Research Article

# Decompositions of Continuity In Simply Extended Topological Spaces

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**ABSTRACT.** In this paper, we obtain some decompositions of continuity in simply extended topological spaces.

## 1. INTRODUCTION

Semi-open, preopen sets, $\alpha$-open, $\beta$-open sets or semi-preopen sets play an important role in the research of generalizations of continuity. By using these sets several authors introduced and studied various types of modifications of continuity in topological spaces. Semi-continuity, precontinuity, $\alpha$-continuity, $\beta$continuity or semi- precontinuity and other forms. In [4–6,17] and [18], the following notions are introduced. $D(\tau,s)$-sets, $D(\tau,p)$-sets, $D(\alpha,s)$-sets, $D(\alpha,p)$sets and $D(\tau,s)$-continuity, $D(\tau,p)$-continuity, $D(\alpha,s)$-continuity, $D(\alpha,p)$-continuity. By using these notions and the notions of semi-continuity, pre-continuity, $\alpha$continuity, $\beta$-continuity some decompositions of continuity are obtained.

The notion of g-closed sets is introduced in [8]. The notion of g-continuity is introduced and studied in [2]. Recently, Murugalingam [10] introduced certain generalizations of g-closed sets in topological spaces.

In [13, 15] and other research articles, the authors introduced and investigated the notions of minimal structures, m-spaces, M-continuity and M*-continuity. The notion of M*-continuity is introduced in [9]. In [12] the author introduced the notions of $D(m_1,m_2)$-sets, where $m_1$ *and* $m_2$ are minimal structures on nonempty set X, and obtain useful results concerning these sets. By using these results we obtain general decompositions of M-continuity. As immediate consequences, generalizations of the results established in [3,5,6,17,18] are

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obtained as new forms of continuity and weak forms of continuity in topological spaces.

In this paper, we obtain some decompositions of continuity in simply extended topological spaces.

## 2. PRELIMINARIES

**Definition 2.1.** *Let X be a non-empty set and Levine [7] defined* $\tau(B) = \{O\cup(O^0\cap B) : O,O^0 \in \tau\}$ *and called it simple extension of* $\tau$ *by B, where* $B \notin \tau$. *We call the pair* $(X,\tau(B))$ *a simply extended topological space (briefly SETS). The elements of* $\tau(B)$ *are called B-open sets and the complements of B-open sets are called B-closed sets. The B-closure of a subset S of X, denoted by Bcl(S), is the intersection of B-closed sets of X including S. The B-interior of S, denoted by Bint(S), is the union of B-open sets of X contained in S.*

**Definition 2.2.** *[11] Let* $(X,\tau(B))$ *be a SETS and* $A \subseteq X$. *Then A is said to be*

(1) *B-semiopen if* $A \subseteq Bcl(Bint(A))$;
(2) *B-preopen if* $A \subseteq Bint(Bcl(A))$; (3) *B-$\alpha$-open if* $A \subseteq Bint(Bcl(Bint(A)))$;
(4) *B-$\beta$-open if* $A \subseteq Bcl(Bint(Bcl(A)))$.

*The complement of B-semiopen (resp. B-preopen, B-$\alpha$-open, B-$\beta$-open) is said to be B-semiclosed (resp. B-preclosed, B-$\alpha$-closed, B-$\beta$-closed).*

*In this paper, let us denote by* $\sigma(\tau(B))$ *(or* $\sigma$*) the class of all B-semiopen sets on X, by* $\pi(\tau(B))$ *(or* $\pi$*) the class of all B-preopen sets on X, by* $\alpha(\tau(B))$ *(or* $\alpha$*) the class of all B-$\alpha$-open sets on X, by* $\beta(\tau(B))$ *(or* $\beta$*) the class of all B-$\beta$-open sets on X.*

**Lemma 2.3.** *[16] Let* $(X,m_X)$ *be an m-space and* $m_X$ *satisfy property* B. *Then for a subset A of X, the following properties hold:*

(1) $A \in m_X$ *if and only if mint(A)=A,*
(2) *A is* $m_X$*-closed if and only if mcl(A)=A,*
(3) $mint(A) \in m_X$ *and mcl(A) is* $m_X$*-closed.*

**Theorem 2.4.** *[12] Let X be a nonempty set and* $m_1,m_2,m_3$ *minimal structures on X such that* $m_1$ *has property* B *and* $m_1 \subset m_2 \subset m_3$. *Then* $m_1 = m_2 \cap D(m_1,m_3)$.

