

Mean- Adjusted Variance Model for Portfolio Optimization

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Abstract: This paper proposes an operational founded model for portfolio optimization. The procedure used is based on the redacting of the asymmetry impact of the variance. This is a new approach that gives assets more accurate risk measures. The risk adjustment is based on the measure of volatility skewness and the goal here is to eliminate noisy risk. Moreover, we give our risk asymmetrical effect, according to the means of each asset.

Keywords: Portfolio Optimization, Asymmetry, Return Adjusted, Volatility Skewness.

1. Introduction

The stock market collapse in 2008 inadvertently highlighted the importance of market risk in portfolio management. Market risk is defined as the risk related to the fluctuation of the market portfolio. The study of portfolio risk management goes back to the fundamental work of Markowitz in 1952. Markowitz was the first to propose the mean-variance optimization framework.

The portfolio optimization problem consists of choosing a specific number of assets that meet investors' objectives, Mansini et al. (2014). The most recent research topic involves calculating the proportion of the initial budget that should be allocated in the available securities.

Since Markowitz mean-variance theory, many papers have been published extending or modifying the basic model (e.g.) Alexander & Baptista (2004); Benati & Rizzi (2007); Mansini et al. (2007); Ogryczak (2000); Sawik (2013a), (2013b), (2012a), (2012b), (2012c), (2011), (2010a), (2010b), (2009a), (2009b), (2009c), (2008); Speranza (1993); Iancu & Trichakis (2014); Terradez et al. (2015) Hamza & Janssen (1998).

The mean-variance model plays an important role in portfolio management. Additionally, the mean-variance analysis is essential to many asset pricing theories. Despite its reputation, dissemination, and adoption in the financial field, the theory of mean-variance optimization has been the subject of strong criticism since its publication. Numerous practitioners have reported some complications in implementing mean-variance analysis.

One of the most common criticisms of the Markowitz theory is its lack of robustness. Among other things, the mean-variance theory is particularly sensitive to the estimation of two parameters (mean and variance-covariance matrix).

Many authors have shown that the parameters generally tend to be unstable when estimated from a sample of historical data. Another criticism of this theory is based on the variance as a risk measure. However, the main drawback of the variance is that it penalizes both upward and downward deviations (i.e. both losses and profits).

Several research fields have developed this theory to try to correct the perceived shortcomings in the assumptions, the empirical results, and extend its application to other areas, Baker & Filbeck (2013).

The purpose of this study is to find the best possible allocation for any portfolio. Our technique attempts to equalize the upwards and downwards risk spread. Nevertheless, in case the downward deviations are lower than the upward ones, this equalization will be done according to the part that represents the real risk for the investor. In this case, the positive deviation does not represent a risk, we also keep the symmetrical shape to give the assets an equal chance for profit and loss. This process will reduce the risk of our portfolio and make it more efficient.

This article is organized as follows. The second section contains the literature review on the different risk optimization theory and its limitations. The third section presents the data, methodology, and techniques used. Then the 4th section covers all the results and discussion while the last section concludes our work.

2. Literature Review

In the capital market, selecting security or asset by the investors has never been a simple matter. Markowitz was the first to formalize the tradeoff between risk and return of a portfolio. He proposed the mean-variance optimal portfolio. Thus, this theory suggests that portfolio risk is measured by variance, and the main goal is to minimize it for a certain average return. Markowitz's mean-variance optimization theory was considered the heart of the modern portfolio theory. Better yet, the study of the correlation between the securities led him to develop the portfolio diversification strategy, which is often more expressive under the quote "Never put all your eggs in one basket".

In pioneering work, Markowitz (1952) combined probability theory and optimization theory to model the investment behavior of economic agents. Accordingly, in his analysis, he stated that the investment return should be quantified as the expected value and risk as to the variance.

Furthermore, Markowitz made his models as follows: minimizing variance subject to a lower limit on expected returns or maximizing expected returns subject to a minimal level of variance. The main contribution of this model is relatively general, allowing its use and simplicity, in terms of theoretical analysis, in many practical situations.

To develop this model, Markowitz relies on a very restrictive assumption. He assumed that the returns on financial assets follow a normal distribution. So, Markowitz uses only the first two moments of the distribution: mean and variance in his model. Thereafter, several researchers Grama & Schyns (2003); Deng, Li, & Wang (2005) took variance to measure the risk in various situations. Although the variance is a very popular measure of risk in the portfolio selection, it has some limitations, Markowitz (1959).

One of the main limitations is that the variance considers extremely high returns as equal to low returns. When security's returns are asymmetric, the selected portfolio based on variance may have a potential anomaly that sacrifices too much-expected return to eliminate both extremely high and low returns. Also, there is some empirical study showing that many security returns are not distributed symmetrically. Liu, Wang, & Qiu (2003); Prakash, Chang & Pactwa (2003); Yan & Li (2009); Guo, Li, Zou, Guo, & Yan (2012).

It was argued that the definition of risk as the variance of expected returns is justified only when expected returns are normally distributed and when investors have quadratic utility functions, these two assumptions are not valid in practice, Feldstein (1969). According to the classic portfolio model, the use of the first two moments of distributions leads to a suboptimal allocation of assets Bawa & Lindenberg (1977); Harlow & Rao (1989); Harlow (1991).

The variance-covariance measurement of risk is somewhat controversial Jeffrey (1984). In this context, the semi-variance was proposed by Markowitz (1959) to replace the variance as a measure of risk in portfolio selection. Markowitz (1959) suggested the use of semi-variance instead of variance in his asset allocation model. In this case, excess returns above the average collapse to zero so that they do not correspond to the variances unnecessarily, resulting in intuitively better asset allocation. Semi-variance proved to be a more accurate measure of risk, not only for asset allocation but also for assessing Mao (1970).

Having established that mean-semi-variance is an approximately correct criterion in the sense that it provides a level of utility highly correlated with the expected utility of an investor. Thereby, we will now consider some additional reasons that support the plausibility of the semi-variance as a measure of risk.

Some of these reasons are practical, some others are empirical. From a practical perspective, first of all, investors do not dislike upside volatility; they only dislike downside volatility. Secondly, semi-variance is more useful than the standard deviation when the underlying distribution of returns is asymmetric and just as useful when the underlying distribution is symmetric i.e. the semi-variance is at least useful as a measure of risk as to the standard deviation. Finally, the semi-variance combines into one measure the information provided by two statistics: variance and skewness. Thus, making it possible to use a one-factor model to estimate the required returns.

