Research Article

Characterizations of $N_{nc}e$-open and $N_{nc}e$-closed Functions

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Abstract: The purpose of this paper is to introduce and investigate several new classes of functions called, $N_{nc}e$-open and $N_{nc}e$-closed functions in N_{nc} topological spaces by using the concept of $N_{nc}e$-open sets. Several new characterizations and fundamental properties concerning of these new types of functions are obtained. Furthermore, these kinds of functions have strong application in the area of image processing and have very important applications in quantum particle physics, high energy physics and superstring theory.

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1 Introduction

Smarandache's neutrosophic system have wide range of real time applications for the fields of Computer Science, Information Systems, Applied Matheamatics, Artifical Intelligence, Mechanics, decision making, Medicine, Electrical & Electronic, and Management Science etc [1, 2, 3, 4, 31, 32]. Topology is a classical subject, as a generalization topological spaces many types of topological spaces introduced over the year. Smarandache [25] defined the Neutrosophic set on three component Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Neutrosophic topological spaces (*nts*'s) introduced by Salama and Alblowi [22]. Lellies Thivagar et.al. [12] was given the geometric existence of N topology, which is a non-empty set equipped with N arbitrary topologies. Lellis Thivagar et al. [13] introduced the notion of N_n-open (closed) sets and N_n topological spaces. Al-Hamido [5] explore the possibility of expanding the concept of neutrosophic crisp topological spaces into N-neutrosophic crisp topological spaces and investigate some of their basic properties. Several generalized forms of open and closed functions in topological spaces have been introduced and investigated over the course of years. Certainly, it is hard to say whether one form is more or less important than another. Functions and of course open and closed functions stand among the most important and most researched points in the whole of mathematical science. Various interesting problems arise when one considers openness and closeness. Its importance is significant in various areas of mathematics and related sciences. In 2008, Erdal Ekici [6] introduced a new class of generalized open sets called e-open sets and studied several fundamental and interesting properties of e-open sets and introduced a new class of continuous functions called e-continuous functions into the field of topology. In 2020, Vadivel and co-authors [27, 28] the concept of N-neutrosophic δ-open, N-neutrosophic δ-semiopen, N-neutrosophic δ-preopen and N-neutrosophic e-open sets are introduced. In this paper, we will continue the study of related functions by involving $N_{nc}e$-open sets. The aim of this paper is to introduce and investigate several new types of N_{nc}-open and N_{nc}-closed functions in topological spaces via $N_{nc}e$-open sets. Some characterizations and several interesting properties of these functions are discussed. Additionally, these kinds of functions have strong application in the area of Image Processing and have very important applications in quantum particle physics, high energy physics and superstring theory.

2 Preliminaries

Salama and Smarandache [24] presented the idea of a neutrosophic crisp set in a set X and defined the inclusion between two neutrosophic crisp sets, the intersection (union) of two neutrosophic crisp sets, the complement of a neutrosophic crisp set, neutrosophic crisp empty (resp., whole) set as more then two types. And they studied some properties related to nutrosophic crisp set operations. However, by selecting only one type, we define the inclusion, the intersection (union), and neutrosophic crisp empty (resp., whole) set again and discover a few properties.

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Definition 2.1 Let X be a non-empty set. Then H is called a neutrosophic crisp set (in short, *ncs*) in X if H has the form
$H = (H_1,H_2,H_3)$, where H_1,H_2, and H_3 are subsets of X,

The neutrosophic crisp empty (resp., whole) set, denoted by ϕ_n (resp., X_n) is an *ncs* in X defined by $\phi_n = (\phi,\phi,X)$ (resp. $X_n = (X,X,\phi)$). We will denote the set of all *ncs*'s in X as $ncS(X)$.

In particular, Salama and Smarandache [23] classified a neutrosophic crisp set as the followings.
A neutrosophic crisp set $H = (H_1,H_2,H_3)$ in X is called a neutrosophic crisp set of Type 1 (resp. 2 & 3) (in short, *ncs*-Type 1 (resp. 2 & 3)), if it satisfies $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ (resp. $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ and $H_1 \cup H_2 \cup H_3 = X$ & $H_1 \cap H_2 \cap H_3 = \phi$ and $H_1 \cup H_2 \cup H_3 = X$). $ncS_1(X)$ ($ncS_2(X)$ and $ncS_3(X)$) means set of all *ncs* Type 1 (resp. 2 and 3).

