Relations of Pre Generalized Regular Weakly Locally Closed Sets in Topological Spaces

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Article History: Received: 10 November 2020; Revised 12 January 2021 Accepted: 27 January 2021; Published online: 5 April 2021

Abstract: In this paper pgrw-locally closed set, pgrw-locally closed*-set and pgrw-locally closed**-set are introduced. A subset A of a topological space (X,t) is called pgrw-locally closed (pgrw-lc) if A=GÇF where G is a pgrw-open set and F is a pgrw-closed set in (X,t). A subset A of a topological space (X,t) is a pgrw-lc* set if there exist a pgrw-open set G and a closed set F in X such that A= GÇF. A subset A of a topological space (X,t) is a pgrw-lc**-set if there exists an open set G and a pgrw-closed set F such that A=GÇF.

The results regarding pgrw-locally closed sets, pgrw-locally closed* sets, pgrw-locally closed** sets, pgrw-lc-continuous maps and pgrw-lc-irresolute maps and some of the properties of these sets and their relation with other lc-sets are established.

Keywords: pgrw-lc, pgrw-lc*, pgrw-lc**-set, pgrw-sub-maximal space, pgrw-lc-continuous maps

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1. Introduction

According to Bourbakia subset A of a topological space X is called locally closed in X if it is the intersection of an open set and a closed set in X. Gangster and Reilly used locally closed sets to define LC-Continuity and LC-irresoluteness. Balachandran, Sundaram and Maki introduced the concept of generalized locally closed sets in topological spaces and investigated some of their properties.

2. Preliminaries:

2.1 Definition: A subset A of a topological space

(X, τ) is called

i. a semi-open set [4] if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.
ii. a pre-open set [5] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.
iii. an α-open set [6] if $A \subseteq int(cl(int(A)))$ and α -closed set if $cl(int(cl(A))) \subseteq A$.
iv. a semi-pre-open set (β-open) [7] if $A \subseteq cl(int(cl(A))))$ and a semi-pre closed set (β-closed) if $int(cl(int(A))) \subseteq A$.
v. a regular open set [8] if A = int(clA)) and a regular closed set if A = cl(int(A)).
vi. δ-closed [9] if $A=cl\delta(A)$, where $cl\delta(A)=\{x \epsilon X : int(cl(U)) \cap A \neq \theta, U \epsilon T \text{ and } x \epsilon U\}$
vii. Regular semi open [10] set if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.
viii. a regular generalized closed set(briefly rg-closed) [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
ix. a generalized semi pre regular closed (gspr-closed) set [12] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in S.
x. a generalized semi-pre closed set(briefly gsp-closed) [13] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
xi. a pre generalized pre regular closed set [14] (pgpr-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is rg-open in X.
xii. a generalized pre closed (briefly gp-closed) set [3] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
xiii. a regular w-closed set (rw-closed) [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in Ş.
xiv. a #regular generalized closed (briefly #rg-closed) set [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is rw-open.

2.2 Definition: A subset A of a topological space (X, τ) is called a pre generalized regular weakly closed set if pcl(A)⊆U whenever A ⊆ U and U is a rw-open set [17].

The complements of the abovementioned closed sets are their open sets respectively.

2.3 Definition: Let (X, τ) be a topological space and A⊆X. The intersection of all closed (resp pre-closed, α-closed and semi-pre-closed) subsets of space X containing A is called the closure (resp pre-closure, α-closure and Semi-preclosure) of A and denoted by cl(A) (resppcl(A), αcl(A), spcl(A)).

3. pgrw-locally-closed sets

3.1 Definition: A subset A of a topological space (X,τ) is pgrw-locally closed (pgrw-lc) if Ą=G∩F where G is a pgrw-open set and F is a pgrw-closed set in (X,τ).

The set of all pgrw-locally closed subsets of (X,τ) is given by PGRWLC(X,τ).