From an empirical perspective, semi-variance was reported to explain the cross-section of emerging market returns Estrada (2000); Harvey (2000), the cross-section of industries in emerging markets Estrada (2001), and the cross-section of Internet stocks return Estrada (2004). Additional support for the semi-variance as an appropriate measure of risk is in Clash (1999); Hamza & Janssen (1998); Hamza & Janssen (2009); Sortino & van der Meer (1991) and Sortino et al (1999) among others.

Many studies have noted that the distribution of asset return is rarely normal and that the first two moments are not enough when we consider portfolio risk. Similarly, numerous studies have focused on skewness, kurtosis, and higher moments in portfolio selection Samuelson (1970); Rockinger & Jondeau (2002).

Tobin (1958) and Rubinstein (1973) showed that higher moments must be taken into account in portfolio selection and that the utility function is not quadratic, along with Samuelson (1970), Rubinstein (1973); Scott & Horvath (1980); Lai (1991); Konno & Suzuki (1995); Chunhachinda et al (1997); Fang & Lai (1997); Prakash et al (2003); Lai et al. (2006).

Empirical evidence given by Arditti (1971) showed that stock return distributions are often positively skewed, and the investor's preference for more skewed distribution rather than less is consistent with the notion of decreasing absolute risk aversion, a positively skewed asset return indicates an elongated right tail in the density function of the asset return Arditti (1967).

Konno et al. (1993) and Konno & Suzuki (1995) argued that the skewness of the returns and the third-order derivative of a utility function play significant roles in choosing an optimal portfolio. Investors would prefer a portfolio with a larger third moment if the mean and variance are the same. They suggested a mean-variance-skewness (MVS) portfolio optimization approach. This could have been able to make it possible to calculate an approximated efficient frontier of mean-variance-skewness and to calculate a portfolio with the maximal expected utility for any decreasingly risk aversion utility function using this model. However, they failed to solve the mean-variance-skewness model but instead reported preliminary computational results.

A non-parametric approach for measuring efficiency in the case of the static portfolio selection problem based on the mean-variance-skewness space was proposed by Bricc et al. (2007). They defined a shortage function that looks for increases in return and skewness and decreases in variance. Overall, optimality is guaranteed for the resulting optimal portfolio.

3. Data and Methodology

This section provides an overview of the data used in this study, as well as the model used to equalize the two semi-variances. This equalization eliminates some unnecessary and detrimental risks from the portfolio.

3.1 The Adjusted Return Risk Portfolio Selection Model

The proposed model in this paper aims to make the risk symmetric concerning the means. Here, we target a buying strategy (no short selling allowed) if the upside risk is greater than the downside risk (up semi-variance greater than the down semi-variance). We will, therefore, proceed to the reduction of the positive risk to equalize the up and down semi-variance given by the assets. To implement this model, the change will be applied to returns, the new adjusted returns (output of our model) will only be used to calculate the variance-covariance matrix, while the portfolio's return will not be changed (using the original returns). The goal behind this change is to remove noisy unwanted shock that can affect the risk in case of a buying strategy.

The purpose of this change is to eliminate the detrimental risk caused by shocks for our portfolio. In the case of a buying strategy, investors consider only the risk of loss, while the risk associated with the asset sometimes obscures its real image. For instance, in Table-1, 'CELG' assets have a positive return, but the down semi-variance is less than the up one. In this case, investors choosing this asset, don't know that their chance of obtaining a positive shock is greater than that of a negative one.

So, the risk of losing is not only related to the downside risk, but there is also the fact of what we can gain from this asset and the chances of having positive shocks. So, we conclude that this asset (CELG) responds more to positive shocks than to negative ones; the upside risk is also important in portfolio assets selection.

The return is calculated according to the following formula:

$$r_i = \log \frac{p_t}{p_{t-1}} \tag{1}$$

Where p_t is the asset's price

Given any combination of our risky assets and a set of weights describing the distribution of our portfolio, the overall expected portfolio return formula is:

$$E(r_p) = \sum_{i=1}^n x_i E(r_i) \tag{2}$$

Where $\sum_{i=1}^n x_i = 1$

- n Is the number of securities;
- x_i Is the proportion of the funds invested in security i ;
- r_i Is the return of security i ;
- r_p Is the portfolio return;

The return computation is nothing more than finding the weighted average return of the securities included in the portfolio.

The variance is the expectation of the squared deviation of a random variable from its mean. Informally, it measures how far a set of (random) numbers is spread out from their average value. The variance is the square of the standard deviation.

The portfolio variance is a measurement of how the combined returns of a set of securities make up a portfolio fluctuate over time. This portfolio variance statistic is calculated using the variance of each security in the portfolio as well as the covariance of each security pair in the portfolio:

$$Var(r_p) = \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij} Cov(r_i, r_j) \quad (3)$$

Covariance can also be expressed in terms of the correlation coefficient:

$$Cov(r_i, r_j) = \rho_{ij} \sigma_i \sigma_j = \sigma_{ij} \quad (4)$$

Whereby ρ_{ij} is the correlation coefficient between returns.

We can measure the degree of returns asymmetry using the Skewness coefficient. Considering that a simple test of the symmetry hypothesis consists in testing the nullity of the central third order moment of the distribution, the Skewness coefficient defined as:

$$skew(X) = \frac{E[X-E(X)]^3}{[varX]^{3/2}} \quad (5)$$

For a significant negative Skewness coefficient, the distribution is asymmetric. This means that the probability of getting below-average values is higher than that of getting higher than average values and vice versa for positive skewness.

The flattening coefficient or Kurtosis of a random variable corresponds to its fourth-order centered moment. Kurtosis is a measure of the thickness of the tails of distributions. This measure is based on the normal distribution, which is considered a flat-tailed distribution and has a flattening coefficient of 3. If the Kurtosis exceeds 3, then the distribution tails are thick and the distribution is called leptokurtic. If the Kurtosis is less than 3, the distribution is called Platykurtic. Kurtosis's formula exclaims as follows:

$$kurt(X) = \frac{E[X-E(X)]^4}{[varX]^2} \quad (6)$$

3.1.1 Semi-variance

The semi-variance of asset returns is a better risk measurement way for several reasons. First of all, investors 'like' more positive volatility than the negative ones. Second, semi-variance is more useful than the variance when the distribution is asymmetric, and it is also beneficial when the distribution is symmetrical. Third, semi-variance provides information about two dimensions: variance and skewness, Estrada (2007). Consequently, investors require higher returns from the shares, which have high downside risk deviations. They do not buy shares, as long as they become cheaper to an appropriate level. In the meantime, the shares with high positive deviation risk are overestimated and acquired for high prices, Ishiba et al. (2012).