**Remark 2.5.** *[11]*

(1) *Every open set is B-open set.*
(2) *Every B-open set is B-preopen.*
(3) *Every B-open set is B-semi-open.*

## 3. **SIMPLE EXTENSION OF TOPOLOGIES**

**Definition 3.1.** *A subset A of a simply extended topological space* ($X,\tau(B)$) *is said to be B-b-open if* $A \subseteq Bcl(Bint(A)) \cup Bint(Bcl(A))$.

*The complement of B-b-open set is called B-b-closed.*

*In this paper the family of all B-open (resp. B-semi-open , B-preopen, B-αopen, B-β-open, B-b-open) sets in a simply extended topological space* ($X,\tau(B)$) *is denoted by B(X) (resp. BSO(X) BPO(X), Bα(X), BβO(X), BbO(X)). The following relations are well-known:*

open −→ B-open −→ B-preopen
↓ ↓
B-semi-open −→ B-b-open −→ B-β-preopen Diagram – I

**Example 3.2.** *Let* $X = \{a,b,c,d,e\}$, $\tau = \{\varphi,X,\{a,b\},\{a,b,c,d\}\}$ *and* $B = \{c,d\}$.
*Then* $\tau(B) = \{\varphi,X,\{a,b\},\{c,d\},\{a,b,c,d\}\}$. *We have*

*(1)* $\{a\}$ *is B-b-open set but not B-semi-open.*
*(2)* $\{a,e\}$ *is B-β-open but not B-b-open.*

**Definition 3.3.** *In this chapter, the intersection of all B-semi-closed (resp. Bpreclosed, B-α-closed, B-b-closed, B-β-closed ) sets of X containing A is called the B-semi-closure (resp. B-preclosure, B-α-closure, B-b-closure, B-β-closure) of A and is denoted by Bscl(A) (resp. Bpcl(A), Bαcl(A), Bbcl(A), Bβcl(A)).*

**Definition 3.4.** *The union of all B-semi-open (resp. B-preopen, Bα-open, Bβopen, Bb-open) sets of X contained in A is called the B-semi-interior (resp. Bpreinterior, Bα-interior , Bβ-interior, Bb-interior ) of A and is denoted by Bsint(A) (resp. Bpint(A), Bαint(A), Bβint(A), Bbint(A)).*

*The family of all B-open (resp. B-semi-open, B-preopen, B-α-open, B-b-open, Bβ-open) sets is denoted by B(X) (resp. BSO(X), BPO(X), Bα(X), BBO(X), Bβ(X)).*

We have the following implications

−→ α
open↓ ↕̸ -open
B-open −→ B-α-open −→ B-preopen
↓ ↓
B-semiopen −→ B-b-open −→ B-β-open Diagram – II

**Example 3.5.** *Let X = {a, b, c}, τ = {φ, X, {a}} and B = {b}. Then τ(B) = {φ, X, {a}, {b}, {a, b}}. Then {a, b} is α-open but not open.*

**Remark 3.6.** *α-openness and B-α-openness are independent.*

**Example 3.7.** *Let X = {a, b, c}, τ = {φ, X, {a}} and B = {b}. Then τ(B) = {φ, X, {a}, {b}, {a, b}}. Then {a,c} is α-open but not B-α-open and {b} is B-α-open but not α-open.*

**Definition 3.8.** *Let* ($X,\tau(B)$) *be a simply extended topological space. A subset A of X is said to be B-g-closed [1] (resp. B-sg-closed, B-pg-closed, B-αg-closed, Bβg-closed, B-bg-closed) if* $Bcl(A) \subseteq U$ *and U is open (resp. semi-open, preopen, α-open, β-open, b-open) in* ($X,\tau$). *The complement of a B-g-closed (resp. B-sgclosed, B-pg-closed, B-αg-closed, B-βg-closed, B-bg-closed) set is a B-g-open (resp. B-sg-open, B-pg-open, B-αg-open, B-βg-open, B-bg-open).*