3.2 Example: X={1,2,3,4} and τ ={X, ϕ,{1},{2},{1,2}, {1,2,3}}.
rw-open sets are X, ϕ,{1},{2},{3},{4},{3,4},{1,2},{1,2,3}.
Pre-closed sets are X, ϕ,{3},{4},{3,4},{2,3,4},{1,3,4}.
pgrw-closed sets are X, ϕ,{3},{4},{2,3},{3,4},{1,4},{2,4}, {2,3,4}, {1,3,4},{1,2,4}.
The set {2,3}={1,2,3}∩{2,3,4} is a pgrw-lc set where {1,2,3} is pgrw-open and {2,3,4} pgrw-closed.

3.3 Remark: In the space of 3.2 the set{3}={1,2,3}∩{3,4} is a pgrw-lc set where {1,2,3} is pgrw open and {3,4}, pgrw-closed and also {3}={1,3}∩{2,3,4} where {1,3} is pgrw open and {2,3,4} is pgrw-closed. Therefore G and F are not unique.

3.4 Theorem: subset A of X is pgrw-lc if and only if its complement A^c is the union of a pgrw-open set and a pgrw-closed set.

Proof: A is a pgrw-lc set in (X,τ).

⇒A=G∩F where G is a pgrw-open set and F is a pgrw-closed set.

⇒ $A^c=(G\cap F)^c = G^c\cup F^c$ where $G^c$is a pgrw-closed set and F^c is a pgrw-open set.

Conversely,A is a subset f (X,τ) such that A^c =G∪F where G is a pgrw-open set and F is a pgrw-closed set.

⇒ $(A^c)^c = (G\cup F)^c$

⇒ A= $G^c\cap F^c = F^c\cap G^c$ where F^c is a pgrw-open set and $G^c$is a pgrw-closed set.

⇒ A is a pgrw-lc set.

3.5 Theorem:

i) Every pgrw-open set in X is pgrw-lc.

ii) Every pgrw-closed set in X is pgrw-lc
Proof: i) A is a pgrw-open set in X.

⇒ A=SA∩X where Ą is pgrw-open and X is pgrw-closed.

⇒ A is pgrw-lc.

ii) A is a pgrw-closed subset of X.

⇒ A=X∩A where X is pgrw-open and A is pgrw-closed.

⇒ A is pgrw-lc.

The converse statements are not true.

3.6 Example: In **3.2,** the set {2,4}=X∩{2,4} is pgrw-lc, but not pgrw-open.The set {1,3}={1,3}∩{1,3,4} is pgrw-lc, but not pgrw-closed.

3.7 Corollary: In X

Everyopen set is pgrw-lc.

i) every closed set is pgrw-lc.
Proof: i) A is open in X.

⇒ Aispgrw-open in X.

⇒ A is pgrw-lc in X.

ii) Ais closed in X.

⇒ A is pgrw-closed in X.

⇒ A is pgrw-lc in X.

The converse statements are not true.

3.8 Example: In 3.2**,**{2,4} is pgrw-lc, but not open and {1,3} is pgrw-lc, but not closed.

3.9 Theorem: Every locally closed set in X is pgrw-lc.

Proof: A is a locally closed subset of X.

⇒ A = G∩H, G is an open set and H is a closed set.

⇒ A = G∩H,G is pgrw-open and H is pgrw-closed.

⇒ A is pgrw-lc in X.

The converse statement is not true.

3.10 Example: In 3.2**,** the set {2, 4} is pgrw-lc, but not alc-set.

3.11 Theorem: In X

i) every locally-δ-closed set is pgrw-lc.
ii) every regular-locally closed set is pgrw-lc.
iii) every α-locally closed set is pgrw-lc.
iv) every #rg-locally closed set is pgrw-lc.
v) Everypgpr-locally-closed set is pgrw-lc.

Proof: i) A is a lδc-set in (X,τ).

⇒ A=G∩F, G is δ-open and F is δ-closed.

⇒ A=G∩F, G is pgrw-open and F is pgrw-closed in X.

⇒ A is a pgrw-lc set in (X,τ).

The other statements may be proved similarly.

The converse statements are not true.

3.12 Example: In 3.2,δ-closed sets in X are X, ϕ,{3,4},{2,3,4},

{1,3,4}. The set {2,4} is pgrw-lc, but not lδc.