The semi-variance can be estimated (using historical data) as follow:

$$\begin{cases} SV^+_j = \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^n \max(0, r_{j,t} - E(r_j))^2 \\ SV^-_j = \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^n \min(0, r_{j,t} - E(r_j))^2 \end{cases} \quad j = 1, 2, \dots, n \text{ and } t=1, 2, \dots, T \quad (7)$$

3.1.2 Volatility Skewness

Volatility Skewness is the ratio of a distribution’s upside variance to its downside variance, where the variances are measured relative to the mean. If the distribution is symmetrical, it has a skewness of 1.00. The values greater than 1.00 indicate positive skewness and values less than 1.00 indicate negative skewness, Sortino (2009).

Some critics of downside risk claim that eliminating the upside returns, from the risk calculation, is incorrect since strong positive returns somehow imply the inevitability of correspondingly strong negative returns. This criticism is anecdotal and is unsupported by any published research Sortino (2009).

As previously stated, it is necessary to fit some distribution to the observed return-data points into accurately calculated downside risk and related statistics. However, this is no different from the traditional method used in the MPT analysis in which the normal distribution is assumed to infer probabilities of loss from the observed data points.

The Volatility of Skewness can be estimated (using historical data) as follows:

$$V_Skew_j = \frac{\frac{1}{T} \sum_{t=1}^T \sum_{j=1}^n \max(0, r_{j,t} - E(r_j))^2}{\frac{1}{T} \sum_{t=1}^T \sum_{j=1}^n \min(0, r_{j,t} - E(r_j))^2} \quad j = 1, 2, \dots, n \quad (8)$$

The method proposed in this paper, as we have mentioned, uses the volatility skewness as a corrective term to adjust the upside risk. Our strategies aim to equalize the down and upside risks. Making the risk of our assets symmetrical will help to reduce and eliminate any unnecessary risk that was involved implicitly in our portfolio.

The method consisted of dividing the upside deviation according to the means of all the returns by the square root of the volatility skewness coefficient of each asset.

$$A^+ = \frac{\max(0, r_{j,t} - E(r_j))}{\sqrt{V_Skew_j}} \quad (9)$$

To recompose the adjusted returns, we recomposed all the elements to obtain a new return for each asset. The new return will be similar to the original ones but with a very small difference that can be noticed in figure 1.

The new returns obtained are given by the formula:

$$r^* = \frac{\max(0, r_{j,t} - E(r_j))}{\sqrt{V_Skew_j}} + \min(0, r_{j,t}) + E(r_{j,t}) \quad r > 0 \text{ and } SV^+_j > SV^-_j \quad (10)$$

This new return (adjusted return) will be used only to calculate the variance-covariance matrix. The variance calculation based on the new returns has equal semi-variances. To implement the change in the mean-variance model, we need to substitute the old variance-covariance matrix with the new matrix calculated by using the adjusted returns.

Then, the optimization program will be written as follow.

$$\text{Minimize} \quad \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij}^*$$

Subject to

$$\begin{aligned} \sum_{j=1}^n x_j E[r_j] &> \rho \\ \sum_{j=1}^n x_j &= 1 \\ x_j &\geq 0 \quad j = 1, \dots, n \end{aligned}$$

This is a quadratic programming problem, having n variables and two linear constraints. The term σ_{ij}^* is calculated using the adjusted returns resulted from our method. These constraints can significantly change the efficient frontier. As a result, x_j can be negative if the assets issued by the investor, or in case of short selling.

This system can be written as the following matrix form:

$$X^T \sum X$$

Minimize

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_j \end{bmatrix} \times \begin{bmatrix} \sigma_{11}^* & \sigma_{12}^* & \dots & \sigma_{1j}^* \\ \sigma_{11}^* & \sigma_{12}^* & \dots & \sigma_{1j}^* \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{11}^* & \sigma_{12}^* & \dots & \sigma_{1j}^* \end{bmatrix} \begin{bmatrix} x_1 x_2 \dots x_j \end{bmatrix}$$

Subject to

$$\begin{aligned} \begin{bmatrix} x_1 x_2 \dots x_j \end{bmatrix} \times \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_j \end{bmatrix} &\geq \rho \\ x_1 + x_2 + \dots + x_j &= 1 \\ x_j &\geq 0 \quad R_j = E[r_j] \quad j = 1, \dots, n \end{aligned}$$

3.2 Presentation and Description of Data

Our study sample includes data corresponding to the historical returns of 24 financial assets from the NYSE market. The data covers the period from 1/5/2017 to 12/28/2017. The returns are based on the price of securities from different activity areas, such as the Industrial sector, Finance, Transport, Services, etc. For our simulation, we used an expected return rate of 0.05% per unit of time (one week). Our work is mostly achieved by using *MATLAB financial toolbox*.

