (resp. \(NZCS(X)\)).

**Definition 3.2** A set \(K\) is said to be a neutrosophic

(i) \(Z\) interior of \(K\) (briefly, \(NZint(K)\)) is defined by \(NZint(K) = \cap \{A : A \subseteq K \& A is a NZos in X\}\).

(ii) \(Z\) closure of \(K\) (briefly, \(NZcl(K)\)) is defined by \(NZcl(K) = \{A : K \subseteq A \& A is a NZcs in X\}\).

**Proposition 3.1** The statements are hold but the converse does not true.

(i) Every \(N\) \(\delta\) \(os\) (resp. \(N\) \(\delta\) \(cs\)) is a \(Nos\) (resp. \(Ncs\)).

(ii) Every \(Nos\) (resp. \(Ncs\)) is a \(N\) \(\delta\) \(os\) (resp. \(N\) \(\delta\) \(cs\)).

(iii) Every \(Nos\) (resp. \(Ncs\)) is a \(NPos\) (resp. \(NPcs\)).

(iv) Every \(N\) \(\delta\) \(so\) (resp. \(N\) \(\delta\) \(sCs\)) is a \(NZos\) (resp. \(NZcs\)).

(v) Every \(N\) \(\delta\) \(os\) (resp. \(N\) \(\delta\) \(cs\)) is a \(NPos\) (resp. \(NPcs\)).

(vi) Every \(NZos\) (resp. \(NZcs\)) is a \(Neos\) (resp. \(Necs\)).

**Proof.** The proof of (i), (ii) & (iii) are studied in [14, 15].

(iv) \(K\) is a \(N\) \(\delta\) \(os\), then \(\subseteq N\) \(\delta\) (\(int\)) \(K\) \(\subseteq N\) \(\delta\) (\(cl\)) \(K\) \(\cup N\) \(\delta\) (\(int\)) \(K\) \(\cup N\) \(\delta\) (\(cl\)) \(K\). \(\therefore K\) is a \(NZos\).

(v) \(K\) is a \(N\) \(\delta\) \(os\), then \(\subseteq N\) \(\delta\) (\(cl\)) \(K\) \(\subseteq N\) \(\delta\) (\(int\)) \(K\) \(\cup N\) \(\delta\) (\(cl\)) \(K\). \(\therefore K\) is a \(NZos\).

(vi) \(K\) is a \(NZos\) then \(\subseteq N\) \(\delta\) (\(cl\)) \(K\) \(\subseteq N\) \(\delta\) (\(int\)) \(K\). \(\therefore K\) is a \(NZos\).

It is also true for their respective closed sets.

**Remark 3.1** The diagram shows \(NZos\)'s in \(Nts\).
Example 3.1 Let \( Y = \{a, b, c\} \) and define \( N \)'s \( Y_1, Y_2, Y_3 \) in \( \mathbb{X} \) are
\[
Y_1 = \langle \mu_a, \mu_b, \mu_c, \sigma_a, \sigma_b, \sigma_c, \nu_a, \nu_b, \nu_c \rangle,
\]
\[
Y_2 = \langle \mu_a, \mu_b, \mu_c, \sigma_a, \sigma_b, \sigma_c, \nu_a, \nu_b, \nu_c \rangle,
\]
\[
Y_3 = \langle \mu_a, \mu_b, \mu_c, \sigma_a, \sigma_b, \sigma_c, \nu_a, \nu_b, \nu_c \rangle,
\]
\[
Y_4 = \langle \mu_a, \mu_b, \mu_c, \sigma_a, \sigma_b, \sigma_c, \nu_a, \nu_b, \nu_c \rangle,
\]
Then we have \( \tau_N = \{0_N, Y_1, Y_2, 1_N\} \) is a \( N \)s in \( \mathbb{X} \), then
(i) \( Y_3 \) is a \( N \)Pos but not \( N \)os,
(ii) \( Y_4 \) is a \( N \)Os but not \( N \)Pos,
(iii) \( Y_5 \) is a \( N \)os but not \( N \)os.