3.13 Example: In 3.2**,** regular-closed sets in X are X, ϕ, {2,3,4},{1,3,4}. The set {2,4} is pgrw-lc, but not regular-lc.

3.14 Example: In X = {1,2,3,4}, τ={X,ϕ,{2,3},{1,2,3}, {2,3,4}}. α-closed sets in X are X, ϕ,{1,4},{1},{4}.

The set {1,3}=X∩{1,3} is pgrw-lc, but not α-lc.

3.15 Example: In 3.2 #rg-closed sets in X are X, ϕ, {4},

{3,4},{1,4},{2,4},{1,3},{2,3,4}, {1,3,4}. The set {1,2,4}=X∩{1,2,4} is pgrw-lc, but not #rg-lc.

3.16 Example: In 3.2 pgpr-closed sets in X are X, ϕ,{3}, {4},{3,4},{1,3,4}, {2,3,4}. The set {1,2}={1,2}∩X is pgrw-lc, but not pgpr-lc.

3.17 Theorem: In X every pgrw-locally closed set is

i) gp-lc ii) gpr-lc iii) gsp-lciv) gspr-lc

Proof: i) A is a pgrw-lc set in X.

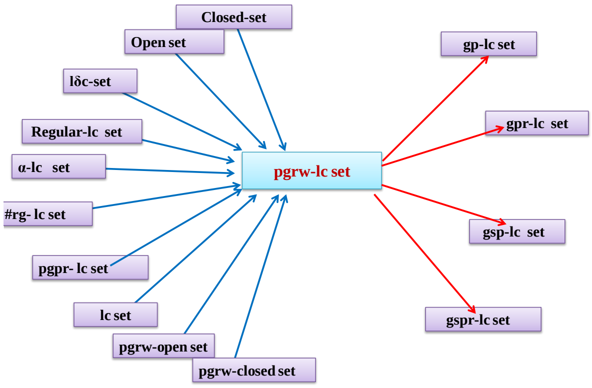
⇒ A=G∩H, G is pgrw-open and H is pgrw-closed.

⇒ A=G∩H, G is gp-open and H is gp-closed.

⇒ A is a gp-lc set in (X,τ).

The other statements may be proved similarly.

3.18 Remark: The above results are shown in the following diagram



4. pgrw-locally closed*-sets

4.1 Definition: A subset Aof a topological space (X,τ) is a pgrw-lc* set if there exist a pgrw-open set G and a closed set F in X such that A= G∩F.

The set of all pgrw-lc* subsets of (X,τ) is denoted by PGRWLC*(X,τ).

4.2 Example: Refer 3.2, {2,3}={1,2,3}∩{ b, c, d} is pgrw-locally closed* set, because {1,2,3} is pgrw-open and {2,3,4} is closed.

4.3 Theorem: Every lc-set of X is a pgrw-lc*-set .

Proof: A is alc-set in X.

⇒ A=G∩C, G is open and C is closed in X.

⇒ A=G∩C, G is pgrw-open and C is closed in X.

⇒ A is a pgrw-lc*-set in X.

The converse statement is not true.

4.4 Example: X={1,2,3}, τ={X, ϕ,{1},{2,3}}.

pgrw-closed sets are all subsets of X. The set {1,2}is pgrw-open and {2,3} is closed. Since {2}={1,2}∩{2,3} is a pgrw-lc*-set, but not a lc-set.

4.5 Theorem: Every pgrw-lc*-set of X is a pgrw-lc set.

Proof: A is a pgrw-lc*-set in X.

⇒ A=G∩C where G is pgrw-open and C is closed in X.

⇒A=G∩C where G is pgrw-open & C is pgrw-closed in X.

⇒ A is a pgrw-lc-set in X.

4.6 Theorem: A subset A of X is pgrw-lc* iff A= G∩cl(A) for some pgrw-open set G.

Proof:A is a pgrwlc*-set in X.

⇒ A=G∩F for a pgrw-open set G and a closed set F in X.

⇒ A⊆G and A⊆F, a closed set.

⇒ A⊆G∩cl(A) and cl(A)⊆F

⇒ A⊆G∩cl(A) and G∩cl(A)⊆G∩F=A.

⇒ A=G∩cl(A).