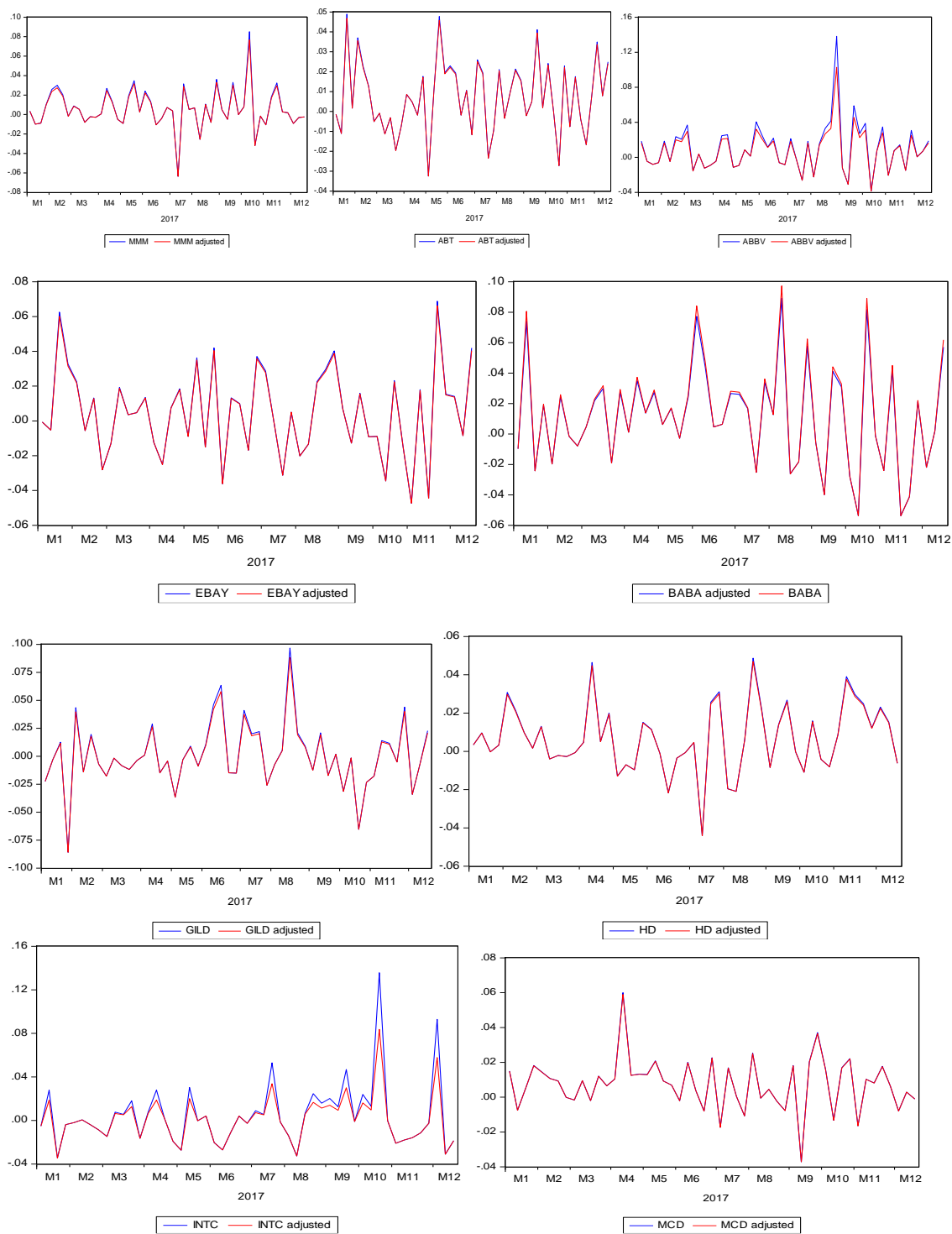
To describe the statistical of our data, table 1 shows the average return, variance, skewness, kurtosis, the skewness as well as kurtosis after adjustment, the negative and positive semi-variance, and finally the volatility skewness for all 24 assets.

According to table 1, we can conclude that the weekly returns distribution of all assets is different from the normal distribution. Additionally, the adjusted volatility has less value than the original one. Accordingly, this confirms that our model has shrunk some of the risks of our assets, which is noticeable in the result of our portfolio optimization. In terms of skewness, almost all the securities (i.e. 24 securities) are asymmetric (Skewness $\neq 0$). For the kurtosis coefficient, we observe that all of the assets have a coefficient above or below three (Kurtosis $\neq 3$). The returns with a high kurtosis coefficient imply that the investor will experience occasional extreme returns (either positive or negative). After applying our model, the kurtosis shows less value than the original one. This will reduce some of the occasional extreme returns. However, in this case, we have reduced the extremely positive return because the Kurtosis coefficient does not differ from the positive to the negative ones. The adjustment of our portfolio returns is done using the volatility skewness coefficient. This adjustment is applied to assets characterized by having a positive semi-variance greater than the negative one.

Table 1. Descriptive Statistics of the data

Assets	average return	variance	variance after adjustment	(-) Semi-variance	(+) Semi-variance	volatility skewness	skewness	Kurtosis	skewness after adjustment	kurtosis after adjustment
BI DU	0.0062	0.0011	0.0011	0.0005479	0.0005316	0.9850	0.0201	3.4308	0.0201	3.4308
MD LZ	-0.0008	0.0002	0.0002	0.0001332	0.0001111	0.9133	-0.1673	2.1539	-0.1673	2.1539
MM	0.0060	0.0005	0.0004	0.0001966	0.0002462	1.1191	0.3986	7.0281	0.1265	6.9571
AA PL	0.0074	0.0006	0.0006	0.0003149	0.0002711	0.9279	0.3042	3.9142	-0.3042	3.9142
AB T	0.0075	0.0003	0.0003	0.0001564	0.0001732	1.0521	0.1572	2.7545	0.1017	2.7423
AB BV	0.0103	0.0008	0.0005	0.0002615	0.0004990	1.3815	1.8191	9.8971	1.1697	7.1997
EB AY	0.0050	0.0007	0.0006	0.0003129	0.0003404	1.0431	0.1813	2.7827	0.1350	2.7577
BA BA	0.0126	0.0013	0.0011	0.0005532	0.0006774	1.1066	0.3729	2.8592	0.2584	2.7753
BM Y	0.0019	0.0008	0.0008	0.0005134	0.0003161	0.7847	1.4073	8.2776	-1.4073	8.2776
KO	0.0026	0.0002	0.0002	0.0001008	0.0000749	0.8617	0.5835	3.4984	-0.5835	3.4984
CO P	0.0023	0.0009	0.0009	0.0004577	0.0004339	0.9736	0.0796	2.6098	-0.0796	2.6098
LL Y	0.0029	0.0005	0.0005	0.0002606	0.0001858	0.8444	0.6721	3.4483	-0.6721	3.4483
XO M	0.0007	0.0002	0.0002	0.0001327	0.0001062	0.8949	0.2734	3.0866	-0.2734	3.0866
GI LD	0.0008	0.0009	0.0008	0.0003964	0.0004780	1.0982	0.2769	5.0598	0.0953	5.0489
HD	0.0069	0.0003	0.0003	0.0001492	0.0001629	1.0448	0.0285	3.4629	-0.0315	3.4999
IN TC	0.0045	0.0009	0.0005	0.0002314	0.0006365	1.6586	2.2048	9.8462	1.3172	6.5171
MC D	0.0075	0.0002	0.0002	0.0001085	0.0001127	1.0192	0.2651	5.5998	0.2268	5.5591
NK E	0.0039	0.0009	0.0009	0.0004850	0.0004354	0.9475	0.2479	4.3049	-0.2479	4.3049
PE P	0.0034	0.0002	0.0002	0.0000888	0.0000791	0.9438	0.1420	2.0572	-0.1420	2.0572
TX N	0.0077	0.0005	0.0004	0.0002092	0.0002350	1.0598	0.2430	2.5868	0.1850	2.5440
UN P	0.0057	0.0005	0.0004	0.0002122	0.0002986	1.1864	0.6164	2.9843	0.4325	2.7561
V	0.0069	0.0002	0.0002	0.0001047	0.0001105	1.0273	0.0422	2.1207	0.0202	2.1249
WB A	-0.0018	0.0007	0.0007	0.0004258	0.0002212	0.7207	1.2372	5.1366	-1.2372	5.1366
W MT	0.0078	0.0005	0.0004	0.0001855	0.0002882	1.2464	0.9000	5.2335	0.5162	4.7299

In our case, all the assets with a volatility skewness greater than 1 have been changed. After recomposing returns using the formula 10, the return of our assets is given by the following figure 1.