*The family of B-g-open (resp. B-sg-open, B-pg-open, B-αg-open, B-βg-open, B-bg-open) is denoted by BGO(X) (resp. BSG(X), BPG(X), BαG(X), BβG(X), BbG(X)).*

open −→ B-βg-open −→ B-pg-open
↓ ↓
B-sg-open −→ B-αg-open −→ B-g-open Diagram – III

**Example 3.9.** *Let X = {a, b, c}, τ = {φ, X} and B = {c}. Then τ(B) = {φ, X, {c}}. Then*

(1) *{c} is a B-βg-open but not open.*
(2) *{a, b} is B-sg-open but not B-βg-open.*
(3) *{a, b} is B-αg-open but not B-pg-open*

**Example 3.10.** *Let X = {a, b, c}, τ ={φ, X,{a}} and B = {b}. Then τ(B) = {φ, X, {a}, {b},{a, b}}. Then {c} is B-g-open but not B-αg-open.*

**Example 3.11.** *Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, X, \{a, d\}\}$ and $B = \{c\}$. Then $\tau(B) = \{\varphi, X, \{c\}, \{a, d\},\{a, c, d\}\}$. Then*

(1) *$\{b\}$ is B-pg-open but not B-βg-open.*
(2) *$\{b\}$ is B-αg-open but not B-sg-open.*

## 4. MINIMAL STRUCTURES

**Remark 4.1.** *Let $(X,\tau)$ be a topological space and $(X,\tau(B))$ be a simply extended topological space. Then*

(1) *The families $\tau$, $SO(X)$, $PO(X)$, $\alpha(X)$, $BO(X)$ and $\beta(X)$ are all m-structures on X.*
(2) *The families $B(X)$, $BSO(X)$, $BPO(X)$, $B\alpha(X)$, $BbO(X)$ and $B\beta O(X)$ are all also m-structures on X.*
(3) *$BGO(X)$, $BSG(X)$, $BPG(X)$, $B\alpha G(X)$, $BbG(X)$ and $B\beta G(X)$ are all also m-structures on X.*

**Remark 4.2.** *Let $(X,\tau)$ be a topological space and $(X,\tau(B))$ be a simply extended topological space and A be a subset of X, then*

(1) *If $m_X = \tau$(resp.$SO(X)$, $PO(X)$, $\alpha(X)$, $BO(X)$, $\beta(X)$), then we have*
  (i) *mcl(A) = cl(A) (resp. scl(A), pcl(A), αcl(A), bcl(A), βcl(A));*
  (ii) *mint(A) = int(A) (resp. sint(A), pint(A), αint(A), bint(A), βint(A)).*
(2) *If $m_X = B(X)$(resp.$BSO(X)$, $BPO(X)$, $B\alpha(X)$, $BbO(X)$, $B\beta O(X)$),*
  (i) *mcl(A)=Bcl(A) (resp. Bscl(A), Bpcl(A), Bαcl(A), Bbcl(A), Bβcl(A));*
  (ii) *mint(A)=Bint(A) (resp.Bsint(A), Bpint(A), Bαint(A), Bbint(A), Bβint(A)).*
(3) *If $m_X = BGO(X)$ (resp. $BSG(X)$, $BPG(X)$, $B\alpha G(X)$, $BbG(X)$, $B\beta G(X)$), then we have*
  (i) *mcl(A) = Bgcl(A) (resp. Bsgcl(A), Bpgcl(A), Bα gcl(A), Bbgcl(A), Bβ gcl(A));*
  (ii) *mint(A) = Bgint(A) (resp. Bsgint(A), Bpgint(A),B α gint(A), Bbgint(A), Bβ gint(A)).*