Definition 2.2 Let $H = (H_1,H_2,H_3),M = (M_1,M_2,M_3) \in ncS(X)$. Then H is said to be contained in (resp. equal to) M, denoted by $H \subseteq M$ (resp. $H = M$), if $H_1 \subseteq M_1,H_2 \subseteq M_2$ and $H_3 \supseteq M_3$ (resp. $H \subseteq M$ and $M \subseteq H$); $H^c = (H_3, H_2^c, H_1)$; $H \cap M = (H1 \cap M1,H2 \cap M2,H3 \cup M3)$; $H \cup M = (H1 \cup M1,H2 \cup M2,H3 \cap M3)$. Let $(Aj)_{j\in J} \subseteq ncS(X)$, where $H_j = (H_j1,H_j2,H_j3)$. Then $\bigcap_{j\in J} H_j$ (simply $\cap H_j$) $= (\cap H_j1,\cap H_j2,\cup H_j3)$; $\bigcup_{j\in J} H_j$ (simply $\cup H_j$), $= (\cup H_j1,\cup H_j2,\cap H_j3)$.

The following are the quick consequence of Definition 2.2.

Proposition 2.1 [7] Let $L,M,O \in ncS(X)$. Then
(i) $\phi_n \subseteq L \subseteq X_n$,
(ii) if $L \subseteq M$ and $M \subseteq O$, then $L \subseteq O$,
(iii) $L \cap M \subseteq L$ and $L \cap M \subseteq M$,
(iv) $L \subseteq L \cup M$ and $M \subseteq L \cup M$,
(v) $L \subseteq M$ iff $L \cap M = L$,
(vi) $L \subseteq M$ iff $L \cup M = M$.

Likewise the following are the quick consequence of Definition 2.2.

Proposition 2.2 [7] Let $L,M,O \in ncS(X)$. Then
(i) $L \cup L = L$, $L \cap L = L$ (Idempotent laws),
(ii) $L \cup M = M \cup L$, $L \cap M = M \cap L$ (Commutative laws),
(iii) (Associative laws) : $L \cup (M \cup O) = (L \cup M) \cup O$, $L \cap (M \cap O) = (L \cap M) \cap O$,
(iv) (Distributive laws:) $L \cup (M \cap O) = (L \cup M) \cap (L \cup O)$, $L \cap (M \cup O) = (L \cap M) \cup (L \cap O)$,
(v) (Absorption laws) : $L \cup (L \cap M) = L$, $L \cap (L \cup M) = L$,
(vi) (DeMorgan's laws) : $(L \cup M)^c = L^c \cap M^c$, $(L \cap M)^c = L^c \cup M^c$,
(vii) $(L^c)^c = L$,
(viii) (a) $L \cup \phi_n = L$, $L \cap \phi_n = \phi_n$,
(b) $L \cup X_n = X_n$, $L \cap X_n = L$,
(c) $X_n{}^c = \phi$, $\phi^c{}_n = X_n$,
(d) in general, $L \cup L^c \neq X_n$, $L \cap L^c \neq \phi_n$.

Proposition 2.3 [7] Let $L \in ncS(X)$ and let $(L_j)_{j\in J} \subseteq ncS(X)$. Then
(i) $(\bigcap L_j)^c = \bigcup L_j^c$, $(\bigcup L_j)^c = \bigcap L_j^c$,
(ii) $L \cap (\cup L_j) = \cup(L \cap L_j)$, $L \cup (\cap L_j) = \cap(L \cup L_j)$.

Definition 2.3 [23] A neutrosophic crisp topology (briefly, *ncts*) on a non-empty set X is a family τ of *nc* subsets of X satisfying the following axioms
(i) $\phi_n, X_n \in \tau$.
(ii) $H_1 \cap H_2 \in \tau$ $\forall$ H_1 & $H_2 \in \tau$.
(iii) $\bigcup_a H_a \in \tau$, for any $\{H_a : a \in J\} \subseteq \tau$.

Then (X,τ) is a neutrosophic crisp topological space (briefly, *ncts*) in X. The τ elements are called neutrosophic crisp open sets (briefly, *ncos*) in X. A *ncs* C is closed set (briefly, *nccs*) iff its complement C^c is *ncos*.