Conversely, A = G∩cl(A) where G is a pgrw-open set.

⇒ A is the intersection of a pgrw-open set and a closed set.

⇒ A is pgrw-lc*.

4.7 Theorem: If for a subset V of X, $V\cup(cl(V))^C$ is pgrw-open, then V is pgrw-lc*.

Proof: ∀ subset V of X.

$V = V\cup\phi$

$= V\cup((cl(V))^c\cap cl(V))$

$= (V\cup(cl(V))^c)\cap(V\cup cl(V))$

$= (V\cup(cl(V))^c\cap cl(V)$, because $V\subseteq cl(V)$.

So if $V\cup(cl(V))^c$ is pgrw-open, then V is the intersection of a pgrw-open set and a closed set. Therefore V is pgrw-lc*.

4.8 Corollary: If for a subset Vof X the set cl(V)–V is pgrw-closed, then A is pgrw-lc*.

Proof: For any subset V of X

$cl(V)-V=cl(V)\cap V^c=(((cl(V))^c\cup V)^c$.

Therefore cl(V)–V is pgrw-closed.

$\Rightarrow V\cup(cl(V))^c$ is pgrw-open.

⇒ V is pgrw-lc*.

5. pgrw-locally closed**-sets

5.1 Definition: A subset A of (X,τ) is a pgrw-lc**-set if there exists an open set G and a pgrw-closed set F such that A=G∩F.

The set of all pgrw-lc**-sets of (X,τ) is denoted by PGRWLC**(X,τ).

5.2 Example: Refer 3.2, {1,2}∩{2,3,4}={2} is pgrw-locally closed**-set, because {1,2} is open and {2,3,4} is pgrw-closed.

5.3 Theorem: Every lc-set of X is a pgrw-lc**-set.

Proof: A is alc-set X.

⇒ A=G∩F where G is open and F is closed in X.

⇒ A=G∩F where G is open and F is pgrw-closed in X.

⇒ A is a pgrw-lc**-set in X.

The converse statement is not true.

5.4 Example: X={1,2,3}, τ={X, ϕ,{1},{2,3}}.

pgrw-closed sets in X are all subsets of X. The set {3}={2,3}∩{3} where {2,3} is open and {3} is pgrw-closed. So {3} is a pgrw-lc**-set. But {3} is not alc-set.

5.5 Theorem: Every pgrw-lc**-set in X is pgrw-lc.

Proof: A is a pgrw-lc**-set in X.

⇒ A=G∩F where G is open and F is pgrw-closed.

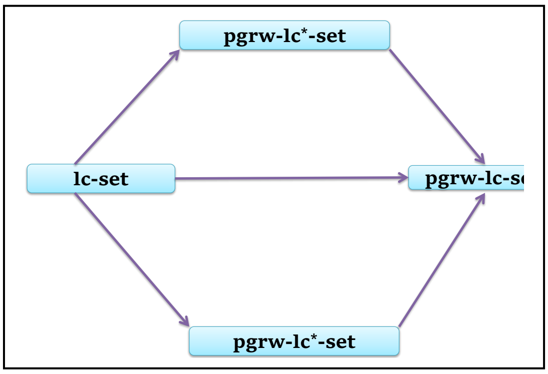
⇒ A=G∩F where G is pgrw-open and F is pgrw-closed.

⇒ A is a pgrw-lc-set.

The converse statement is not true.

5.6 Example: X={1,2,3}, τ={X, ϕ,{1},{1,3}} pgrw-closed sets are X, ϕ,{2},{3},{2,3}.{1,2} is apgrw-lc set, but not pgrw- lc**.

5.7 Remark: The following diagram shows the relation between lc-set, pgrw-lc-set, pgrw-lc*-set and pgrw**-set.



5.8 Theorem:

i) If A∈PGRWLC*(X, τ) and B is closed in (X, τ), then A∩B∈ PGRWLC*(X, τ).
ii) If A∈PGRWLC**(X, τ) and B is open in (X, τ), then A∩B∈ PGRWLC**(X, τ).

Proof: i) A∈PGRWLC*(X, τ) and B is closed in X.