**Remark 4.3.** *Let $(X,\tau(B))$ be a simply extended topological space. Then*

(1) *The families $SO(X)$, $PO(X)$, $\alpha(X)$, $BO(X)$ and $\beta(X)$ are m-structures with property* B.
(2) *The families $B(X)$, $BSO(X)$, $BPO(X)$, $B\alpha(X)$, $BbO(X)$ and $B\beta O(X)$ are m-structures with property* B.
(3) *The families $BGO(X)$, $BSG(X)$, $BPG(X)$, $B\alpha G(X)$, $BbG(X)$,$B\beta G(X)$) do not have property* B *in general.*

## 5. $D(m_1,m_2)$-SETS

**Definition 5.1.** *Let $(X,\tau(B))$ be a simply extended topological space, then we define the following:*

(1) $D(\tau,B\alpha) = \{A \subset X : int(A) = B\alpha int(A)\}$,
(2) $D(B,B\alpha) = \{A \subset X : Bint(A) = B\alpha int(A)\}$,
(3) $D(B\alpha,Bs) = \{A \subset X : B\alpha int(A) = Bsint(A)\}$,
(4) $D(Bs,Bp) = \{A \subset X : Bsint(A) = Bpint(A)\}$.

**Remark 5.2.** *Let $(X,\tau(B))$ be a simply extended topological space, then we have the following:*

(1) $D(\tau,m) = \{A \subset X : int(A) = mint(A)\}$, *where m = Bβg, Bsg, Bpg, Bαg or Bβg.*
(2) $D(\beta g,m) = \{A \subset X : \beta gint(A) = mint(A)\}$, *where m =Bsg, Bpg, Bαg or B-g.*
(3) $D(Bsg,m) = \{A \subset X : Bsgint(A) = mint(A)\}$, *where m = Bpg, Bαg or B-g.*
(4) $D(Bpg,m) = \{A \subset X : Bpgint(A) = mint(A)\}$, *where m = Bαg or BBg.*
(5) $D(B\alpha g,Bg) = \{A \subset X : B\alpha gint(A) = Bgint(A)\}$.

*$D(\tau,B\alpha)$, $D(B,B\alpha)$, $D(B\alpha,Bs)$ and $D(Bs,Bp)$ are defined in Definition 5.1.*

**Remark 5.3.** *Let $(X,\tau(B))$ be a simply extended topological space, then we have the following:*

(1) $D(B,m) = \{A \subset X : Bint(A) = mint(A)\}$, *where m = B-g, Bsg, Bpg, Bαg, Bbg or Bβg.*
(2) $D(B,m) = \{A \subset X : Bint(A) = mint(A)\}$, *where m = Bα, Bp, Bs, Bb or Bβ.*
(3) $D(B\alpha,m) = \{A \subset X : B\alpha int(A) = mint(A)\}$, *where m = Bp, Bs, Bb or Bβ.*
(4) $D(Bs,m) = \{A \subset X : Bsint(A) = mint(A)\}$, *where m = Bp, Bb or Bβ.*
(5) $D(Bp,m) = \{A \subset X : Bpint(A) = mint(A)\}$, *where m = Bb or Bβ.*
(6) $D(Bb,B\beta) = \{A \subset X : Bbint(A) = B\beta int(A)\}$.

**Theorem 5.4.** *Let X be a nonempty set and $m_1$,$m_2$ minimal structures on X such that $m_1$ has property* B *and $m_1 \subset m_2$. Then $m_1 = m_2 \cap D(m_1,m_2)$.*

*Proof.* Let $V \in m_1$, *then* $V \in m_2$ *and* $V = m_2int(V)$. Since $V \in m_1$,$V = m_1int(V)$ and hence $V = m_1int(V) = m_2int(V)$. Therefore, we have $V \in m_2 \cap D(m_1,m_2)$ and hence $m_1 \subset m_2 \cap D(m_1,m_2)$.

Conversely, suppose $V \in m_2 \cap D(m_1,m_2)$. Since $V \in m_2, V = m_2int(V)$. Since $V \in D(m_1,m_2), m_1int(V) = m_2int(V)$ and hence $V = m_1int(V)$. Since $m_1$ has property B, by Lemma 2.3 we have $V \in m_1$ *and* $m_2 \cap D(m_1,m_2) \subset m_1$.