Definition 2.4 [5] Let X be a non-empty set. Then ${}_{nc}\tau_1, {}_{nc}\tau_2, \cdots, {}_{nc}\tau_N$ are N-arbitrary crisp topologies defined on X and the collection $N_{nc}\tau = \{A \subseteq X : A = (\bigcup_{j=1}^{N} H_j) \cup (\bigcap_{j=1}^{N} L_j), H_j,L_j \in {}_{nc}\tau_j\}$ is called N neutrosophic crisp (briefly, N_{nc})-topology on X if the axioms are satisfied:

(i) $\phi_n, X_n \in N_{nc}\tau$.

(ii) $\bigcup_{j=1}^{\infty} A_j \in N_{nc}\tau \; \forall \{A_j\}_{j=1}^{\infty} \in N_{nc}\tau$.

(iii) $\bigcap_{j=1}^{n} A_j \in N_{nc}\tau \; \forall \{A_j\}_{j=1}^{n} \in N_{nc}\tau$.

Then $(X, N_{nc}\tau)$ is called a N_{nc}-topological space (briefly, $N_{nc}ts$) on X. The $N_{nc}\tau$ elements are called N_{nc}-open sets ($N_{nc}os$) on X and its complement is called N_{nc}-closed sets ($N_{nc}cs$) on X. The elements of X are known as N_{nc}-sets ($N_{nc}s$) on X.

Definition 2.5 [5] Let $(X, N_{nc}\tau)$ be any $N_{nc}ts$. Let H be an $N_{nc}s$ in $(X, N_{nc}\tau)$. Then H is said to be a N_{nc}-regular open [26] set (briefly, $N_{nc}ros$) if $H = N_{nc}int(N_{nc}cl(H))$. The complement of an $N_{nc}ros$ is called an N_{nc}-regular closed set (briefly, $N_{nc}rcs$) in X.

The family of all $N_{nc}ros$ (resp. $N_{nc}rcs$) of X is denoted by $N_{nc}ROS(X)$ (resp. $N_{nc}RCS(X)$).

Definition 2.6 [27] A set H is said to be a

(i) $N_{nc}\delta$ interior of H (briefly, $N_{nc}\delta int(H)$) is defined by $N_{nc}\delta int(H) = \cup\{A : A \subseteq H \;\&\; A \text{ is a } N_{nc}ros\}$.

(ii) $N_{nc}\delta$ closure of H (briefly, $N_{nc}\delta cl(H)$) is defined by $N_{nc}\delta cl(H) = \cup\{x \in X : N_{nc}int(N_{nc}cl(L)) \cap H \neq \phi, x \in L \;\&\; L \text{ is a } N_{nc}os\}$.

Definition 2.7 A set H is said to be a

(i) $N_{nc}\delta$- open (briefly, $N_{nc}\delta o$) set [27] if $H = N_{nc}\delta int(H)$.

(ii) $N_{nc}e$-open (briefly, $N_{nc}eo$) set [28] if $H \subseteq N_{nc}cl(N_{nc}\delta int(H)) \cup N_{nc}int(N_{nc}\delta cl(H))$.

The complement of an $N_{nc}\delta os$ (resp. $N_{nc}eos$) is called an $N_{nc}\delta$ (resp. $N_{nc}e$) closed set (briefly, $N_{nc}\delta cs$ (resp. $N_{nc}ecs$)) in X.

The family of all $N_{nc}eos$ (resp. $N_{nc}ecs$) of X containing a point $x \in X$ is denoted by $N_{nc}eOS(X,x)$ (resp. $N_{nc}eCS(X,x)$). The family of all $N_{nc}\delta os$ (resp. $N_{nc}\delta cs$, $N_{nc}eos$ and $N_{nc}ecs$) of X is denoted by $N_{nc}\delta OS(X)$ (resp. $N_{nc}\delta CS(X)$, $N_{nc}eOS(X)$ and $N_{nc}eCS(X)$).

Definition 2.8 A function $f : (X, N_{nc}\tau) \to (Y, N_{nc}\tau^*)$ is said to be $N_{nc}e$-continuous (briefly, $N_{nc}eCts$) [29], if $f^{-1}(V)$ is $N_{nc}eo$ in X for every $N_{nc}o$ set V of Y.

Definition 2.9 A space $(X, N_{nc}\tau)$ is said to be:

(i) $N_{nc}e$-T_1 [30] if for each pair of distinct points x and y of X, there exist $N_{nc}eo$ sets A and B containing x and y, respectively, such that $x \notin B$ and $y \notin A$.

