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RESEARCH ON THE MECHANISM OF INVOLUTION FROM THE PERSPECTIVE OF SUPERMODULAR GAMES

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ABSTRACT

By constructing two-player and multi-player game models and incorporating strategic complementarity analysis from supermodular games, this paper reveals how individuals increase their effort levels to cope with competition, leading to overall system resource waste. On this basis, the paper further analyzes how different game parameters affect equilibrium stability, showing how involution gradually forms through dynamic game evolution, and validates the model through simulation experiments. The novelty of this paper lies in the first-time application of supermodular games to the study of involution, providing a new theoretical perspective and a detailed exploration of the formation mechanism of involution through multi-level game models and simulation experiments. Furthermore, this paper proposes policy recommendations to address involution, emphasizing the optimization of institutional design and adjustments to incentive mechanisms to break the involution trap.

KEYWORDS: Involution; Supermodular Games; Strategic Complementarity; Policy Intervention; Resource Allocation

I. INTRODUCTION

The phenomenon of "involution" has emerged as a pervasive economic and social issue, widely present in fields such as education, the workplace, and business competition. It is typically characterized by individuals continuously escalating their investments in pursuit of minor relative advantages, only to fall into a trap of resource waste and reduced efficiency [1]. This phenomenon not only exacerbates societal competition pressures but also undermines the efficiency of resource allocation [2]. The study of involution was first introduced by Geertz to describe agricultural involution [1]. Over time, the concept of involution has been extended to other social and economic fields, including academic competition in education, work pressures in the workplace, and more [3]. Scholars in economics, sociology, and management have recognized the ubiquity of involution and have begun to use game theory and evolutionary games to analyze how individuals intensify their investments in competitive processes [4], [5]. Supermodular games, as a game theory model with strategic complementarity, provide an effective way to describe how participants increase their effort levels in response to competition, thereby leading to overall system resource waste [4]. Although the phenomenon of involution has received widespread attention across various fields, existing research has primarily focused on qualitative analysis and lacks systematic quantitative models and theoretical support. Traditional studies on involution tend to emphasize descriptive analysis and case studies, often approached from the perspectives of sociology, economics, or education, overlooking the system dynamics and complexity of interactions between individuals in game theory. For example, while some studies explore the relationship between involution and cultural capital, academic competition, and other factors, these studies have generally failed to delve deeply into how individual behaviors, through strategic complementarity in games, drive the intensification of involution. From the perspective of game theory, although existing literature has attempted to use game models to analyze competitive behavior in involution, most studies have not fully utilized supermodular games, a game model with strategic complementarity. Supermodular games can effectively describe how strategies among participants enhance each other [6], particularly in contexts where competition and cooperation coexist, which is especially important in the case of involution. However, current models are often oversimplified, neglecting nonlinear features and dynamic evolution processes in individual decision-making, which prevents them from fully reflecting the complexity of real-world involution phenomena. Moreover, although some studies attempt to explore the involution phenomenon through experiments and case analyses [7], most empirical studies lack systematic mathematical models, failing to reveal the comprehensive impact of parameter changes on system behavior. Current research is weak in terms of quantitative analysis of involution, particularly in exploring how strategy adjustments and institutional design can alleviate or reverse the involution phenomenon, and a universally applicable theoretical framework has not yet been formed. The innovation of this paper lies in the introduction of the supermodular game model based on existing theoretical and empirical research, proposing a new framework for involution games. Through this framework, the paper not only deeply analyzes the formation mechanism of involution but also validates the model's effectiveness through simulation experiments, providing quantitative tools and policy recommendations for addressing involution. This theoretical extension fills the gap in existing research regarding the modeling and governance of involution phenomena, and offers new perspectives and methodological support for future research.

II RELATED WORK

II-A RESEARCH ON INVOLUTION

The concept of involution was first used by the American anthropologist Alexander Goldenweiser to describe the phenomenon where a cultural pattern, once reaching a certain form, is unable to evolve into a new one [8]. Geertz applied this concept to analyze the agricultural economy of Java [1]. Tversky and Kahneman's "prospect theory" offers important insights into the involution phenomenon, especially in the field of behavioral economics, where individuals' loss aversion and overconfidence about future outcomes can lead to irrational investments in competition [9]. Prasenjit Duara and Huang Zongzhi used the concept of "involution" to analyze the development patterns of Chinese agricultural societies [10], [11]. Fei Xiaotong explored the complexity of rural Chinese society, analyzing the uneven distribution of resources and inefficiency in rural society, and proposed that due to the lack of external transformation, rural society falls into a self-reinforcing process of involution, thereby revealing the manifestations of involution in different social structures [12]. In recent years, the phenomenon of involution has gained further attention in fields such as education, the workplace, and business competition. Xu proposed the relationship between involution and cultural capital. Cultural capital is typically expressed in forms such as family background, educational resources, and social networks. The excessive pursuit of cultural capital is one of the root causes of societal involution. When society overly emphasizes the accumulation of cultural capital, it leads to increased invisible competitive pressures, thus resulting in involution. This phenomenon not only restricts individual development space but also affects overall social fairness and mobility [13]. Gan examined the negative impact of involution on students' learning motivation and psychological state from an ecological perspective, pointing out that excessive academic pressure often stems from competition for social cultural capital. In education, excessive competition leading to involution not only weakens students' intrinsic motivation but also exacerbates academic anxiety [3]. Chen further explored the manifestation of educational involution under the "Double Reduction Policy" (which aims to reduce students' academic burdens and extracurricular training). Despite the policy's intention to alleviate student pressure, excessive competition and societal expectations continue to exacerbate academic anxiety, fully demonstrating the complexity of educational involution and the difficulty of policy implementation. When faced with competitive pressures, students often adopt excessive efforts, leading to the intensification of involution [14]. Xia researched the involution phenomenon among finance students, noting that these students continually increase their efforts in pursuit of academic and employment advantages, ultimately falling into a vicious cycle of competition without obtaining corresponding returns [15]. Wang studied Chinesestyle competitive behavior and argued that Chinese students exhibit a self-reinforcing cycle in academic competition, leading to collective involution, making it difficult for individuals to break through their own limitations [16]. Wang and others studied the involution in the governance reform of the sports industry, using the example of a youth training base in S City, analyzing how the sports industry faces involution in cooperation between schools and clubs. The research shows that involutionary competition is not only reflected in academic fields but also impacts the allocation of resources in sports and other industries, leading to excessive competition and resource waste, which in turn affects the professional development of young athletes [17]. Cai et al. studied the "public examination involution" phenomenon among Chinese university students in the post-pandemic era, arguing that the pandemic has intensified competition among students. In civil service exams and the college entrance examination, the involution phenomenon has become more prominent. This behavior may seem to help improve personal social status in the short term, but in the long run, it weakens the overall creativity and development potential of society [18]. The above studies provide a multidimensional analysis of the involution phenomenon but still have some limitations. First, most existing studies focus on a single field or case analysis, lacking a cross-field, systematic theoretical framework. Second, quantitative analysis is limited, and there is a lack of research that uses mathematical models and experiments to verify the mechanisms of involution.