⇒ A = P∩F where P is a pgrw-open set and F is a closed set in X and B is closed.

⇒A∩B=(P∩F)∩B =P∩(F∩B), where P is pgrw-open and

(F∩B) is closed.

⇒A∩B∈ PGRWLC*(X, τ).

ii) A∈PGRWLC**(X, τ) and B is open in X.

⇒A=P∩F where P is an open set and F is a pgrw-closed

set in X and B is open.

⇒A∩B=(P∩F)∩B=(P∩B)∩F, where (P∩B) is open and F is pgrw-closed.

⇒A∩B∈PGRWLC**(X,τ).

5.9 Theorem: If every pgrw-closed set is closed in (X, τ), then PGRWLC (X, τ) = LC(X, τ).

Proof: obvious.

6. pgrw-lc-continuous maps

6.1 Definition: A map f: (X,τ)→(Y,σ) is called pgrw-lc-continuous(pgrw-lc*-continuous, pgrw-lc**-continuous resp.) if ∀ V∈σf^{-1} (V)∈PGRWLC(X,τ), (f^{-1}(V)∈PGRWLC*(X,τ), f^{-1}(V)∈ PGRWLC**(X,τ) resp.)

6.2 Example: For (X,τ) refer 3.2, Y={1,2,3,4} σ = {Y, ϕ, {1,2}, {3,4}}. Define a map f by f(1)=2, f(2)=3, f(3)=4, f(4)=1. Pre-images X, ϕ,{1,4},{2,3} of σ-open sets belong to PGRWLC(X,τ) (PGRWLC*(X,τ), PGRWLC**(X,τ)). So f is a pgrw-lc continuous (pgrw-lc*-continuous, pgrw-lc**-continuous) map.

6.3 Theorem:

i) Every pgrw-lc*-continuous function is pgrw-lc- continuous.

ii) Every pgrw-lc**-continuous function is pgrw-lc- continuous.

Proof: i) A map f is pgrw-lc*-continuous.

⇒∀V∈σ, f^{-1}(V) ∈PGRWLC*(X,τ).

⇒∀V∈σ, f^{-1}(V) ∈PGRWLC (X,τ).

⇒ f is pgrw-lc-continuous.

Similarly (ii) may be proved.

6.4 Theorem:

i) If f is alc-continuous function, then f is pgrw-lc-continuous (pgrw-lc*-continuous, pgrw-lc**-continuous).
ii) If f is lδc-continuous, then f is pgrw-lc-continuous.
iii) If f is regular-lc-continuous, then f is pgrw-lc-continuous.
iv) If f is #rg-lc-continuous, then f is pgrw-lc-continuous.
v) If f is α-lc-continuous, then f is pgrw-lc-continuous.

Proof: i) A map f is lc-continuous.

$\Rightarrow \forall V \in \sigma f^{-1}(V) \in LC\ (X,\tau)$.

$\Rightarrow \forall V \in \sigma f^{-1}(V) \in PGRWLC\ (X,\tau)$

⇒ f is pgrw-lc-continuous.

Similarly, the other statements may be proved.

The converse statements are not true.

6.5 Example: For (X,τ) refer 3.2, Y={1,2,3,4}, σ ={X, ϕ,{1},{3,4},{1,3,4}. Define a map f by f(1)=2, f(2)=4, f(3)=1, f(4)=3. Pre-images X, ϕ,{3},{2,4},{2,3,4} of σ-open sets are pgrw-lc in X. So f is pgrw-lc continuous.

δ-closed sets in X are X,ϕ,{3,4},{2,3,4},{1,3,4}.

Regular-closed sets in X are X,ϕ, {2,3,4},{1,3,4}.

α-closed sets in X are X,ϕ,{2},{1,2},{2,3,4}.

The set {3,4} is σ-open. $f^{-1}(\{3,4\})=\{2,4\}$ is

i) not a lc-set. Therefore f is not lc-continuous.
ii) not a lδc-set. Therefore f is not lδc-continuous.
iii) not a regular-lc-set. Therefore f is not regular-lc-continuous.
iv) not a α-lc-set.Therefore, f is not α-lc-continuous.