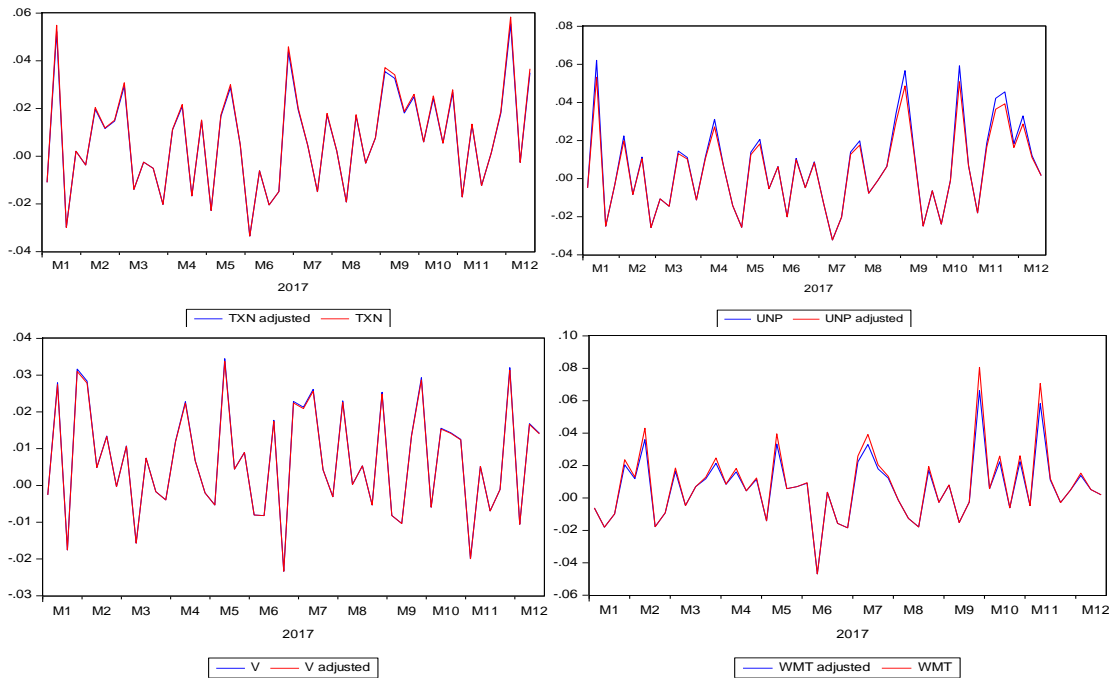


Figure 1. Evolution of portfolio assets' return and their adjustment.

These returns will be used only to calculate the variance-covariance matrix, while the original returns value will be used to calculate the expected return of our portfolio, hence, only the risk of our portfolio will change.

4. Results and Discussion

In this section, we preside to applying the original mean-variance model introduced by Markowitz, by using an adjusted variance-covariance matrix. Therefore, through observing the variance-covariance matrix of original and adjusted returns, we can conclude that the adjusted variance-covariance matrix has fewer risk Tables (2) and (3).

Table 2. Variance - Covariance matrix

Assets	BIDU	MDLZ	MMM	AAPL	ABT	ABBV	EBAY	BABA	BMJ	KO	COP	LLY	XOM	GILD	HD	INTC	MCD	NKE	PEP	TXN	UNP	V	WBA	WMT	
BIDU	0.001101																								
MDLZ	5.57E-05	0.000249																							
MMM	-2.22E-05	6.46E-05	0.000452																						
AAPL	0.000168	9.14E-06	-1.08E-04	0.000598																					
ABT	3.00E-05	-3.01E-05	6.50E-05	6.25E-05	0.000336																				
ABBV	4.75E-05	6.81E-08	7.00E-05	3.76E-05	0.000154	0.000776																			
EBAY	9.16E-05	-2.82E-05	-9.23E-05	0.00029	9.40E-05	0.000166	0.000666																		
BABA	0.000111	-3.53E-05	-0.00017	0.000127	-1.22E-05	0.000314	0.000131	0.001255																	
BMJ	4.71E-05	-2.81E-05	0.000188	-6.13E-05	0.000174	0.0003	-5.05E-06	-2.20E-05	0.000846																
KO	1.75E-05	4.55E-05	-1.13E-05	-8.20E-06	-1.37E-05	5.58E-05	1.87E-05	5.72E-05	-7.71E-05	0.000179															
COP	0.000128	2.81E-06	-2.10E-05	2.87E-05	-8.43E-05	1.53E-05	3.13E-05	4.46E-05	7.28E-06	-6.67E-05	0.000909														
LLY	-7.45E-05	-2.13E-05	0.000178	5.58E-05	0.000133	0.000202	0.000134	-0.00011	0.000271	-6.09E-05	7.40E-05	0.000455													
XOM	5.32E-06	1.96E-05	3.99E-05	-2.76E-05	-2.58E-05	0.000153	7.18E-05	3.85E-05	9.46E-05	-6.10E-05	0.000271	9.44E-05	0.000244												
GILD	8.32E-05	-0.00013	7.32E-05	-3.36E-05	0.000143	0.000137	4.47E-05	-0.00011	0.000181	-9.66E-05	-3.58E-05	0.000201	9.43E-05	0.000892											
HD	8.14E-05	2.84E-05	0.000185	-7.45E-06	2.73E-05	1.02E-04	2.19E-05	-8.88E-05	7.45E-05	-2.81E-05	9.04E-05	0.000123	8.85E-05	4.63E-05	0.000318										
INTC	6.35E-06	5.60E-05	-1.99E-05	0.000221	-5.38E-05	4.69E-05	5.30E-05	6.92E-05	-0.00025	4.01E-05	0.000377	-1.32E-04	5.70E-05	-0.00024	7.78E-05	0.000885									
MCD	6.76E-05	4.50E-05	1.07E-05	0.000106	3.36E-06	3.43E-05	3.98E-05	7.73E-05	5.68E-05	3.62E-05	-4.65E-05	-1.73E-05	-1.42E-06	-4.22E-05	-1.92E-05	1.76E-05	0.000226								
NKE	0.000109	4.87E-05	0.000186	5.50E-06	0.000104	5.70E-05	-7.82E-05	-0.00015	0.000174	-6.61E-05	0.000145	0.000181	5.38E-05	1.98E-05	0.000131	7.26E-05	-7.42E-05	0.00094							
PEP	-2.13E-05	7.62E-05	-5.29E-05	9.72E-06	-8.35E-06	-5.33E-05	-2.73E-05	-1.27E-05	-5.39E-05	9.24E-05	-6.15E-05	-6.30E-05	-4.87E-05	-0.00014	9.21E-07	-3.47E-05	4.58E-05	2.89E-05	0.0001713						
TXN	0.00013	6.89E-05	4.95E-05	0.000131	6.65E-05	1.97E-05	4.94E-05	2.52E-05	-0.00014	2.12E-05	0.000115	-1.56E-05	6.10E-06	-3.18E-05	5.48E-05	0.000225	-1.97E-06	7.26E-05	1.08E-05	0.000453					
UNP	-0.000205	6.85E-05	4.10E-05	-1.47E-06	-6.87E-05	2.68E-05	6.10E-05	7.90E-05	-0.00013	-4.28E-05	0.000269	-2.14E-05	8.17E-05	-3.43E-05	1.06E-04	0.000258	2.02E-05	0.00013	9.24E-06	0.000176	0.000521				
V	0.000119	0.00011	3.45E-05	0.00012	1.48E-05	1.15E-04	7.12E-05	0.000154	5.73E-05	4.32E-05	3.40E-05	-4.70E-06	3.59E-05	4.53E-05	-1.56E-05	5.74E-05	7.72E-05	8.86E-05	2.47E-05	0.000116	7.59E-05	0.00022			
WBA	1.03E-05	4.37E-06	0.00015	2.97E-06	0.00013	2.86E-05	-7.43E-05	-8.96E-05	0.000168	-6.14E-05	2.21E-05	0.000107	1.23E-05	0.000118	0.000153	-6.37E-05	-0.000118	0.00038	-1.38E-05	-7.65E-06	1.65E-05	1.84E-05	0.00066		
WMT	0.000307	0.000136	5.34E-05	1.11E-04	4.91E-05	3.66E-06	-3.90E-05	-1.08E-05	7.62E-05	3.58E-05	1.03E-04	-2.10E-06	3.75E-05	-5.86E-05	6.76E-05	4.46E-05	7.30E-05	0.00019	1.05E-04	7.23E-05	-1.54E-05	1.08E-04	6.95E-05	0.00048	