II-B RESEARCH ON SUPERMODULAR GAMES

Supermodular games, as an important branch of game theory, have been widely applied in various fields, such as economics, industrial organization, social choice theory, and mechanism design. The core feature of supermodular games is that the strategies of the players exhibit strategic complementarity, meaning that one player's strategy choice increases the marginal benefit of other participants' strategies. Supermodular games provide a theoretical framework for analyzing interdependencies in market competition, social interactions, and decision-making. Lazear, in his study of workplace incentive mechanisms, proposed an analytical framework related to supermodular games, discussing how individual behaviors interact in the workplace through external incentive mechanisms, especially in terms of strategic complementarity in collective competition and cooperation [19]. Fudenberg and Maskin studied the application of supermodular games in repeated games, analyzing how to maintain game stability under incomplete information. Their work provided important theoretical support for dynamic games, especially in discussions on auction mechanisms and market competition, where long-term interactions achieve stable equilibria [6]. Tirole applied supermodular games to analyze firms' strategic behavior in the market, revealing how firms respond to competitors' strategies by increasing output, lowering prices, and other tactics [20]. Building on this, Milgrom and Roberts introduced some important properties of supermodular games, including the monotonicity of best response functions, the existence of Nash equilibria, and the constructability of equilibria. They first applied supermodular games to economic competition in manufacturing, discussing how prices and outputs in markets depend on each other [7]. Topkis, in his classic work, defined the basic framework of supermodular games and explored the role of strategic complementarity in games [4]. These studies laid the foundation for later theoretical developments and applications. In social choice theory, supermodular games have also been widely applied. Akerlof and Kranton studied the role of supermodular games in social norms and social choice, proposing that participants' behaviors are driven not only by economic interests but also by social norms and cultural factors [2]. Their research helps to understand how to adjust behavior in society through mechanism design to maximize social welfare. Maskin and Sjoöström applied supermodular games in auction design, proposing how to improve bidder bids through the design of suitable auction mechanisms. Their work provided theoretical support for optimizing auction mechanisms by analyzing strategic complementarity among bidders [5]. Bergemann and Morris combined risk-sensitive preferences with supermodular games to study how to optimize participants' decisions under incomplete information through appropriate game design [21]. Clark and Gertler, from the perspective of competitive markets, studied the performance of supermodular games in market behavior, especially how to change market participants' strategic decisions by improving product or service quality, thereby affecting market stability [22]. Choi explored the application of supermodular games in market structure, proposing how changes in market structure influence participants' strategy choices and the evolution of outcomes [23]. In evolutionary games, Guth and Klose studied the application of supermodular games in cooperative games, proposing how cooperation can be maintained through strategy evolution in long-term games, revealing the role of strategic complementarity in cooperative games [24]. Kreps, in his classic work "Game Theory and Economic Modelling," combined supermodular games with economic modeling to explore how game theory can be used to describe and predict participants' behavior in markets [25]. Pereira and Sandholm studied the application of supermodular games in artificial intelligence, proposing how agents can use supermodular game theory to achieve optimal strategies in multi-agent systems, thus improving overall system efficiency [26]. Chakraborty and Vohra further explored the design of mechanisms with supermodular game preferences, analyzing how to use supermodular game theory in auction and contract design to enhance the effectiveness of strategies [27]. Shapley and Whinston applied the concept of supermodular games to network systems, studying how cooperation in network games can be optimized using supermodular game theory, especially in terms of cooperation behavior and strategic interactions in complex systems [28]. Sobel provided an overview of the application of supermodular games in market design, especially in multi-party games and network games, offering important theoretical support for the design of modern market mechanisms [29]. As theoretical research continues to deepen, the application of supermodular games is expanding. In recent years, scholars have gradually applied it to complex environments such as information asymmetry, heterogeneous participants, and network games. Xu explored the application of supermodular games in environments with information asymmetry, analyzing how participants choose optimal strategies under incomplete information [30]. Her research broadens the perspective of supermodular game applications. Zhang and Wang studied the application of supermodular games in social networks, proposing how to optimize resource allocation by improving network structures, reducing efficiency losses in competition [31]. They analyzed the mutual influence of individual behavior in social networks and its impact on network stability. Li et al. further extended the dynamic model of supermodular games, proposing how to adjust game outcomes through policy interventions in multi-round games [32]. Their research emphasizes the strategic behavior of participants in long-term interactions and its impact on social welfare. These studies not only expand the theoretical application scope of supermodular games but also provide powerful tools for decision-making in practical problems. Whether in industrial organization, auction mechanisms, or evolutionary games, supermodular games play a vital role. With the development of big data and artificial intelligence, the potential of supermodular games in machine learning and data science remains worthy of further exploration.