6.6 Example: Consider the spaces in 6.5, #rg-closed sets in X are X,ϕ,{4},{3,4},{1,4},{2,4},{1,3},{2,3,4},{1,3,4}. Define a map f:(X,τ)→(Y,σ) by f(1)=1, f(2)=3, f(3)=2, f(4)=4. Pre-images ofσ-open sets are X, ϕ, {1}, {2, 4}, {1, 2, 4} which are pgrw-lc-sets. So f is pgrw-lc-continuous. But {1,3,4} is σ-open and $f^{-1}(\{1,3,4\}) =\{1,2,4\}$ is not #rg-lc set. Therefore f is not #rg-lc-continuous.

6.7 Theorem: If f is pgrw-lc-continuous, then it is

i) gp-lc-continuous. ii) gpr-lc-continuous.

iii) gsp-lc-continuous iv) gspr-lc-continuous

Proof: i) A map f is pgrw-lc-continuous.

$\Rightarrow \forall V \in \sigma f^{-1}(V) \in PGRWLC\ (X,\tau)$.

$\Rightarrow \forall V \in \sigma f^{-1}(V) \in GPLC\ (X,\tau)$.

⇒ f is gp-lc-continuous.

Similarly the other statements may be proved

6.8 Theorem: If X is a door space, then every map i is

i. pgrw-lc-continuous.
ii. pgrw-lc*-continuous
iii. pgrw-lc**-continuous

Proof : i) X is a door space and f is a map.

$\Rightarrow \forall A \in \sigma f^{-1}(A)$ is either open or closed in X.

$\Rightarrow \forall A \in \sigma f^{-1}(A)$ is either pgrw-open or pgrw-closed in X.

$\Rightarrow \forall A \in \sigma f^{-1}(A)=f^{-1}(A)\cap X$ where $f^{-1}(A)$ is pgrw-open and X is pgrw-closed or $f^{-1}(A) = X\cap f^{-1}(A)$ where X is pgrw-open and $f^{-1}(A)$ is pgrw-closed.

$\Rightarrow \forall A\in\sigma f^{-1}(A)$ is a pgrw-lc set in X.

$\Rightarrow$ f is pgrw-lc-continuous.

Similarly the other statements may be proved.

6.9 Theorem: IfX is pgrw-sub-maximal, then every function f is pgrw-lc*-continuous.

Proof: X is a pgrw-sub-maximal space.

$\Rightarrow PGRWLC^*(X,\tau) = P(X)$, the power set of X.

$\Rightarrow$ for any map ff$^{-1}(V)\in PGRWLC^*(X,\tau)\ \forall V\subseteq Y$.

$\Rightarrow f^{-1}(V) \in PGRW\text{-}LC^*(X,\tau)\ \forall V\in\sigma$.

$\Rightarrow$ f is pgrw-lc*-continuous.

6.10 Corollary: If X is pgrw-sub-maximal, thenevery function f is pgrw-lc-continuous.

Proof: obvious.

6.11 Theorem: If f is a pgrw-lc-continuous (resp. pgrw-lc*-continuous, pgrw-lc**-continuous) map and g is a continuous map, then goi:$(X,\tau)\rightarrow(Z,\eta)$ is pgrw-lc-continuous (resp. pgrw-lc*-continuous, pgrw-lc**-continuous).

Proof: g is continuous and f is pgrw-lc-continuous.

$\Rightarrow\forall\eta$-open set $U \in Z$ $g^{-1}(U)$ is open in (Y,σ) and $f^{-1}(g^{-1}(U))$ is pgrw-lc in X.

$\Rightarrow\forall\eta$-open set $U \in Z$ $(gof)^{-1}(U)))$ is pgrw-lc in X.

$\Rightarrow$gof:$(X,\tau)\rightarrow(Z,\eta)$ is pgrw-lc-continuous.

Similarly the other statements may be proved.

6.12 Definition: A function g is sub-pgrw-lc*-continuous if there is a basis β for (Y,σ) such that $f^{-1}(U)\in PGRWLC^*(X,\tau)\ \forall U\in\beta$.