Table 3. Variance - Covariance matrix after adjustment.

Assets	BIDU	MDLZ	MMM	AAPL	ABT	ABBV	EBAY	BABA	BMJ	KO	COP	LLY	XOM	GILD	HD	INTC	MCD	NKE	PEP	TXN	UNP	V	WBA	WMT	
BIDU	0.001101																								
MDLZ	5.57E-05	0.000249																							
MMM	-1.96E-05	6.01E-05	0.0004																						
AAPL	0.000168	9.14E-06	-9.98E-05	0.000598																					
ABT	3.03E-05	-2.97E-05	5.95E-05	5.99E-05	0.000319																				
ABBV	2.76E-05	-5.28E-06	4.14E-05	4.65E-05	0.000128	0.000526																			
EBAY	9.12E-05	-2.81E-05	-8.16E-05	0.000286	8.83E-05	0.000135	0.000638																		
BABA	0.000114	-3.07E-05	-0.00016	0.000125	-9.04E-06	0.000263	0.000128	0.001127																	
BMJ	4.71E-05	-2.81E-05	0.000177	-6.13E-05	0.000171	0.00025	-4.66E-06	-1.47E-05	0.000846																
KO	1.75E-05	4.55E-05	-1.08E-05	-8.20E-06	-1.32E-05	4.04E-05	1.85E-05	5.82E-05	-7.71E-05	0.000179															
COP	0.000128	2.81E-06	-2.16E-05	2.87E-05	-8.12E-05	1.05E-05	3.28E-05	3.41E-05	7.28E-06	-6.67E-05	0.000999														
LLY	-7.45E-05	-2.13E-05	0.000172	5.58E-05	0.00013	0.000167	0.000131	-0.0001	0.000271	-6.09E-05	7.40E-05	0.000455													
XOM	5.32E-06	1.96E-05	3.70E-05	-2.76E-05	-2.48E-05	0.000129	7.09E-05	3.33E-05	9.46E-05	-6.10E-05	0.000271	9.44E-05	0.000244												
GILD	7.99E-05	-0.00012	6.73E-05	-3.91E-05	0.000135	0.00011	3.68E-05	-0.0001	0.000166	-9.11E-05	-3.76E-05	0.000189	8.95E-05	0.000808											
HD	7.72E-05	2.78E-05	-4.82E-05	-7.57E-06	-3.62E-05	7.68E-05	2.00E-05	-8.68E-05	7.19E-05	-2.75E-05	8.81E-05	0.000121	8.59E-05	4.32E-05	0.000304										
INTC	3.12E-05	4.38E-05	9.87E-08	0.000158	-3.40E-05	3.13E-05	3.72E-05	1.53E-05	-0.00017	3.79E-05	0.000266	-8.42E-05	4.09E-05	-0.00015	5.51E-05	0.000457									
MCD	6.62E-05	4.41E-05	1.11E-05	0.000105	3.22E-06	3.78E-05	3.95E-05	7.73E-05	5.64E-05	3.59E-05	-4.64E-05	-1.70E-05	-1.46E-06	-4.08E-05	-2.00E-05	1.15E-05	0.000221								
NKE	0.000109	4.87E-05	0.000172	5.50E-06	0.000103	4.37E-05	-7.72E-05	-0.00014	0.000174	-6.61E-05	0.000145	0.000181	5.38E-05	1.76E-05	0.000129	8.38E-05	-7.32E-05	0.00094							
PEP	-2.13E-05	7.62E-05	-4.82E-05	9.72E-06	-8.14E-06	-2.68E-05	-8.09E-06	-5.39E-05	9.24E-05	-6.15E-05	-6.30E-05	-4.87E-05	-0.00014	6.38E-07	-2.37E-05	4.55E-05	2.89E-05	0.0001713							
TXN	0.000129	6.77E-05	4.66E-05	0.000129	6.40E-05	1.64E-05	4.95E-05	2.32E-05	-0.00014	2.16E-05	0.000111	-1.47E-05	5.86E-06	-2.92E-05	5.30E-05	0.000175	-1.30E-06	7.15E-05	1.10E-05	0.000427					
UNP	-0.000187	6.19E-05	4.11E-05	-3.14E-06	-5.77E-05	2.35E-05	5.45E-05	5.99E-05	-0.00011	-4.03E-05	0.000248	-1.31E-05	7.78E-05	-2.44E-05	9.63E-05	0.000161	1.64E-05	0.00012	7.07E-06	0.000154	0.000431				
V	0.000117	0.000109	3.06E-05	0.000118	1.50E-05	9.54E-05	6.91E-05	0.000148	5.66E-05	4.26E-05	3.35E-05	-4.43E-06	3.56E-05	4.07E-05	-1.51E-05	5.30E-05	7.53E-05	8.71E-05	2.41E-05	0.000113	7.04E-05	0.000214			
WBA	1.03E-05	4.37E-06	0.000139	2.97E-06	0.000127	2.62E-05	-7.49E-05	-8.64E-05	0.000168	-6.14E-05	2.21E-05	0.000107	1.23E-05	0.000113	6.000149	-3.50E-05	-0.000117	0.00038	-1.38E-05	-7.14E-06	2.53E-05	1.87E-05	0.00066		
WMT	0.000261	0.00012	4.41E-05	9.20E-05	4.41E-05	8.84E-06	-3.56E-05	-1.47E-05	7.02E-05	2.97E-05	9.72E-05	-3.27E-06	3.55E-05	-5.28E-05	5.98E-05	4.67E-05	6.09E-05	0.00018	9.20E-05	6.04E-05	-2.55E-06	9.35E-05	7.67E-05	0.00038	