Example 3.2 Let \( Y = \{a, b, c\} \) and define \( N \)'s \( Y_1, Y_2, Y_3 \) in \( \mathbb{X} \) are
\[
Y_1 = \langle \mu_a, \mu_b, \mu_c, \sigma_a, \sigma_b, \sigma_c, \nu_a, \nu_b, \nu_c \rangle,
\]
\[
Y_2 = \langle \mu_a, \mu_b, \mu_c, \sigma_a, \sigma_b, \sigma_c, \nu_a, \nu_b, \nu_c \rangle,
\]
\[
Y_3 = \langle \mu_a, \mu_b, \mu_c, \sigma_a, \sigma_b, \sigma_c, \nu_a, \nu_b, \nu_c \rangle,
\]
Then we have \( \tau_N = \{0_N, Y_1, Y_2, Y_3\} \) is a \( N \)s in \( \mathbb{X} \), then \( Y_3 \) is a \( N \)Os but not \( N \)os.

The other implications are shown in [14].

Theorem 3.1 Let \((X, \tau_N)\) be a \( N \)s. Then if \( M \in N\delta OS(X) \) and \( M \in N\delta OS(X) \), then \( H \cap M \) is \( N \)Zo.

Proof. Suppose that \( H \in N\delta OS(X) \). Then \( H = N\alpha_{\delta}(H) \). Since \( M \in N\delta OS(X) \), then \( M \subseteq NcI(N\alpha_{\delta}(M)) \cup N\alpha_{\delta}(NcI(M)) \)

and hence
\[
H \cap M \subseteq N\alpha_{\delta}(H) \cap (NcI(N\alpha_{\delta}(M)) \cup N\alpha_{\delta}(NcI(M)))
= (N\alpha_{\delta}(H) \cap NcI(N\alpha_{\delta}(M))) \cup (N\alpha_{\delta}(H) \cap N\alpha_{\delta}(NcI(M)))
\subseteq NcI(N\alpha_{\delta}(H) \cap N\alpha_{\delta}(M)) \cup N\alpha_{\delta}(NcI(H) \cap M) \subseteq NcI(N\alpha_{\delta}(H) \cap M) \cup N\alpha_{\delta}(NcI(H) \cap M).
\]

Thus \( H \cap M \subseteq NcI(N\alpha_{\delta}(H) \cap M) \cup N\alpha_{\delta}(NcI(H) \cap M) \). Therefore, \( H \cap M \) is \( N \)Zo.

Proposition 3.2 Let \((X, \tau_N)\) be a \( N \)s. Then the closure of a \( N \)zo set of \( X \) is \( NS \).

Proof. Let \( H \in N\delta OS(X) \). Then
\[
NcI(H) \subseteq NcI(N\alpha_{\delta}(H)) \cup N\alpha_{\delta}(NcI(H)) \subseteq NcI(N\alpha_{\delta}(H)) \cup N\alpha_{\delta}(NcI(H)) = N\alpha_{\delta}(NcI(H)).
\]

Therefore, \( NcI(H) \) is \( NS \).

Theorem 3.2 The statements are true.

(i) \( NPcI(K) \supseteq K \cup N\alpha_{\delta}(NcI(K)) \).
(ii) \( NPcI(K) \subseteq K \cap N\alpha_{\delta}(NcI(K)) \).
(iii) \( N\delta Scl(K) \supseteq K \cup N\alpha_{\delta}(NcI(K)) \).
(iv) \( N\delta Scl(K) \subseteq K \cap N\alpha_{\delta}(NcI(K)) \).

Proof. (i) Since \( NPcI(K) \) is \( NPcI(K) \), we have
\[
NcI(N\alpha_{\delta}(K)) \subseteq NcI(N\alpha_{\delta}(NPcI(K))) \subseteq NPcI(K).
\]

Thus \( K \cup NcI(N\alpha_{\delta}(K)) \subseteq NPcI(K) \).

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N. Moogambigai, A. Vadivel, and S. Tamilselvan
The other cases are similar.