6.13 Example: For (X,τ) and pgrw-open sets in X refer 3.2.

Y={1,2,3}, σ ={Y, ϕ, {1},{2},{1,2}}; β={Y, ϕ,{1},{2}} is a basis for (Y, σ). Define a function f by f(a)=3, f(2)=1, f(3)=2, f(4)=3. Pre-images of elements of β are X, ϕ, {2}, {3} and are pgrw-lc* sets. So f is sub-pgrw-lc*-continuous.

6.14 Theorem: If f is sub-lc-continuous, then it is sub-pgrw-lc*-continuous.

Proof: Follows from $LC(X,\tau)\subseteq PGRWLC(X,\tau)$.

The converse statement is not true.

6.15 Example: For (X,τ) refer 3.2, Y={1,2,3}, σ ={Y, ϕ, {1},{2},{1,2}}; β={Y, ϕ,{1},{2}} is a basis for σ. Define a function f:X$\rightarrow$Y by f(1)=3, f(2)=1, f(3)=2, f(4)=3. Pre-images of elements of β are X, ϕ,{2},{3} and are pgrw-lc*-sets. So f is sub-pgrw-lc*-continuous. Then f is not sub-lc-continuous, because $\{2\}\in\beta$, $f^{-1}(\{2\})=\{3\}$ is not a lc-set in X.

6.16 Theorem: If f is pgrw-lc*-continuous, then it is sub-pgrw-lc*-continuous.

Proof:f is pgrw-lc*-continuous.

$\Rightarrow V\in\sigma f^{-1}(V)\in PGRWLC^*(X,\tau)$.

$\Rightarrow V\in\beta$, a basis, $f^{-1}(V))\in PGRWLC^*(X,\tau)$, because $\beta\subset\sigma$.

$\Rightarrow$ f is sub-pgrw-lc*-continuous.

6.17 Theorem: If f is sub-pgrw-lc*-continuous, then there is a sub-basis S for (Y,σ) such that $f^{-1}(V) \in$ PGRWLC*(X, τ), ∀ V∈S.

Proof: If f is sub-pgrw-lc*-continuous, then there is a basis β for (Y,σ) such that $i^{-1}(U) \in$PGRWLC*(X,τ) for each U∈β. Since β is also a sub-basis for (Y,σ) the proof is obvious.

6.18 Remark: The composition of a sub-pgrw-lc*-continuous function and a continuous function need not be a sub-pgrw-lc*-continuous.

Proof: Take a sub-pgrw-lc*-continuous function f which is not pgrw-lc*-continuous**.** Hence there is a set V∈σ such that $f^{-1}(V) \notin$pgrw-lc*(X,τ). Let η={Y, ϕ, V}. Then η is a topology on Y and the identity function g is continuous. But the composition gof:(X,τ)→(Y,η) is not sub-pgrw-lc*-continuous.

7. pgrw-lc-irresolute maps

7.1 Definition: A map f:(X,τ)→(Y,σ) is called pgrw-lc irresolute if ∀pgrw-lc-set V in Y. $f^{-1}(V)$ is pgrw-lc in X.

Similarly pgrw-lc*-irresolute and pgrw-lc**-irresolute functions are defined.

7.2 Example: X={1, 2, 3}=Y, τ={X, ϕ, {1}, { 1, 3 }}; σ ={Y, ϕ,{1},{2,3}}.

pgrw-closed sets in X are X, ϕ,{2},{3},{2,3}. pgrw-closed sets in Y are all subsets of Y. Define a map f:X→Y by f(1)=2, f(2)=3, f(3)=1. f is pgrw-lc-irresolute.

7.3 Theorem: A map f is

i. pgrw-irresolute ⇒ f is pgrw-lc-irresolute.
ii. pgrw-lc-irresolute⇒ f is pgrw-lc-continuous.
iii. pgrw-lc*-irresolute ⇒ f is pgrw-lc*-continuous.
iv. pgrw-lc**-irresolute ⇒ f is pgrw-lc**-continuous.

Proof: ∀ map f and for sets U, F∈Y,

$f^{-1}(U \cap F) = f^{-1}(U) \cap f^{-1}(F)$.

i) V∈ PGRW-LC(Y,σ) and f ispgrw-irresolute.