**Corollary 5.5.** *Let* $(X,\tau(B))$ *be a simply extended topological space. Then the following properties hold:*

(1) $\tau(B) = B(X) \cap D(\tau,B) = B\alpha(X) \cap D(\tau,B\alpha) = BPO(X) \cap D(\tau,BP)$
$= BSO(X) \cap D(\tau,BS) = BBO(X) \cap D(\tau,Bb) = B\beta(X) \cap D(\tau,B\beta)$,
(2) $B(X) = B\alpha(X) \cap D(B,B\alpha) = BPO(X) \cap D(B,Bp) = BSO(X) \cap$
$D(B,Bs)$
$= BBO(X) \cap D(B,Bb) = B\beta(x) \cap D(B,B\beta)$,
(3) $B\alpha(X) = BPO(X) \cap D(B\alpha,Bp) = BSO(X) \cap D(B\alpha,Bs) = BBO(X) \cap D(B\alpha,Bb) = B\beta(X) \cap D(B\alpha,B\beta)$,
(4) $BPO(X) = BBO(X) \cap D(Bp,Bb) = B\beta(x) \cap D(Bp,B\beta)$, (5) $BSO(X) = BBO(X) \cap D(Bs,Bb) = B\beta(X) \cap D(Bs,B\beta)$,
(6) $BBO(X) = B\beta(X) \cap D(Bb,B\beta)$.

*Proof.* This is an immediate consequence of Theorem 5.4 and Diagram II.

**Corollary 5.6.** *Let* $(X,\tau(B))$ *be a simply extended topological space. Then the following properties hold:*

(1) $\tau(B) = B\beta G(X) \cap D(\tau(B),B\beta g) = BSG(X) \cap D(\tau(B),Bsg) = BSG(X) \cap D(\tau(B),Bpg) = B\alpha G(X) \cap D(\tau(B),B\alpha g) = BGO(X) \cap$
$D(\tau(B),Bg)$.
(2) $BSG(X) = B\alpha G(X) \cap D(Bsg,B\alpha g) = BGO(X) \cap D(Bsg,Bg)$.

*Proof.* This is an immediate consequence of Theorem 5.4 and Diagram III.

**Corollary 5.7.** *Let* $(X,\tau(B))$ *be a simply extended topological space. Then the following properties hold:*

(1) $\tau = B(X) \cap D(\tau,m)$, *where* $m = B\alpha, Bs, Bp, Bb$ *or* $B\beta$
$= B\alpha(X) \cap D(\tau,m)$, *where* $m = Bs, Bp, Bb$ *or* $B\beta$
$= BSO(X) \cap D(\tau,Bb) = BSO(X) \cap D(\tau,B\beta) = BPO(X) \cap D(\tau,Bb) = BPO(X) \cap D(\tau,B\beta) = BBO(X) \cap D(\tau,B\beta)$.
(2) $B(X) = B\alpha(X) \cap D(B,m)$, *where* $m = Bs, Bp, Bb$ *or* $B\beta = BPO(X) \cap D(B,Bb) = BPO(X) \cap D(B,B\beta) = BSO(X) \cap D(B,Bb) = BSO(X) \cap D(B,B\beta) = BBO(X) \cap D(B,B\beta)$.
(3) $B\alpha(X) = BPO(X) \cap D(B\alpha,Bb) = BPO(X) \cap D(B\alpha,B\beta)$
$= BSO(X) \cap D(B\alpha,Bb) = BSO(X) \cap D(B\alpha,B\beta) = BBO(X) \cap D(B\alpha,B\beta)$.
(4) $BPO(X) = BBO(X) \cap D(Bp,B\beta)$.
(5) $BSO(X) = BBO(X) \cap D(Bs,B\beta)$.
(6) $BBO(X) = B\beta(X) \cap D(Bb,B\beta)$.

*Proof.* This is an immediate consequence of Theorem 2.4 and Diagram II.