(ii) $N_{nc}e$-T_2 [30] if for each pair of distinct points x and y in X, there exist disjoint $N_{nc}eo$ sets A and B in X such that $x \in A$ and $y \in B$.

Definition 2.10 A space $(X, N_{nc}\tau)$ is said to be:

(i) $N_{nc}e$-compact [30] if every cover of X by $N_{nc}eo$ sets has a N_{nc} finite sub cover.

(ii) $N_{nc}e$-Lindelöf [30] if every cover of X by $N_{nc}eo$ sets has a countable subcover.

Definition 2.11 A space $(X, N_{nc}\tau)$ is said to be $N_{nc}e$-connected [30] if X cannot be written as the union of two nonempty disjoint $N_{nc}eo$ sets.

3 Characterizations of $N_{nc}e$-open and $N_{nc}e$-closed functions

In this section, we obtain some characterizations and several properties concerning $N_{nc}e$-open functions and $N_{nc}e$-closed functions via $N_{nc}eo$ and $N_{nc}ec$ sets.

Definition 3.1 A function $f : (X, N_{nc}\tau) \to (Y, N_{nc}\tau^*)$ is said to be $N_{nc}e$-open (briefly. $N_{nc}eO$) if $f(U) \in N_{nc}eOS(Y)$ for every $N_{nc}o$ set U in X.

Theorem 3.1 A function $f : (X, N_{nc}\tau) \to (Y, N_{nc}\tau^*)$ is $N_{nc}eO$ iff for each $x \in X$ and each $N_{nc}o$ set U in X with $x \in U$, there exists a set $V \in N_{nc}eOS(Y)$ containing $f(x)$ such that $V \subseteq f(U)$.

Proof. The proof is follows immediately from Definition 3.1. ■

Theorem 3.2 Let $f : (X, N_{nc}\tau) \to (Y, N_{nc}\tau^*)$ be $N_{nc}eO$. If $V \subseteq Y$ and M is a $N_{nc}c$ set of X containing $f^{-1}(V)$, then there exists a set $F \in N_{nc}eCS(Y)$ containing V such that $f^{-1}(F) \subseteq M$.

Proof. Let $F = Y - f(X - M)$. Then, $F \in N_{nc}eCS(Y)$, since $f^{-1}(V) \subseteq M$, we have, $f(X - M) \subseteq (Y - V)$ and so $V \subseteq F$. Also $f^{-1}(F) = X - f^{-1}(f(X - M)) \subseteq X - (X - M) = M$. ■

Theorem 3.3 A function $f : (X, N_{nc}\tau) \to (Y, N_{nc}\tau^*)$ is $N_{nc}eO$ iff $f(N_{nc}int(A)) \subseteq N_{nc}eint(f(A))$, for every $A \subseteq X$.

Proof. Let $A \subseteq X$ and $x \in N_{nc}int(A)$. Then there exists an $N_{nc}o$ set U_x in X such that $x \in U_x \subseteq A$. Now $f(x) \in f(U_x) \subseteq f(A)$, since f is $N_{nc}eO$, $f(U_x) \in N_{nc}eOS(Y)$. Then, $f(x) \in N_{nc}eint(f(A))$. Thus $f(N_{nc}int(A)) \subseteq N_{nc}eint(f(A))$. Conversely, let U be an $N_{nc}o$ set in X. Then by assumption, $f(N_{nc}int(U)) \subseteq N_{nc}eint(f(U))$. Since $N_{nc}eint(f(U)) \subseteq f(U)$, $f(U) = N_{nc}eint(f(U))$. Thus $f(U) \in N_{nc}eOS(Y)$. So f is $N_{nc}eO$. ■

Remark 3.1 The equality in the Theorem 3.3 need not be true as shown in the following Example.

Example 3.1 Let $X = \{a,b,c,d,e\}$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N\}$. $A = \langle\{c\},\{\phi\},\{a,b,d,e\}\rangle$, $B = \langle\{a,b\},\{\phi\},\{c,d,e\}$ $C = \langle\{a,b,c\},\{\phi\},\{d,e\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, A, B, C\}$. Define $f : (X, 2_{nc}\tau) \to (X, 2_{nc}\tau)$ be an identity function.