III SUPERMODULAR GAME THEORY

Game theory studies the decisions made by participants who directly interact with each other in competition, aiming to maximize individual utility and the equilibrium of their decisions. Its core elements include participants, strategy sets, and payoff functions. Nash equilibrium is widely used to describe the stable state of a game. In models such as the prisoner's dilemma and the tragedy of the commons, although all participants make individually optimal choices, the collective outcome remains suboptimal, illustrating the paradox of "individual rationality leading to collective irrationality." The characteristic of supermodular games is that in such games, participants' strategies have complementarity, meaning that the marginal utility caused by increasing a participant's strategy increases as the opponent's strategy increases. Supermodular games have pure strategy Nash equilibria. The upper bound of a participant's Nash equilibrium strategy exists, and this upper bound is an optimal response to the upper bound of their opponent's Nash equilibrium strategy. Similarly, the same applies to the lower bound.

Definition III.1. Let the strategy set S_i for each participant i be a subset of the finite-dimensional Euclidean space R^{m_i} , then

$S \equiv \times^{I}_{i=1} S_i$ is a subset of R^m, where $m = \sum_{i=1}^{I} m_i$. Let x and y represent two vectors in some Euclidean space R^K, and we denote $x \geq y$ to mean that for all $k = 1,2, \cdots, K$, $x_k \geq y_k$. We denote $x > y$ to mean $x \geq y$ and there exists a k such that $x_k > y_k$.

Define:

$$x \wedge y \equiv (\min(x_1,y_1), \cdots, \min(x_K,y_K)) \quad (1)$$

$$x \vee y \equiv (\max(x_1,y_1), \cdots, \max(x_K,y_K)) \quad (2)$$

If $s \in S$ and $\tilde{s} \in S$, then $s \wedge \tilde{s} \in S$ and $s \vee \tilde{s} \in S$, meaning that S is a sublattice of R^m.

Definition III.2. A supermodular function is a function $f: S \rightarrow R$ from a sublattice $S \subseteq R^m$ to the real numbers, if for all $x,y \in S$, it satisfies:

$$F(x) + f(y) \leq f(x \wedge y) + f(x \vee y) \qquad (3)$$

Such a function is called a supermodular function, or simply a supermodular function on S.

Definition III.3. If for all $(s_i, \tilde{s}_i) \in S_i^2$ and $(s_{-i}, \tilde{s}_{-i}) \in S_{-i}^2$,

where $s_i \geq \tilde{s}_i$ and $s-i \geq \tilde{s}-i$, we have

$$u_i(s_i, s_{-i}) - u_i(\tilde{s}_i, s_{-i}) \geq u_i(s_i, \tilde{s}_{-i}) - u_i(\tilde{s}_i, \tilde{s}_{-i}), \qquad (4)$$

then $u_i(s_i, s_{-i})$ exhibits increasing differences in (s_i, s_{-i}) . Increasing differences indicate that the increase ın the opponent's strategy increases the participant's own strategy.

Definition III.4. A super modular game is defined as follows: for every i, the strategy set S_i is a sublattice of, R^{m_i}, the utility function u_i exhibits increasing differences in (s_i, s_{-i}), and u_i is super modular in (s_i, s_{-i}).

Corollary III.1. If $S_i = R^{m_i}$ and if u_i is twice continuously differentiable with respect to s_i, then u_i is supermodular in s_i if and only if for any two components of s_i , say s_{ik} and s_{il} $(k \neq l)$

we have:

$$\frac{\partial^2 u_i}{\partial s_{ik} \partial s_{il}} \geq 0 \qquad (5)$$

Topkis and Milgrom & Roberts pointed out that supermodular games have the following properties [4], [7]:

- The best response function is monotonically increasing;
- Nash equilibria exist and can construct maximum/minimum equilibria;
- It is easy to analyze the comparative static responses of equilibria to parameters.

Supermodular Game Model of Involution

IV SUPERMODULAR GAME MODEL OF INVOLUTION

IV-A TWO-PLAYER COMPETITIVE GAME FRAMEWORK

Consider a basic two-player competitive structure, where the participants are $i = 1,2$, and their strategies are effort levels $e_i \in [0,\infty)$. The participants determine their share of a resource or payoff R based on their relative effort levels. The relative payoff function is defined as:

$$P_i(e_i, e_j) = \frac{e_i^\beta}{e_i^\beta + e_j^\beta}, \quad \beta \geq 1 \qquad (6)$$

where β represents the intensity of competition or the incentive amplification coefficient. This structure reflects the real-world logic of " effort equals reward," and as β increases, small differences are amplified.

Each participant's utility function is:

$U_i(e_i,e_j) = P_i(e_i,e_j) \cdot R - ce^2{}_i$ (7)

where $c > 0$ is the marginal cost coefficient. This function reflects the typical competitive reward structure seen in fields such as education, the workplace, and research, where the competition payoff is reduced by the cost of effort.

To find the optimal response function for participant i, we take the first derivative of U_i with respect to e_i. Let $i,j \in \{1,2\}$ and $i \neq j$:

$$\frac{\partial U_i}{\partial e_i} = \frac{\partial P_i}{\partial e_i} \cdot R - 2ce_i \quad (8)$$

The first derivative of P_i with respect to e_i is:

$$\frac{\partial P_i}{\partial e_i} = \beta e_i^{\beta-1} \cdot \frac{e_j^\beta}{(e_i^\beta + e_j^\beta)^2} \quad (9)$$

Thus:

$$\frac{\partial U_i}{\partial e_i} = \beta e_i^{\beta-1} \cdot \frac{e_j^\beta R}{(e_i^\beta + e_j^\beta)^2} - 2ce_i \quad (10)$$

Setting $\frac{\partial U_i}{\partial e_i} = 0$, we obtain the optimal response function for

player j, $BR_j(e_i)$, which satisfies:

$$\beta e_i^{\beta-1} \cdot \frac{e_j^\beta R}{(e_i^\beta + e_j^\beta)^2} = 2ce_i \quad (11)$$

This equation is implicitly defined but can be studied numerically or qualitatively.