**Corollary 5.8.** *Let* $(X,\tau(B))$ *be a simply extended topological space. Then the following properties hold:*

(1) $\tau(B) = B\beta G(X) \cap D(\tau(B),m)$, *where* $m = Bsg, Bpg, B\alpha g$ *or* $Bg$
$= BSG(X) \cap D(\tau(B),B\alpha g) = BSG(X) \cap D(\tau(B),Bg) = BPG(X) \cap D(\tau(B),B\alpha g) = B\alpha G(X) \cap D(\tau(B),Bg) = B\alpha G(X) \cap D(\tau(B),Bg)$.
(2) $BSG(X) = B\alpha G(X) \cap D(Bsg,Bg)$.

*Proof.* This is an immediate consequence of Theorem 2.4 and Diagram III.

## 6. DECOMPOSITIONS OF CONTINUITY

**Remark 6.1.** *Let* $(X,\tau(B))$ *be a simply extended topological space and* $m_X$ *an m-structure on X.*

(1) *If* $m_X = \tau(B)$ *(resp.* $BSO(X)$, $BPO(X)$, $B\alpha(X)$, $BBO(X)$, $B\beta(X)$*)*, $m_Y = \sigma$ *is a simply extended topology for Y and* $f : (X,m_X) \to (Y,m_Y)$ *is*
*M-continuous, then f is B-continuous (resp. B-semi-continuous, B-precontinuous, Bα-continuous, B-b-continuous, B-β-continuous).*
(2) *If* $m_X = Bg(X)$ *(resp.* $BSG(X), BPG(X), B\alpha G(X), B\beta G(X)$*)*, $m_Y = \sigma$
*is*
*a simply extended topology for Y and* $f : (X,m_X) \to (Y,m_Y)$ *is M*-*
*continuous, then f is Bg-continuous (resp. Bsg-continuous, B-pg-continuous,*
*Bαg-continuous,*
*Bβg-continuous).*

**Definition 6.2.** *Let X be a nonempty set and* $m_1,m_2$ *two minimal structures on X.*

*A function $f : (X,D(m_1,m_2)) \to (Y,m_Y)$ is said to be $D(m_1,m_2)$-continuous if f is M*-continuous, equivalently if the inverse image of each $m_Y$-open set of Y is a $D(m_1,m_2)$-set of X.*

**Theorem 6.3.** *Let X be a nonempty set and $m_1,m_2$ minimal structures on X such that $m_1$ has property* B *and $m_1 \subset m_2$. Then a function $f : (X,m_1) \to (Y,m_Y)$ is M-continuous if and only if*

(1) *$f : (X,m_2) \to (Y,m_Y)$ is M*-continuous and*
(2) *$f : (X,D(m_1,m_2)) \to (Y,m_Y)$ is $D(m_1,m_2)$-continuous.*

*Proof.* The proof follows immediately from Theorem 5.4.

**Corollary 6.4.** (1) *For a function $f : (X,\tau(B)) \to (Y,\sigma(B))$, the following are equivalent:*

(a) *f is continuous;*
(b) *f is B-continuous and $D(\tau,\tau(B))$-continuous;*
(c) *f is Bα-continuous and $D(\tau,B\alpha)$-continuous;*
(d) *f is Bp-continuous and $D(\tau,Bp)$-continuous;*
(e) *f is Bs-continuous and $D(\tau,Bs)$-continuous;* (f) *f is Bb-continuous and $D(\tau,Bb)$-continuous;*
(g) *f is Bβ-continuous and $D(\tau,B\beta)$-continuous.*

(2) *For a function $f : (X,\tau(B)) \to (Y,\sigma(B))$, the following are equivalent:* (a) *f is B-continuous;*

(b) *f is Bα-continuous and $D(B,B\alpha)$-continuous;*
(c) *f is Bp-continuous and $D(B,Bp)$-continuous;*
(d) *f is Bs-continuous and $D(B,Bs)$-continuous;*
(e) *f is Bb-continuous and $D(B,Bb)$-continuous;*
(f) *f is Bβ-continuous and $D(B,B\beta)$-continuous.*

*Proof.* This is an immediate consequence of Corollary 5.5 and Theorem 6.3.