**Theorem 3.3** Let $K$ is a NZos iff $K = NPint(K) \cup N\bar{o}N\bar{i}nt(K)$.

**Proof.** Let $K$ is a NZos. Then $K \subseteq Ncl(N\bar{o}int(K)) \cup N\bar{i}nt(Ncl(K))$. By Theorem 3.2, we have

\[
NPint(K) \cup N\bar{o}N\bar{i}nt(K) = K \cap (N\bar{i}nt(Ncl(K)) \cup (K \cap Ncl(N\bar{o}int(K))) = K \cap (N\bar{i}nt(Ncl(K)) \cup Ncl(N\bar{o}int(K)) = K.
\]

Conversely, if $K = NPint(K) \cup N\bar{o}N\bar{i}nt(K)$ then, by Theorem 3.2
\[
K = NPint(K) \cup N\bar{o}N\bar{i}nt(K)
= (K \cap N\bar{i}nt(Ncl(K))) \cup (K \cap Ncl(N\bar{o}int(K)))
= K \cap (N\bar{i}nt(Ncl(K)) \cup Ncl(N\bar{o}int(K)) \subseteq N\bar{i}nt(Ncl(K)) \cup Ncl(N\bar{o}int(K))
\]

and hence $K$ is a NZos.

**Theorem 3.4** The union (resp. intersection) of any family of NZOS(X) (resp. NZCS(X)) is a NZOS(X) (resp. NZCS(X)).

**Proof.** Let $\{K_a : a \in \tau_N\}$ be a family of NZos’s. For each $a \in \tau_N$, $K_a \subseteq Ncl(N\bar{o}int(K_a)) \cup N\bar{i}nt(Ncl(K_a))$.
\[
\bigcup_{a \in \tau_N} K_a \subseteq \bigcup_{a \in \tau_N} Ncl(N\bar{o}int(K_a)) \cup N\bar{i}nt(Ncl(K_a))
\subseteq Ncl(N\bar{o}int(\bigcup K_a)) \cup N\bar{i}nt(Ncl(\bigcup K_a))
\]

The other case is similar.

**Remark 3.2** The intersection of two NZos’s need not be NZos.

**Example 3.3** Let $Y = \{a, b\}$ and define $N\bar{s}$‘s $Y_1, Y_2, Y_3$ in $X$ are
\[
Y_1 = \langle Y, \left(\begin{array}{ccc}
\mu_a & \mu_b \\
0.2 & 0.1 \\
0.5 & 0.5
\end{array}\right), \left(\begin{array}{ccc}
\sigma_a & \sigma_b \\
0.7 & 0.5 \\
0.5 & 0.5
\end{array}\right) \rangle,
Y_2 = \langle Y, \left(\begin{array}{ccc}
\mu_a & \mu_b \\
0.3 & 0.5 \\
0.5 & 0.5
\end{array}\right), \left(\begin{array}{ccc}
\sigma_a & \sigma_b \\
0.7 & 0.2 \\
0.5 & 0.5
\end{array}\right) \rangle,
Y_3 = \langle Y, \left(\begin{array}{ccc}
\mu_a & \mu_b \\
0.1 & 0.2 \\
0.5 & 0.5
\end{array}\right), \left(\begin{array}{ccc}
\sigma_a & \sigma_b \\
0.7 & 0.5 \\
0.5 & 0.5
\end{array}\right) \rangle.
\]

Then we have $\tau_N = \{0, Y_1, Y_2, Y_3\}$ is a $N\bar{s}$ in $X$, then $Y_2$ & $Y_3$ are NZos but $Y_2 \cap Y_3$ is not NZos.

**Proposition 3.3** Let $K$ is a NZos, that is
\[K \subseteq Ncl(N\bar{o}int(K)) \cup N\bar{i}nt(Ncl(K)) = 0_N \cup N\bar{i}nt(Ncl(K)) = N\bar{i}nt(Ncl(K))\]

Hence $K$ is a NPos.

(ii) Let $K$ be a NZos, that is
\[K \subseteq Ncl(N\bar{o}int(K)) \cup N\bar{i}nt(Ncl(K)) = N\bar{i}nt(Ncl(K)) \cup 0_N = Ncl(N\bar{o}int(K))\]

Hence $K$ is a N\bar{O}Sos.