To prove that this game is a supermodular game, we need to verify if it satisfies the ”monotonic response” condition, meaning that an increase in the opponent’s strategy will encourage the participant to increase their own effort.

We examine the cross partial derivative:

$$\frac{\partial^2 U_i}{\partial e_j \partial e_i} = \frac{\partial}{\partial e_j}\left(\beta e_i^{\beta-1} \cdot \frac{e_j^\beta R}{(e_i^\beta + e_j^\beta)^2}\right) \quad (12)$$

Since $\beta e^\beta{}_i{}^{-1}$ and R are constants with respect to e_j, we can factor out the constants, denoted as:

$$= \beta e_i^{\beta-1} R \cdot \frac{\partial}{\partial e_j}\left(\frac{e_j^\beta}{(e_i^\beta + e_j^\beta)^2}\right) \quad (13)$$

Let:

$u = e^\beta{}_j\, , v = (e^\beta{}_i + e^\beta{}_j\,)^2$ (14)

Then:

$$\frac{d}{de_j}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2} \quad (15)$$

where:

$$u' = \frac{d}{de_j}(e_j^\beta) = \beta e_j^{\beta-1}, \quad v' = 2(e_i^\beta + e_j^\beta) \cdot \frac{d}{de_j}(e_j^\beta) = 2\beta e_j^{\beta-1}(e_i^\beta + e_j^\beta) \quad (16)$$

Substituting into the equation, we get:

$$\frac{d}{de_j}\left(\frac{e_j^\beta}{(e_i^\beta + e_j^\beta)^2}\right) = \beta e_j^{\beta-1} \cdot \frac{(e_i^\beta + e_j^\beta)^2 - 2e_j^\beta(e_i^\beta + e_j^\beta)}{(e_i^\beta + e_j^\beta)^4} \quad (17)$$

$$= \beta e_j^{\beta-1} \cdot \frac{(e_i^\beta + e_j^\beta)(e_i^\beta - e_j^\beta)}{(e_i^\beta + e_j^\beta)^4} = \beta e_j^{\beta-1} \cdot \frac{e_i^\beta - e_j^\beta}{(e_i^\beta + e_j^\beta)^3} \quad (18)$$

Finally, we obtain:

$$\frac{\partial^2 U_i}{\partial e_j \partial e_i} = \beta^2 R e_i^{\beta-1} e_j^{\beta-1} \cdot \frac{e_i^\beta - e_j^\beta}{(e_i^\beta + e_j^\beta)^3} \quad (19)$$

We can see that:

- If $e_i < e_j$, then $\frac{\partial^2 U_i}{\partial e_j \partial e_i} < 0$; - If $e_i > e_j$, then $\frac{\partial^2 U_i}{\partial e_j \partial e_i} > 0$.

This shows that the game does not satisfy the supermodular condition globally, but it has a" conditional supermodular structure" in certain parameter ranges or local regions. This is consistent with the real-world phenomenon of involution, where "competition amplifies after a certain threshold." For non-global supermodularity, we can introduce the following definition:

Definition IV.1. If a game satisfies the supermodular condition in a certain subset of the strategy space, and this subset contains all feasible Nash equilibrium points, it is called a locally supermodular game.

In this model, when $e_i \approx e_j$ and β is large, $\frac{\partial^2 U_i}{\partial e_i \partial e_j} > 0$ holds, i.e., locally satisfying strategic complementarity, and thus locally satisfying the supermodular structure. Therefore, we introduce the following:

Strategic Complementarity Interval: $\{(e_i, e_j) \in \mathbb{R}_+^2 \mid |e_i - e_j| < \epsilon\}$ (20)

In this region, there exists an equilibrium construction sequence

(the intersection points of the increasing best response functions):

$$e_i^{(t+1)} = BR_i(e_j^{(t)}), \quad e_j^{(t+1)} = BR_j(e_i^{(t)}) \quad (21)$$

According to Tarski's theorem, as long as the best response functions are monotonically increasing and continuous, this sequence will converge to a stable point (e_1^*, e_2^*) in the compact set, i.e., reaching an involution equilibrium. However, involution leads to social resource waste. A Pareto improvement can be used to measure the social cost of involution. Let there exist a low-effort configuration $(\bar{e}_1, \bar{e}_2)$, which satisfies:

$$U_1(\bar{e}_1, \bar{e}_2) > U_1(e_1^*, e_2^*), \quad U_2(\bar{e}_1, \bar{e}_2) > U_2(e_1^*, e_2^*) \quad (22)$$

Then the current equilibrium (e_1^*, e_2^*) is a non-Pareto optimal state. To quantify this gap, we introduce the social loss function:

$$L = \sum_{i=1}^{2} \left(ce_i^{*2} - ce_i^{\circ 2}\right) \quad (23)$$

where e°_i is the ideal optimal effort level (the individual optimum without competitive pressure). If $L > 0$, it indicates the "purely resource loss caused by involution," that is, the social cost of "effort waste."