**Remark 6.5.** *By Corollary 5.5(3)-(6) and Theorem 6.3, we can obtain several decompositions of Bα-continuity, Bp-continuity, Bs-continuity and Bb-continuity.*

**Corollary 6.6.** (1) *For a function $f : (X,\tau(B)) \to (Y,\sigma(B))$, the following are equivalent:*

(a) *f is continuous;*
(b) *f is Bβg-continuous and $D(\tau,B\beta g)$-continuous;*
(c) *f is Bsg-continuous and $D(\tau,Bsg)$-continuous;*
(d) *f is Bpg-continuous and $D(\tau,Bpg)$-continuous;*
(e) *f is Bαg-continuous and $D(\tau,B\alpha g)$-continuous;* (f) *f is Bg-continuous and $D(\tau,Bg)$-continuous.*

(2) *For a function $f : (X,\tau(B)) \to (Y,\sigma(B))$, the following are equivalent:* (a) *f is Bsg-continuous;*
(b) *f is Bαg-continuous and $D(Bsg,B\alpha g)$-continuous;* (c) *f is Bg-continuous and D(Bsg, Bg)-continuous.*

*Proof.* The proof follows immediately from Theorem 6.3 and Corollary 5.6.

**Theorem 6.7.** *Let X be a nonempty set and $m_1,m_2,m_3$ minimal structures on X such that $m_1$ has property* B *and $m_1 \subset m_2 \subset m_3$. Then a function $f : (X,m_1) \to (Y,m_Y)$ is M-continuous if and only if*

(1) *$f : (X,m_2) \to (Y,m_Y)$ is M*-continuous and*
(2) *$f : (X,D(m_1,m_3)) \to (Y,m_Y)$ is $D(m_1,m_3)$-continuous.*

*Proof.* The proof follows immediately from Theorem 2.4.

**Corollary 6.8.** (1) *For a function $f : (X,\tau(B)) \to (Y,\sigma(B))$, the following are equivalent:*

(a) *f is continuous;*
(b) *f is B-continuous and $D(\tau,m)$-continuous, where m=Bα, Bs, Bp, Bb or Bβ;*
(c) *f is Bα-continuous and $D(\tau,m)$-continuous, where m=Bs, Bp, Bb or Bβ;*
(d) *f is Bs-continuous and $D(\tau,m)$-continuous, where m= Bb or Bβ;* (e) *f is Bp-continuous and $D(\tau,m)$-continuous, where m= Bb or Bβ;* (f) *f is Bb-continuous and $D(\tau,B\beta)$-continuous.*

(2) *For a function $f : (X,\tau(B)) \to (Y,\sigma(B))$, the following are equivalent:* (a) *f is B-continuous;*

(b) *f is Bα-continuous and $D(\tau(B),m)$-continuous, where m = Bs, Bp, Bb or Bβ;*
(c) *f is Bp-continuous and $D(\tau(B),m)$-continuous, where m= Bb or Bβ;*
(d) *f is Bs-continuous and $D(\tau(B),m)$-continuous, where m= Bb or Bβ;* (e) *f is Bb-continuous and $D(\tau(B),B\beta)$-continuous.*

*Proof.* This is an immediate consequence of Corollary 5.7 and Theorem 6.7.

**Remark 6.9.** *By Corollary 5.7 and Theorem 6.7, we can obtain several decompositions of Bα-continuity, Bp-continuity and Bs-continuity.*

**Corollary 6.10.** (1) *For a function* $f : (X,\tau(B)) \to (Y,\sigma(B))$, *the following are equivalent:*

(a) *f is continuous;*

(b) *f is Bβg-continuous and* $D(\tau,m)$*-continuous, where m = Bαg or Bg*

(c) *f is Bsg-continuous and* $D(\tau,m)$*-continuous, where m = Bαg or Bg;* (d) *f is Bpg-continuous and* $D(\tau,m)$*-continuous, where m = Bαg or Bg;* (e) *f is Bαg-continuous and* $D(\tau,Bg)$*-continuous.*

(2) *A function* $f : (X,\tau(B)) \to (Y,\sigma(B))$ *is sg-continuous if and only if f is Bαg-continuous and D(Bsg, Bg)-continuous.*

*Proof.* The proof follows immediately from Theorem 6.7 and Corollary 6.8.

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