To prove this, we proceed to the simulation of both models using the MATLAB financial toolbox package. We obtained a 10-equidistance portfolio, which expresses the efficient frontier for both our portfolio's original and adjusted one. The results are given by both tables 4 and 5.

Table 4. Optimal 10-equidistance portfolio using original variance-covariance matrix

Assets	port 1	port 2	port 3	port 4	port 5	port 6	port 7	port 8	port 9	port 10	Annualized risk
BIDU	0.02207	0.02178	0.00599	0	0	0	0	0	0	0	0.23928
MDLZ	0.01941	0	0	0	0	0	0	0	0	0	0.11385
MMM	0.04464	0.07810	0.08486	0.06909	0.03468	0	0	0	0	0	0.15326
AAPL	0.02590	0.03217	0.03344	0.02411	0.00507	0	0	0	0	0	0.17630
ABT	0.05722	0.09530	0.12211	0.14108	0.16192	0.1171	0.0616	0	0	0	0.13222
ABBV	0	0	0.0007	0.0216	0.0358	0.1234	0.2182	0.3017	0.3340	0	0.20084
EBAY	0	0	0.0027	0	0	0	0	0	0	0	0.18614
BABA	0.0207	0.0462	0.0728	0.094	0.125	0.230	0.347	0.476	0.645	1	0.25548
BMJ	0.00978	0	0	0	0	0	0	0	0	0	0.20975
KO	0.16748	0.11362	0.07025	0.00813	0	0	0	0	0	0	0.09653
COP	0	0.00473	0.03044	0.02159	0	0	0	0	0	0	0.21746
LLY	0.03451	0.02178	0	0	0	0	0	0	0	0	0.15387
XOM	0.14447	0.10200	0.0235	0	0	0	0	0	0	0	0.11257
GILD	0.06196	0.05324	0.0481	0.0195	0	0	0	0	0	0	0.21536
HD	0	0.02154	0.08856	0.13544	0.18129	0.13107	0.04655	0	0	0	0.12865
INTC	0.03592	0.02689	0.01030	0	0	0	0	0	0	0	0.21455

MCD	0.067 7	0.099 3	0.152 8	0.206 6	0.268 2	0.187 6	0.077 8	0	0	0	0.10829
NKE	0	0	9E- 05	0	0	0	0	0	0	0	0.22095
PEP	0.204 9	0.212 6	2E- 01	0.143 5	0.009 1	0	0	0	0	0	0.09439
TXN	0	0.012 78	4E- 02	0.066 5	0.097 9	0.105 9	0.102 5	0.068 0	0	0	0.15351
UNP	0.041 78	0.044 58	3E- 02	0.015 9	0.010 3	0	0	0	0	0	0.16459
V	0	0	0	0.017 4	0.007 2	0	0	0	0	0	0.10684
WBA	0.041 53	0.013 46	0	0	0	0	0	0	0	0	0.18524
WMT	0	0	0	0.015 2	0.063 6	0.105 0	0.146 7	0.154 6	0.021 1	0	0.15851
Return	0.213 07	0.290 98	0.368 89	0.446 80	0.524 72	0.602 63	0.680 54	0.758 45	0.836 36	0.914 27	
Variance	0.005 64	0.005 91	0.006 64	0.007 75	0.009 28	0.012 10	0.016 45	0.021 48	0.027 28	0.035 43	

The table 4 shows the top 10-equidistance optimal portfolio consists of 24 assets.

Table 5. Optimal 10-equidistance portfolio using adjusted variance-covariance matrix

Assets	port 1	port 2	port 3	port 4	port 5	port 6	port 7	port 8	port 9	port 10	Annualized risk
BIDU	0.018 83	0.021 57	0.005 68	0	0	0	0	0	0	0	0.23928
MDL Z	0.017 91	0	0	0	0	0	0	0	0	0	0.11385
MM M	0.050 46	0.084 56	0.097 30	0.077 63	0.044 88	0	0	0	0	0	0.14431
AAPL	0.018 50	0.025 32	0.019 20	0.016 21	0.003 51	0	0	0	0	0	0.17630
ABT	0.054 28	0.096 91	0.108 95	0.126 38	0.142 40	0.091 44	0.025 70	0	0	0	0.12879
ABB V	0	0	0.021 4	0.049 5	0.073 9	0.180 8	0.304 2	0.393 6	0.372 7	0	0.16532
EBA Y	0.004 45	0	0.010 7	0	0	0	0	0	0	0	0.18215
BAB A	0.025 55	0.052 56	0.074 70	0.094 65	0.119 02	0.204 05	0.303 87	0.432 25	0.627 26	1	0.24205
BMY	0.009 9	0	0	0	0	0	0	0	0	0	0.20975
KO	0.142 5	0.089 92	0.033 52	0	0	0	0	0	0	0	0.09653
COP	0	0	0.026 38	0.010 16	0	0	0	0	0	0	0.21746
LLY	0.033 09	0.023 21	0	0	0	0	0	0	0	0	0.15387
XOM	0.130 25	0.091 44	0.006 30	0	0	0	0	0	0	0	0.11257
GILD	0.068 48	0.058 34	0.057 81	0.026 87	0	0	0	0	0	0	0.20494
HD	0	0.020 28	0.073 22	0.111 73	0.154 02	0.120 95	0.036 94	0	0	0	0.12578
INTC	0.067 06	0.061 94	0.049 93	0.028 04	0	0	0	0	0	0	0.15413
MCD	0.067 97	0.093 54	0.140 71	0.185 23	0.235 91	0.163 53	0.051 78	0	0	0	0.10726