(iii) Let $K$ be a NZos and N\bar{O}cs, that is
\[K \subseteq Ncl(N\bar{o}int(K)) \cup N\bar{i}nt(Ncl(K)) = N\bar{i}nt(Ncl(K)) \cup N\bar{i}nt(Ncl(K)) = N\bar{i}nt(Ncl(K))\]

Hence $K$ is a N\bar{O}Sos.

(iv) Let $K$ be a N\bar{O}Sos and N\bar{O}cs, that is
\[K \subseteq Ncl(N\bar{o}int(K)) \subseteq Ncl(N\bar{o}int(K)) \cup N\bar{i}nt(Ncl(K))\]

Hence $K$ is a NZos.

**Theorem 3.5** Let $K$ be a NZcs (resp. NZos) iff $K = NZcl(K)$ (resp. $K = NZint(K)$).

**Proof.** Suppose $K = NZcl(K) = \cap \{A : K \subseteq A & A is a NZcs\}$. This means $K \subseteq \cap \{A : K \subseteq A & A is a NZcs\}$ and hence $K$ is NZcs.

Conversely, suppose $K$ be a NZcs in $X$. Then, we have $K \subseteq \cap \{A : K \subseteq A & A is a NZcs\}$. Hence, $K \subseteq A$ implies $K = \cap \{A : K \subseteq A & A is a NZcs\} = NZcl(K)$.

Similarly for $K = NZint(K)$.
(ii) \( K \subseteq K \cup L \) or \( L \subseteq K \cup L \). Hence \( \text{NZcl}(K) \subseteq \text{NZcl}(K \cup L) \) or \( \text{NZcl}(L) \subseteq \text{NZcl}(K \cup L) \). Therefore, \( \text{NZcl}(K \cup L) \supseteq \text{NZcl}(K) \cup \text{NZcl}(L) \). The other one is similar.

(iii) \( K \subseteq K \cup L \) or \( L \subseteq K \cup L \). Hence \( \text{Nint}(K) \subseteq \text{Nint}(K \cup L) \) or \( \text{Nint}(L) \subseteq \text{Nint}(K \cup L) \). Therefore, \( \text{Nint}(K \cup L) \supseteq \text{Nint}(K) \cup \text{Nint}(L) \). The other one is similar.

Remark 3.3 The equality of (ii) in Proposition 3.4 can not be true in the given example.

Example 3.4 Let \( Y = \{a, b, c, d\} \) and define \( N \)'s \( Y_1, Y_2, Y_3 \) & \( Y_4 \) in \( X \) are

\[
Y_1 = \langle Y, (\mu_a, \mu_b, \mu_c, \mu_d; 0.5, 0.5, 0.5, 0.5), (\nu_a, \nu_b, \nu_c, \nu_d; 0, 0, 0, 0) \rangle
\]

\[
Y_2 = \langle Y, (\mu_a, \mu_b, \mu_c, \mu_d; 0, 0, 0, 0), (\nu_a, \nu_b, \nu_c, \nu_d; 0, 0, 0, 0) \rangle
\]

\[
Y_3 = \langle Y, (\mu_a, \mu_b, \mu_c, \mu_d; 1, 0, 0, 0), (\nu_a, \nu_b, \nu_c, \nu_d; 0, 0, 0, 0) \rangle
\]

\[
Y_4 = \langle Y, (\mu_a, \mu_b, \mu_c, \mu_d; 0, 0, 0, 0), (\nu_a, \nu_b, \nu_c, \nu_d; 0, 0, 0, 0) \rangle
\]

Then we have \( r_0 = \{0, Y_1, Y_2, Y_3 \cup Y_4\} \) is a Nts in \( X \), then \( \text{NZcl}(Y_3) \cup \text{NZcl}(Y_4) \).

Proposition 3.5 Let \( K \) be a neutrosophic set in a neutrosophic topological space \( X \). Then \( \text{Nint}(K) \subseteq \text{NZint}(K) \subseteq K \subseteq \text{NZcl}(K) \subseteq \text{Ncl}(K) \).

Proof. It follows from the definitions of corresponding operators.

Theorem 3.6 Let \( K \) and \( L \) in \( X \), then the \( \text{NZint} \) sets have

(i) \( \text{NZint}(0_N) = 0_N, \text{NZint}(1_N) = 1_N \).