IV-B EXTENSION OF THE TWO-PLAYER MODEL

Let the set of participants be $i = 1,2,...,N$, and each

participant chooses an effort level $e_i \in [0,\infty)$. The utility is:

$$U_i(e_1, e_2, \cdots, e_N) = \frac{e_i}{\sum_{j=1}^{N} e_j} \cdot R - ce_i^2 \quad (24)$$

We can further generalize the effort level e_i to a function of effort $g(e_i)$, then the utility is:

$$U_i(e_1, e_2, \cdots, e_N) = \frac{g(e_i)}{\sum_{j=1}^{N} g(e_j)} \cdot R - ce_i^2 \quad (25)$$

where $g(e_i)$ can be a linear function or an S-shaped function, etc. If we set $g(e_i) = \tanh(Be_i)$, $B > 0$ controls the intensity of marginal incentives. The derivative is calculated as follows:

$$g'(e) = B \cdot \mathrm{sech}^2(Be), \quad \text{where } \mathrm{sech}(x) = \frac{1}{\cosh(x)} \quad (26)$$

Substitute this into the partial derivative expression:

(27) $$\frac{d}{de_i}\left[\frac{g(e_i)}{g(e_i) + (N-1)g(e^*)}\right]$$

(28) $$= \frac{g'(e_i)(g(e_i) + (N-1)g(e^*)) - g(e_i)g'(e_i)}{(g(e_i) + (N-1)g(e^*))^2}$$

Simplify:

$$\frac{R}{(g(e_i) + (N-1)g(e^*))^2} \cdot g'(e_i) \cdot [(N-1)g(e^*)] = 2ce_i \quad (29)$$

Substitute $g(e_i) = \tanh(Be_i)$ and $g'(e_i) = B \cdot \mathrm{sech}^2(Be_i)$ into the above equation:

$$\frac{R}{(\tanh(Be_i) + (N-1)\tanh(Be_j))^2} \cdot B \cdot \mathrm{sech}^2(Be_i) \quad (30)$$

$$\cdot [(N-1)\tanh(Be_j)] = 2ce_i \quad (31)$$

This expression can be used to solve for the stable effort level $e*$, which has consistency and feedback structure. Implicit differentiation of the given equation can be used to verify.

Taking the derivative of both sides with respect to e_j:

(32) $$\frac{\partial}{\partial e_j}\left[\frac{R}{(\tanh(Be_i)+(N-1)\tanh(Be_j))^2}\right.$$

(33) $$\cdot B\cdot \operatorname{sech}^2(Be_i)\cdot[(N-1)\tanh(Be_j)]$$

(34) $$=\frac{\partial}{\partial e_j}[2ce_i]$$

By the chain rule, we ultimately obtain:

$$\frac{\partial e_i}{\partial e_j}>0 \quad (35)$$

This shows that as the effort strategy e_j of other participants increases, the effort strategy e_i of participant i also increases. The involution phenomenon typically manifests as individuals' increasing input in resources, effort, etc., but the returns or benefits they receive gradually diminish. In this case, the Sshaped function can effectively characterize this phenomenon. The characteristic of the S-shaped function is that as the input increases, the rate of increase in output gradually diminishes. When the input is at low values, the function value increases rapidly, and as the input increases, the increment slows down and eventually approaches an upper limit (i.e., saturation). In the context of involution, this characteristic can simulate the diminishing effect of individuals or organizations' investments in competition.

IV-C DYNAMIC MODEL ANALYSIS

In reality, involution is not a one-time decision but a dynamic evolutionary process that gradually forms through long-term repeated interactions. Therefore, consider the following structure of an infinite repeated game model:

Let the set of participants be $i = 1,2,...,N$, and each participant chooses an effort level $e^t{}_i \in [0,\infty)$ at each stage $t = 0,1,2,...$. Their current utility is:

$$u_i^t=\frac{g(e_i^t)}{\sum_{j=1}^N g(e_j^t)}\cdot R-c(e_i^t)^2 \quad (36)$$

where $g(x)$ is a monotonically increasing incentive function, typically an S-shaped function like $g(x) = \tanh(Bx)$, with $B > 0$ controlling the strength of marginal incentives.

The total utility of the participant is the sum of the discounted utilities over all stages:

$$U_i=\sum_{t=0}^{\infty}\delta^t u_i^t,\quad \delta\in(0,1) \quad (37)$$

where δ is the discount factor, representing the participant's

degree of future utility valuation.

The participant's strategy is a mapping: $\sigma_i : H^t \to e^t{}_i$, where H^t represents the history information set. For simplicity, we limit the analysis to the Markov Perfect Equilibrium (MPE) framework, where the current strategy only depends on the previous period's behavior levels. Assuming all participants adopt symmetric strategies and there exists a stable state $e^t{}_i = e*$, such that for all t:

$$e_i^* = \arg\max_{e_i \geq 0} \left\{ \frac{g(e_i)}{g(e_i) + (N-1)g(e^*)} \cdot R - ce_i^2 + \delta U_i(e^*) \right\} \quad (38)$$

This is equivalent to solving the following fixed-point problem:

$$\frac{d}{de_i} \left[\frac{g(e_i)}{g(e_i) + (N-1)g(e^*)} \cdot R - ce_i^2 \right] = 0 \quad \text{at } e_i = e^* \quad (39)$$

Considering the perturbation of the initial state $e^0{}_i$ deviating from $e*$, the system follows the update path:

$e_i^{t+1} = BR(e_{-i}^t)$, where $BR(\cdot)$ is the best response function (40)

If strategic complementarity exists (i.e., BR is monotonically increasing), then:

$e^{t+1} \geq e^t \Rightarrow e^t \nearrow e^\infty$ if the initial level is too high (41)

To prove the existence of strategic complementarity, we need to show that the best response function $BR(\cdot)$ is monotonically increasing. Specifically, we need to prove that for all participants $j \neq i$, the best response function $BR(e-_i)$ is increasing with respect to the other participant's effort e_j, i.e.:

$$\frac{\partial BR(e_{-i})}{\partial e_j} > 0 \quad (42)$$

IV-D INVOLUTION TRAP AND INSTITUTIONAL DESIGN

This monotonically increasing dynamic path will lead to involution cascading, meaning that individual behaviors are driven by feedback, continuously escalating, eventually reaching a nonoptimal stable state.

Definition IV.2. (Involution Trap): If there exists a stable effort level $e\infty > e\circ$, where $e\circ$ is the socially optimal effort level, and all individuals cannot unilaterally escape from this state, the system is said to be trapped in the involution trap.