PEP	0.213 32	0.219 07	0.214 69	0.142 35	0.015 12	0	0	0	0	0	0.09439
TXN	0	0.003 41	0.027 99	0.058 35	0.091 25	0.103 4	0.099 7	0.032 3	0	0	0.14894
UNP	0.038 91	0.048 86	0.024 51	0.026 54	0.023 85	0	0	0	0	0	0.14967
WBA	0.038 50	0.009 07	0	0	0	0	0	0	0	0	0.18524
WMT	0	0	0.006 94	0.046 37	0.096 13	0.135 89	0.177 77	0.141 83	0	0	0.13982
return	0.220	0.297	0.374	0.451	0.528	0.605	0.682	0.759	0.837	0.914	
n	01	15	29	43	57	71	85	99	13	27	
variance	0.005	0.005	0.006	0.007	0.008	0.011	0.015	0.019	0.025	0.033	
nce	53	80	50	54	93	38	19	75	28	57	

Table 5 gives us the optimal composition of the 10-equidistance portfolio obtained by the simulation of the adjusted variance-covariance matrix. We notice that the optimization leads to a portfolio consisting of 22 securities. The assets ‘NKE’ and ‘V’ did not take part in this portfolio.

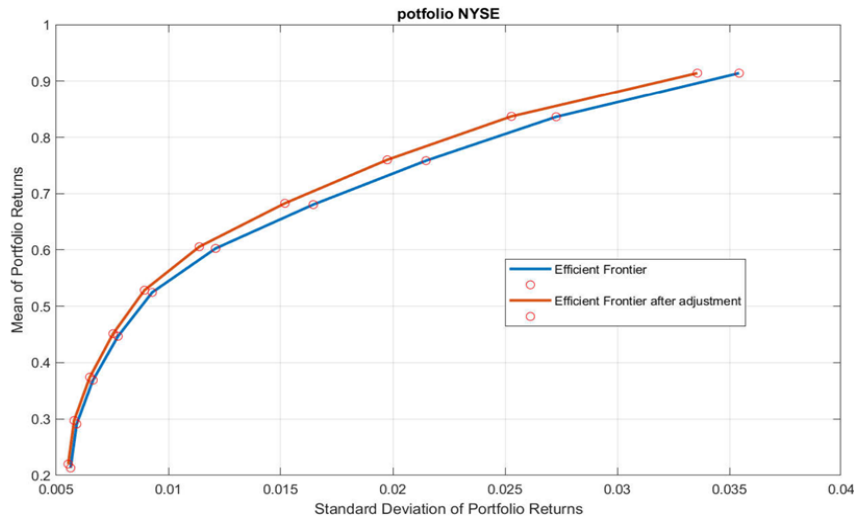


Figure 2. Efficiency frontier before and after adjustment of variance- covariance matrix.

After plotting the efficient frontier for both portfolios, we can observe that the adjusted efficient frontier has less risky and higher returns. This proves that the adjustment that we applied has changed the composition of our portfolio, whereas our adjustment only changed the positive semi-variance. In our case, the positive semi-variance does not represent a risk (no short selling allowed).

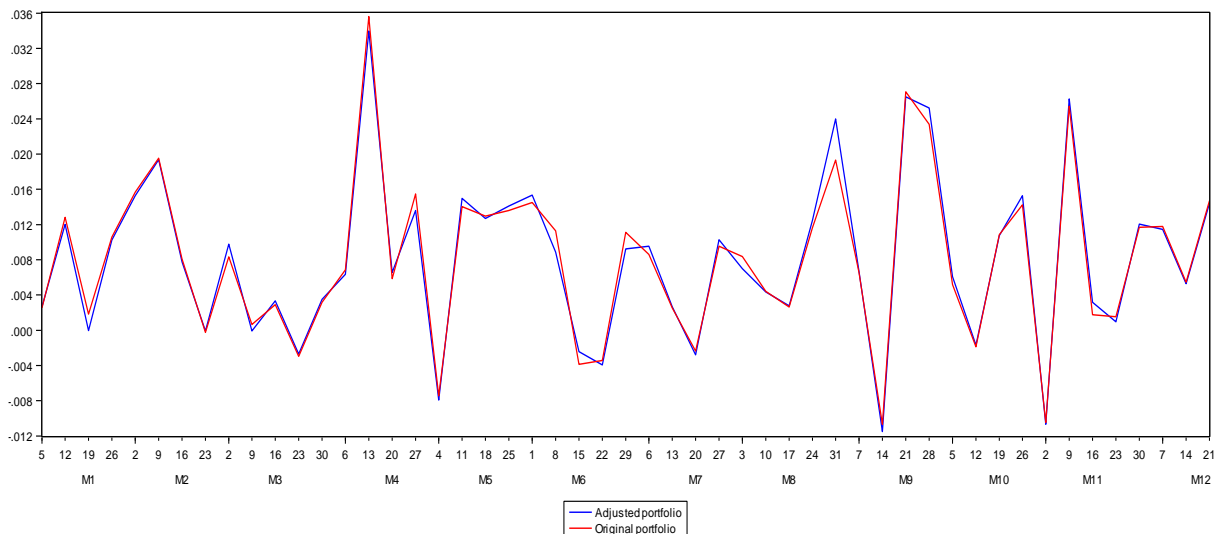


Figure 3. Evolution of portfolio N°5 return before and after adjustment

Figure 3 shows that the adjusted portfolio tends to have more positive returns than the original one. Also, table 6 confirms our result, the return and risk of the fifth portfolio show much difference between the original and adjusted portfolio.

Table 6. Risk and return of the fifth portfolio before and after adjustment

adjusted portfolio	return	0.52857
	variance	0.00893
original portfolio	return	0.52472
	variance	0.00928

5. Conclusion

Since the security market is intricate, some assets hide their true risk. Thereby, in this paper, we try to examine the symmetry of risk, since taking the same risk for both upward and downward shocks is rarely observed in the market. Accordingly, we adjust the variance-covariance matrix by equalizing the two (up and down) semi variances. This procedure removes a part of risk which is unnecessary and only disturbs our optimization.

After the numerical computation of our model using both mean-variance models and mean adjusted variance models, the obtained result shows that the adjusted model has more return and less risk than the classical mean-variance model, this portfolio will reflect the true value of risk to investors. In our future research, we will be adding the transaction cost, as well as some investment restrictions trying to test the flexibility and usability of this model.

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