Then f is a $2_{nc}eO$. Let $U = \langle\{c,d\},\{\phi\},\{a,b,e\}\rangle \subseteq X$. Then $f(2_{nc}int(U)) = f(2_{nc}int(\langle\{c,d\},\{\phi\}, \{a,b,e\}\rangle)) = f(\langle\{c\},\{\phi\},\{a,b,d,e\}\rangle) = \langle\{c\},\{\phi\},\{a,b,d,e\}\rangle$. But $2_{nc}eint(f(U)) = 2_{nc}eint(f(\langle\{c,d\},\{\phi\},\{a,b,e\}\rangle)) = 2_{nc}eint(\langle\{c,d\},\{\phi\},\{a,b,e \langle\{c,d\},\{\phi\},\{a,b,e\}\rangle$. Thus $f(2_{nc}int(U)) = 2$□ $nceint(f(U))$.

Theorem 3.4 A function $f : (X,N_{nc}\tau) \to (Y,N_{nc}\tau^*)$ is $N_{nc}eO$ iff $N_{nc}int(f^{-1}(B)) \subseteq f^{-1}(N_{nc}eint(B))$ for every $B \subseteq Y$.

Proof. Let B be any N_{nc} set of Y. Then $f(N_{nc}int(f^{-1}(B))) \subseteq f(f^{-1}(B)) \subseteq B$. But $f(N_{nc}int(f^{-1}(B))) \in N_{nc}eOS(Y)$, since $N_{nc}int(f^{-1}(B))$ is $N_{nc}o$ in X and f is $N_{nc}eO$. Hence, $f(N_{nc}int(f^{-1}(B))) \subseteq N_{nc}eint(B)$. Therefore $N_{nc}int(f^{-1}(B)) \subseteq f^{-1}(N_{nc}eint(B))$.

Conversely, let A be any N_{nc} set of X. Then $f(A) \subseteq Y$. Hence by assumption, we have, $N_{nc}int(A) \subseteq N_{nc}int(f^{-1}(f(A)) \subseteq f^{-1}(N_{nc}eint(f(A))))$. Thus, $f(N_{nc}int(A)) \subseteq N_{nc}eint(f(A))$, for every $A \subseteq X$. Hence, by Theorem 3.3, f is $N_{nc}eO$. ■

Theorem 3.5 A function $f : (X,N_{nc}\tau) \to (Y,N_{nc}\tau^*)$ is $N_{nc}eo$ iff $f^{-1}(N_{nc}ecl(B)) \subseteq N_{nc}cl(f^{-1}(B))$ for every $B \subseteq Y$.

Proof. Suppose that f is $N_{nc}eO$ and $B \subseteq Y$ and let $x \in f^{-1}(N_{nc}ecl(B))$. Then, $f(x) \in N_{nc}ecl(B)$. Let U be an $N_{nc}o$ set in X such that $x \in U$. Since f is $N_{nc}eo$, then $f(U) \in N_{nc}eOS(Y)$. Therefore $B \cap f(U)$ □$= \phi$. Then, $U \cap f^{-1}(B)$ □$= \phi$. Hence $x \in N_{nc}cl(f^{-1}(B))$. Therefore we have $f^{-1}(N_{nc}ecl(B^{))} \subseteq N_{nc}cl(f^{-1}(B))$.

Conversely, let $B \subseteq Y$, then $(Y - B) \subseteq Y$. By assumption, $f^{-1}(N_{nc}ecl(Y - B)) \subseteq N_{nc}cl(f^{-1}(Y - B))$ this implies, $X - N_{nc}cl(f^{-1}(Y - B)) \subseteq X - f^{-1}(N_{nc}ecl(Y - B))$. Hence $X - N_{nc}cl(X - f^{-1}(B)) \subseteq f^{-1}((Y - N_{nc}ecl(Y - B)))$.

Now $X - N_{nc}cl(X - f^{-1}(B)) = N_{nc}int(X - (X - f^{-1}(B)) = N_{nc}int(f^{-1}(B)))$. Then, we have $Y - N_{nc}ecl(Y - B) = N_{nc}eint(Y - (Y - B)) = N_{nc}eint(B)$. Then $N_{nc}int(f^{-1}(B)) \subseteq f^{-1}(N_{nc}eint(B))$. By Theorem 3.4 we have f is $N_{nc}eo$. ■

Now we introduce some characterizations concerning $N_{nc}e$-closed functions.