Based on the model analysis, to break the involution trap and suppress or reverse the trend of involution, the following measures should be taken:

1) Break the positive feedback loop: β can be understood as the relative ranking weight in the game. By adjusting β, the competitive pressure can be made more balanced, reducing the incentive effect amplified by small differences. An absolute threshold reward and punishment mechanism reduces the intense competition caused by relative comparisons (refer to the simulation experiments section).

2) Increase marginal cost awareness: Theoretically, an individual's effort will have "increasing marginal costs," meaning that as effort increases, the marginal benefit decreases. With the amplification of small differences, individuals may over-invest and ignore cost effects. 1) According to the economic law of diminishing marginal utility, as an individual increases their effort (such as work hours, study time, etc.), the utility or return they receive gradually decreases. Without appropriate institutional or mechanism guidance, individuals are prone to fall into the "over-effort" trap with no corresponding reward. 2) Due to the "cognitive dissonance" principle, individuals may not clearly perceive the cost of their efforts or may underestimate the loss caused by excessive effort.

Therefore, institutional design or social advocacy is required to enhance individual awareness of these costs and avoid over-investment. 3) From the perspective

of the ”consumerism trap” in sociological theory, excessive competition will lead individuals into endless efforts, rather than genuinely pursuing self-value improvement.

3) Control institutional competition structure: For example, reduce excessive performance comparisons in the workplace, and promote a diversified evaluation system in education. In the workplace, the assessment can be expanded to include personal growth and team collaboration, rather than just quantitative outcomes. In education, ”individual progress” can be promoted instead of merely ranking competition.

4) Introduce cooperative mechanisms: Design cooperative incentive functions (e.g., team-shared rewards) to shift individual behavior from zero-sum games to collaborative games, increasing the motivation for win-win cooperation. Collaborative games, through common goals and shared rewards, can effectively reduce the negative effects of pure competition.

5) Institutional punishment for excessive effort: For example, setting a maximum working hour limit, prohibiting mandatory overtime, setting limits on academic workload, etc., to prevent physical and mental health problems caused by over-effort. Additionally, through legislation or policy implementation, ensure that these restrictions are effectively enforced.

The above measures can be understood as ”structural regulation” of the parameters in the game model, weakening or eliminating the positive feedback pressure caused by strategic complementarity. Further, an involution governance model based on mechanism design theory can be generated. Let the institutional designer be the leader, and their goal is to maximize the social welfare function $W = \sum_i U_i$. The following optimization problem can be considered:

$$\max_{\Gamma \in \mathcal{G}} \sum_{i=1}^{N} \mathbb{E}^{\Gamma}[U_i(e_i, e_{-i})] \quad \text{s.t.} \quad e_i = \arg\max_{e_i'} U_i(e_i', e_{-i}; \Gamma) \quad (43)$$

where Γ represents the institutional constraint mechanism (such as taxes, incentives, controls, etc.), and G is the space of implementable institutional mechanisms. This model has multiagent information game characteristics and requires an in-depth solution considering conditions such as incentive compatibility and implementability.

V SIMULATION EXPERIMENTS

To verify the explanatory power and effectiveness of the supermodular game model for the involution phenomenon, a series of simulation experiments were designed. These experiments explore the impact of different parameters and heterogeneous participants on involution equilibrium.

V-A EXPERIMENTAL DESIGN

The main objective of the experiments is to explore the evolution process and stability of the involution phenomenon under different parameter combinations, with particular focus on the following aspects:

- Parameter Sensitivity Analysis: By varying the competition incentive coefficient (β) and individual effort cost coefficient (c), we analyze how these parameters affect the involution equilibrium.
- Heterogeneous Participant Analysis: Simulate heterogeneous participants, i.e., differences in the objective functions and cost structures of different individuals, and observe how these differences affect the involution mechanism.

V-B EXPERIMENTAL PROCESS

1) PARAMETER SENSITIVITY ANALYSIS

In this part of the experiment, the effects of different incentive strengths (β) and individual effort cost coefficients (c) on the involution equilibrium were explored. By setting different values for β (β = 1.5,2.0,3.0,⋯ ,9.0) and c (c = 0.1,0.5,1.0,⋯ ,9.0), the relative payoff structure and optimal response functions were calculated. Specifically, by adjusting these parameters, we observed the regularities of strategy choices and equilibrium changes and plotted the relative payoff structure graph (e.g., Figure 1) and the optimal response function graph (e.g., Figure 2). Figure 1 shows the relative payoff structure under different incentive strengths β, while Figure 2 displays the best response curve of participant i to competitor j's effort ej under different values of the effort cost coefficient c.

From Figure 1, it can be observed that when β is small (e.g., β = 1.5), the payoff structure is relatively smooth, and the difference