⇒ V = U∩F for a pgrw-open set U and a pgrw-closed set F and

$f^{-1}(V) = f^{-1}(U) \cap f^{-1}(F)$, $f^{-1}(U)$ is pgrw-open and $f^{-1}(F)$ is pgrw-closed in (X,τ).

⇒∀V∈ PGRW-LC (Y,σ), $f^{-1}(V) \in$PGRW-LC(X,τ).

⇒f is pgrw-lc-irresolute.

ii) V∈σ and f is pgrw-lc-irresolute.

⇒ V∈PGRW-LC(Y,σ) and f is pgrw-lc-irresolute.

⇒ $f^{-1}(V) \in$ PGRW-LC (X,τ).Thus ∀V∈ σ, $f^{-1}(V) \in$ PGRW-LC (X,τ). Therefore f is pgrw-lc-continuous.

Similarly (iii) and (iv) follow.

7.4 Example: In 7.2, f is pgrw-lc-irresolute. As {16} is pgrw-closed in Y and $f^{-1}(\{2\}) = \{1\}$ is not pgrw-closed in X. So f is not pgrw-irresolute.

7.5 Theorem: If X is a door space, then every map f is pgrw-lc-irresolute.

Proof: X is a door space and f is a map.

⇒ $f^{-1}(A)$ is either open or closed ∀A in Y.

⇒ $f^{-1}(A)$ is either pgrw-open or pgrw-closed ∀Ain Y.

⇒ $f^{-1}(A) = f^{-1}(A) \cap X$ where $f^{-1}(A)$ is pgrw-open and X is pgrw-closed or $f^{-1}(A) = X \cap f^{-1}(A)$ where X is pgrw-open and $f^{-1}(A)$ is pgrw-closed. Thus ∀A in Y, $f^{-1}(A)$ is pgrw-lc in (X,τ) and so∀ V∈PGRW-LC(Y,σ), $f^{-1}(A)$ is pgrw-lc in (X,τ). ⇒f is pgrw-lc-irresolute.

7**.6 Theorem:** f and g are two functions.

f and g are pgrw-lc-irresolute

⇒gof is pgrw-lc-irresolute.

f is pgrw-lc-irresolute and g is pgrw-lc-continuous

⇒gof: $(X,\tau) \rightarrow (Z,\eta)$ is pgrw-lc-continuous.

Proof: i) The functions g and f are pgrw-lc-irresolute.

$\Rightarrow \forall V \in$ PGRW-LC(Z,η), $g^{-1}(V) \in$ PGRW-LC(Y,σ) and

$f^{-1}(g^{-1}(V)) \in$ PGRW-LC(X,τ).

$\Rightarrow \forall V \in$ PGRW-LC(Z,η), $(gof)^{-1}(V) \in$ PGRW-LC(X,τ).

⇒gof:$(X,\tau) \rightarrow (Z,\eta)$ is pgrw-lc-irresolute.

ii) g is pgrw-lc-continuous and f is pgrw-lc-irresolute.

$\Rightarrow \forall V \in \eta$, $g^{-1}(V) \in$ PGRW-LC(Y,σ) and

$f^{-1}((g^{-1}(V) \in$ PGRW-LC(X,τ)

$\Rightarrow \forall V \in \eta$, $(gof)^{-1}(V) \in$ PGRW-LC(X,τ)

⇒gof:$(X,\tau) \rightarrow (Z,\eta)$ is pgrw-lc-continuous.

7.7 Theorem: f and g are two functions.

i) f and g are pgrw-lc*-irresolute

⇒gof is pgrw-lc*-irresolute.
ii) f is pgrw-lc*-irresolute and g is pgrw-lc*-continuous
⇒gof is pgrw-lc*-continuous.

Proof: Similar to7.6.

7.8 Theorem: f and j are two functions.

i) f and g are pgrw-lc**-irresolute

⇒gof is pgrw-lc**-irresolute.
ii) f is pgrw-lc**-irresolute and g is pgrw-lc**-continuous
⇒gof is pgrw-lc**-continuous.
Proof: Similar to7.6

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