Definition 3.2 A function $f : (X,N_{nc}\tau) \to (Y,N_{nc}\tau^*)$ is said to be $N_{nc}e$-closed (briefly, $N_{nc}eC$) if $f(M) \in N_{nc}eCS(Y)$ for every $N_{nc}c$ set M in X.

Example 3.2 Let $X = \{a,b,c,d,e\}, {}_{nc}\tau_1 = \{\phi_N,X_N,A,B,C\}, {}_{nc}\tau_2 = \{\phi_N,X_N\}.A = \langle\{c\},\{\phi\},\{a,b,d,e\}\rangle, B = \langle\{a,b\},\{\phi\},\{c,d,e\}$ $C = \langle\{a,b,c\},\{\phi\},\{d,e\}\rangle$, then we have $2_{nc}\tau = \{\phi_N,X_N,A,B,C\}$. Define $f : (X,2_{nc}\tau) \to (X,2_{nc}\tau)$ be an identity function. Then f is a $2_{nc}eC$.

Theorem 3.6 A function $f : (X,N_{nc}\tau) \to (Y,N_{nc}\tau^*)$ is $N_{nc}eC$ iff $N_{nc}ecl(f(A)) \subseteq f(N_{nc}cl(A))$ for every $A \subseteq X$.

Proof. Let f be $N_{nc}eC$ function and let A be any N_{nc} set of X. Then $f(N_{nc}cl(A)) \in N_{nc}eCS(Y)$. But $f(A) \subseteq f(N_{nc}cl(A))$. Then $N_{nc}ecl(f(A)) \subseteq f(N_{nc}cl(A))$.

Conversely, let A be a $N_{nc}c$ set of X. Then by assumption, $N_{nc}ecl(f(A)) \subseteq f(N_{nc}cl(A)) = f(A)$. This shows that $f(A) \in N_{nc}eCS(X)$. Hence f is $N_{nc}eC$. ■

Corollary 3.1 Let $f : (X,N_{nc}\tau) \to (Y,N_{nc}\tau^*)$ be $N_{nc}eC$ and let $A \subseteq X$. Then, $N_{nc}eint(N_{nc}ecl(f(A)) \subseteq f(N_{nc}cl(A)))$.

Theorem 3.7 Let $f : (X,N_{nc}\tau) \to (Y,N_{nc}\tau^*)$ be a surjective function. Then f is $N_{nc}eC$ iff for each subset B of Y and each $N_{nc}o$ set U in X containing $f^{-1}(B)$, there exists a set $V \in N_{nc}eOS(Y)$ containing B such that $f^{-1}(V) \subseteq U$.

Proof. Let $V = Y - f(X - U)$, then $V \in N_{nc}eOS(Y)$. Since $f^{-1}(B) \subseteq U$, then we have $f(X - U) \subseteq Y - B$ so $B \subseteq V$. Also, $f^{-1}(V) = X - f^{-1}(f(X - U)) \subseteq X - (X - U) = U$.

Conversely, let M be a $N_{nc}c$ set in X and $y \in Y - f(M)$. Then, $f^{-1}(y) \subseteq X - f^{-1}(f(M)) \subseteq X - M$ and $X - M$ is $N_{nc}o$ in X. Hence by assumption, there exists a set $V_y \in N_{nc}eOS(Y,y)$ such that $f^{-1}(V_y) \subseteq X - M$. This implies that $y \in V_y \subseteq Y - f(M)$. Thus $Y - f(M) = \cup\{V_y : y \in Y - f(M)\}$. Hence $Y - f(M) \in N_{nc}eOS(Y)$. Thus $f(M) \in N_{nc}eCS(Y)$. ■

Theorem 3.8 Let $f : X \to Y$ be a bijective. Then the following are equivalent:

(i) f is $N_{nc}eC$, (ii) f is $N_{nc}eO$,

(iii) f^{-1} is $N_{nc}eCts$.

Proof. (i) ⇒ (ii): Let U be an $N_{nc}o$ set of X. Then $X - U$ is $N_{nc}c$ in X. By (i), $f(X - U) \in N_{nc}eCS(Y)$. But $f(X - U) = f(X) - f(U) = Y - f(U)$. Thus $f(U) \in N_{nc}eOS(Y)$.

(ii) ⇒ (iii): Let U be an $N_{nc}o$ set of X. Since f is $N_{nc}eo$. Then, $f(U) = (f^{-1})^{-1}(U) \in N_{nc}eOS(Y)$. Hence f^{-1} is $N_{nc}eCts$.