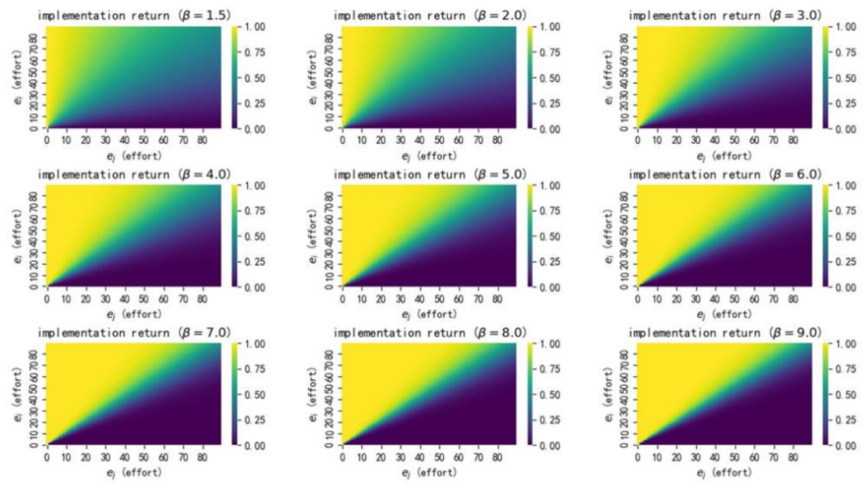

Fig. 1. Heatmap showing the impact of different incentive strengths β

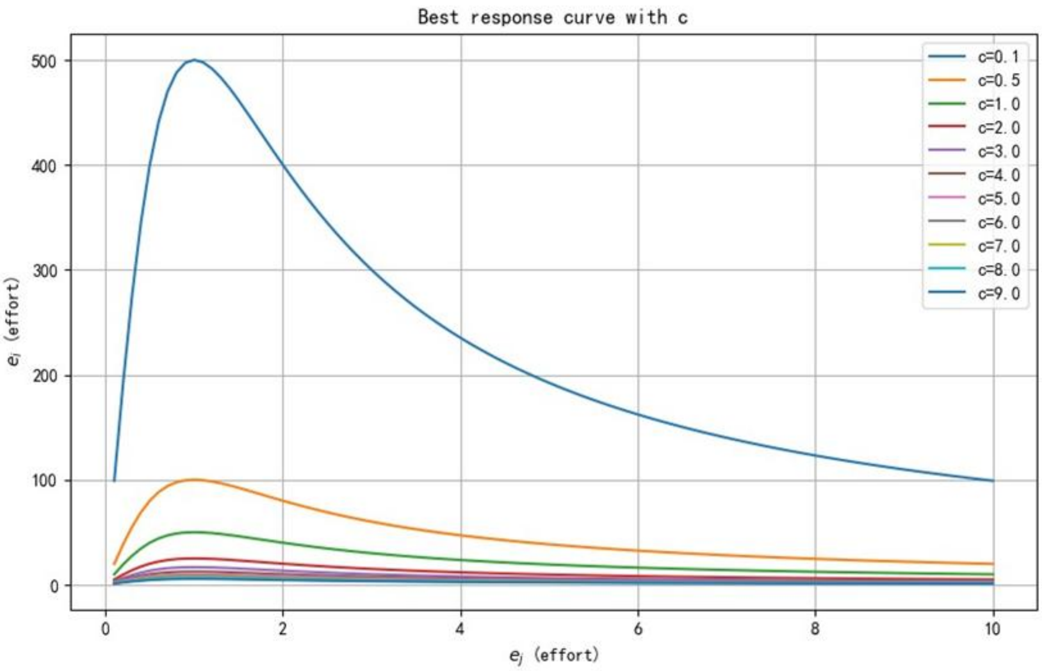

Fig. 2. Best response curve under different c values

in effort has a small impact on the payoff, which is reflected in the relatively uniform color distribution. As β increases, the payoff structure becomes more asymmetric, especially when ej is large, where the relative payoff for ei significantly decreases, reflected in more purple regions. When the β value reaches a high level (e.g., β = 9.0), the difference in effort has a more significant impact on the relative payoff, with the payoff curve changing sharply. Overall, increasing β strengthens the effect of differences in participants' effort levels, so that under certain conditions, participants with lower effort levels will face lower payoffs.

From Figure 2, it can be observed that for small values of ej, particularly when c = 0.1, the best response ei of participant i shows a large value and decreases rapidly as ej increases, indicating that participant i responds with higher effort to lowereffort participant j. As c increases, especially when c reaches a large value (e.g., c = 8.0 and c = 9.0), the best response curve for participant i flattens. This indicates that when c is higher, participant i's effort level becomes less sensitive to ej, and the effort level tends to a smaller and more stable value. When ej increases further, all best response curves show a decreasing trend, suggesting that regardless of the value of c, participant i will choose a lower effort level when ej is high, demonstrating a certain "defensive" strategy. Overall, the variation in c reflects participant i's response mechanism to participant j's effort level, with lower values of c leading to stronger reactions in the early stages, while higher values of c make participant i adopt a more conservative strategy.

Further analysis of the relative payoff function for g(ei) = tanh(Bei), B > 0, and the effect of different values of B on the involution phenomenon is shown in Figure 3.

From Figure 3, it can be observed that when the value of B is small (B = 0.1,0.2,0.3), the change in the tanh function is relatively smooth, and the relative payoff changes slowly with respect to ei. In this case, the effort level has a small impact on the relative payoff. As the effort level increases, the change in relative payoff is small, and the color variation is relatively uniform. As B increases (B = 0.4,0.5,0.6), the slope of the tanh function increases, and the color changes in the heatmap become more noticeable, indicating that the relative payoff becomes more sensitive to changes in effort level. In this case, when ei is small, the relative payoff changes slowly, but as ei increases, the growth rate of the relative payoff increases, and the color gradient in the chart becomes more pronounced. When B increases to higher values (B = 0.7,0.8,0.9), the slope of the tanh function becomes very steep, and the heatmap shows that the relative payoff is extremely sensitive to changes in ei. At this point, when ei is small (close to zero), the relative payoff tends to zero, while for large ei, the relative payoff approaches 1. The heatmap displays a very strong color gradient, indicating that effort level has a dramatic impact on relative payoff.

Overall, as B increases, the sensitivity of the payoff function to effort level increases, which is reflected in the more intense color changes in the heatmap. This phenomenon reflects the typical feature of involution, where competition intensifies, ultimately leading to efficiency reduction.

2) HETEROGENEOUS PARTICIPANT ANALYSIS

Heterogeneous participants are introduced to simulate diversity in real-world scenarios. Specifically, it is assumed that each participant has differences in their objective functions and cost structures, which can be simulated by introducing the parameter α to model individual heterogeneity. For example, participants i and j differ in their effort costs, with α controlling the impact of cost on individual behavior. To better understand how heterogeneity affects participant behavior in the game, we visualize the effect of different heterogeneity parameters α and competitors' efforts ej on the optimal response function ei through heatmaps, as shown in Figure 4.