(ii) \( \text{NZint}(K) \) is a \( \text{NZcs} \) in \( X \).

(iii) \( \text{NZint}(K) \subseteq \text{NZint}(L) \) if \( K \subseteq L \).

(iv) \( K \subseteq \text{NZint}(K) \).

(v) \( K \) is \( \text{Nz} \) set in \( X \iff \text{NZint}(K) = K \).

(vi) \( \text{NZint}(\text{NZint}(K)) = \text{NZint}(K) \).

Proof. The proofs (i) to (iv) and (vi) are directly from definitions of \( \text{NZcl} \) set.

Let \( K \) be \( \text{NZc} \) set in \( X \). By using Proposition 3.4, \( K \) is \( \text{NZo} \) set in \( X \). By Proposition 3.4,

\[
\text{NZint}(\overline{K}) = \overline{\text{K}} \iff \text{NZcl}(K) = K \iff \text{NZcl}(K) = K.
\]

Theorem 3.7 Let \( K \) and \( L \) in \( X \), then the \( \text{Nint} \) sets have

(i) \( \text{Nint}(0_N) = 0_N, \text{Nint}(1_N) = 1_N \).

(ii) \( \text{Nint}(K) \) is a \( \text{NZos} \) in \( X \).

(iii) \( \text{Nint}(K) \subseteq \text{Nint}(L) \) if \( K \subseteq L \).

(iv) \( \text{Nint}(\text{Nint}(K)) = \text{Nint}(K) \).

Proof. The proofs are directly from definitions of \( \text{NZint} \) set.

Proposition 3.6 If \( K \) and \( L \) is in \( X \), then (i) \( \text{NZcl}(K) \supseteq K \cup \text{NZcl}(\text{NZint}(K)) \).

(ii) \( \text{Nint}(K) \subseteq K \cap \text{Nint}(\text{NZcl}(K)) \).

(iii) \( \text{Nint}(\text{NZcl}(K)) \supseteq \text{NZint}(\text{NZcl}(K)) \).

Proof. (i) By Theorem 3.6 \( K \subseteq \text{NZcl}(K) \rightarrow (1) \). Again using Theorem 3.6, \( \text{Nint}(K) \subseteq \text{NZcl}(K) \rightarrow (2) \). By (1) and (2) we have, \( K \cup \text{Nz}(\text{Nint}(K)) \subseteq \text{NZcl}(K) \).

(ii) By Theorem 3.6, \( \text{Nint}(K) \subseteq K \rightarrow (1) \). Again using Theorem 3.6, \( K \subseteq \text{NZcl}(K) \rightarrow (2) \). By (1) and (2) we have, \( \text{Nint}(K) \subseteq K \cup \text{Nint}(\text{NZcl}(K)) \).

(iii) By Theorem 3.6, \( \text{Nz}(K) \subseteq \text{Ncl}(K) \), we get \( \text{Nint}(\text{NZcl}(K)) \subseteq \text{Nint}(\text{Ncl}(K)) \). Hence (iii).

(iv) By (i), \( \text{Nz}(K) \supseteq K \cup \text{Nz}(\text{Nint}(K)) \). We have, \( \text{Nint}(\text{NZcl}(K)) \supseteq \text{Nint}(K \cup \text{Nz}(\text{Nint}(K))) \). Since \( \text{Nint}(K \cup L) \supseteq \text{Nint}(K) \cup \text{Nint}(L) \), \( \text{Nint}(\text{NZcl}(K)) \supseteq \text{Nint}(K) \cup \text{Nint}(\text{NZcl}(K)) \). Hence (ii).

(v) 4 Conclusion

We have studied about neutrosophic Z-open set and neutrosophic Z-closed set and their respective interior and closure operators of neutrosophic topological space in this paper. Also studied some of their fundamental properties along with examples in \( \text{Nts} \). Also, we have discussed a near open sets of neutrosophic Z-open sets in \( \text{Nts} \). In future, we can be extended to neutrosophic Z continuous mappings, neutrosophic Z-open mappings and neutrosophic Z-closed mappings in \( \text{Nts} \).
References
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