(iii) ⇒ (i): Let M be an arbitrary $N_{nc}c$ set in X. Then $X - M$ is $N_{nc}o$ in X. Since f^{-1} is $N_{nc}eCts$, then $(f^{-1})^{-1}(X - M) \in N_{nc}eOS(Y)$. But $(f^{-1})^{-1}(X - M) = f(X - M) = Y - f(M)$, thus $f(M) \in N_{nc}eCS(Y)$. ■

Theorem 3.9 If $f : (X,N_{nc}\tau) \to (Y,N_{nc}\tau^*)$ is $N_{nc}eO$ bijection. Then the following hold:

(i) If X is $N_{nc}T_1$ then Y is $N_{nc}e$-T_1. (ii) If X is $N_{nc}T_2$ then Y is $N_{nc}e$-T_2.

Proof. (i) Let y_1 and y_2 be any distinct points in Y. Then there exist x_1 and x_2 in X such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is $N_{nc}T_1$ then, there exist two $N_{nc}o$ sets U and V in X with $x_1 \in U$, $x_2 \notin U$ and $x_2 \in V$, $x_1 \notin V$. Now $f(U)$ and $f(V)$ are $N_{nc}eo$ in Y with $y_1 \in f(U)$, $y_2 \notin f(U)$ and $y_2 \in f(V)$, $y_1 \notin f(V)$.

(ii) It is similar to (i). Thus is omitted. ■

Theorem 3.10 If $f : (X,N_{nc}\tau) \to (Y,N_{nc}\tau^*)$ is $N_{nc}eO$ bijective. Then the following properties are hold:

(i) If Y is $N_{nc}e$-compact, then X is compact.

(ii) If Y is $N_{nc}e$-Lindelo¨f, then X is Lindelo¨f.

Proof. (i) Let $U_1 = \{U_\lambda : \lambda \in \Delta\}$ be an $N_{nc}o$ cover of X. Then $K_1 = \{f(U_\lambda) : \lambda \in \Delta\}$ is a cover of Y by $N_{nc}eo$ sets in Y. Since Y is $N_{nc}e$-compact, Then K_1 has a N_{nc} finite subcover $K_2 = \{f(U_{\lambda 1}),f(U_{\lambda 2}),\cdots ,f(U_{\lambda n})\}$ for Y. Then $U_2 = \{U_{\lambda 1},U_{\lambda 2},\cdots ,U_{\lambda n}\}$ is a N_{nc} finite subcover of U for X.

(ii) It is similar to (i). Thus is omitted. ■

Theorem 3.11 If a function $f : (X,N_{nc}\tau) \rightarrow (Y,N_{nc}\tau^*)$ is an $N_{nc}eO$ surjective and Y is $N_{nc}e$-connected. Then X is N_{nc}connected.

Proof. Suppose that X is not N_{nc}-connected. Then there exist two non-empty disjoint $N_{nc}o$ sets U and V in X such that $X = U \cup V$. Then $f(U)$ and $f(V)$ are non-empty disjoint $N_{nc}eo$ sets in Y with $Y = f(U) \cup f(V)$ which contradicts the fact that Y is $N_{nc}e$connected. ■

4 Conclusion

Generalized open and closed sets play a very a prominent role in general Topology and it applications. And many topologists worldwide are focusing their researches on these topics and this mounted to many important and useful results. Indeed a significant theme in General Topology, Real analysis and many other branches of mathematics concerns the variously modified forms of continuity, separation axioms etc., by utilizing generalized open and closed sets. One of the well-known notions and that expected it will has a wide applying in physics and Topology and their applications is the notion of $N_{nc}e$-open sets. The importance of general topological spaces rapidly increases in both the pure and applied directions; it plays a significant role in data mining [21]. One can observe the influence of general topological spaces also in computer science and digital topology [8, 9, 10], computational topology for geometric and molecular design [14], particle physics, high energy physics, quantum physics, and Superstring theory [11, 15, 16, 17, 18, 19, 20]. In this paper we have introduced and investigated the notions of new classes of functions which may have very important applications in quantum particle physics, high energy physics and superstring theory. Furthermore, the fuzzy topological version of the concepts and results introduced in this paper are very important. Since El-Naschie has shown that the notion of fuzzy topology has very important applications in quantum particle physics especially in related to both string theory and ϵ^{∞} theory.

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