From Figure 4, it can be seen that as ej increases, the optimal response function ei with respect to ej shows a clear increasing trend. Especially at higher values of ej, the optimal response function increases more significantly, indicating that as competition pressure increases, participants also increase their effort accordingly, thereby intensifying the involution phenomenon. Furthermore, as the heterogeneity parameter α increases, individual strategy choices become more sensitive. This suggests that different types of participants show more divergent behavior in the game, leading to greater instability in the overall game and further promoting involution.

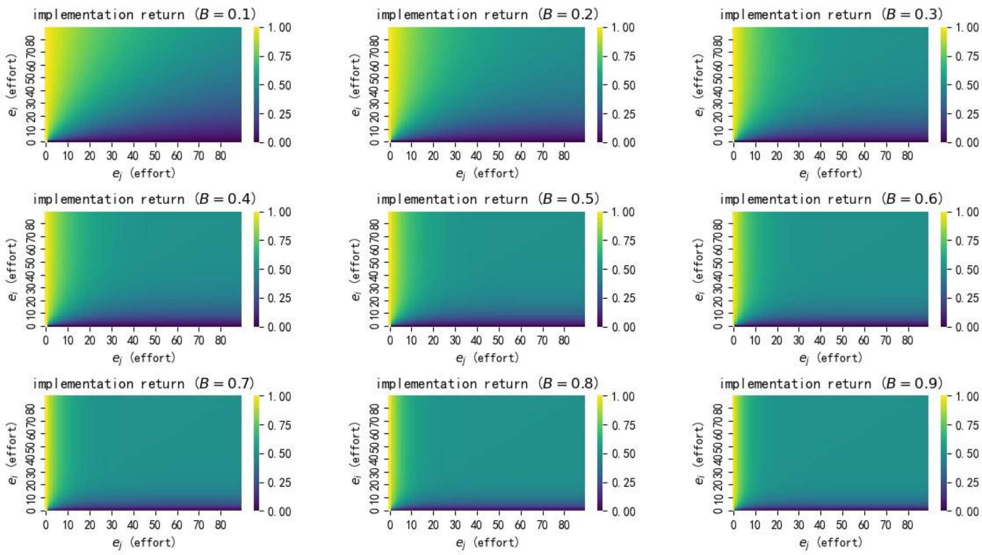

Fig. 3. Relative payoff heatmap under different B values

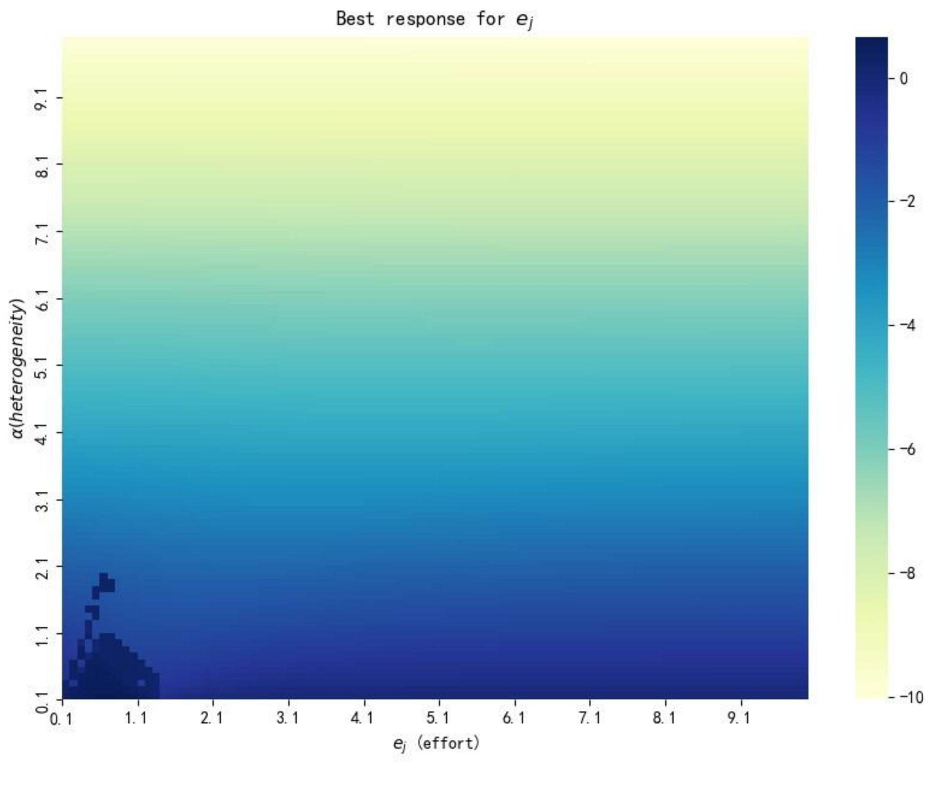

Fig. 4. Heatmap of the optimal response function *ei* with respect to *ej* for heterogeneous participants

V-C Experimental Results Explanation

The main experimental results obtained from the above experiments are as follows:

- Impact of Incentive Strength: As the competition incentive coefficient β increases, the effort differences among participants are rapidly amplified, and the system's involution significantly intensifies. The system exhibits stronger competition reinforcement effects. As c increases, individual effort input gradually decreases, which affects the equilibrium state of the entire game system. At lower values of c, participants tend to increase their effort to gain a competitive advantage, while at higher values of c, the marginal cost of effort becomes too high, leading individuals to choose a lower effort level, ultimately entering a lower involution equilibrium.
- Impact of Heterogeneous Participants: After introducing heterogeneous participants, the system's stability and equilibrium changed. Differences in objective functions and cost structures led to greater strategy differences among participants, which further affected the formation and evolution of involution.

VI CONCLUSION AND OUTLOOK

VI-A MODEL SUMMARY AND THEORETICAL CONTRIBUTIONS

Based on supermodular game theory, various forms of involution models were constructed, including two-player or multi-player static games as well as dynamic repeated game models. Through rigorous mathematical derivations and structural analysis, the simulation experiments further validated the model's predictive capability and demonstrated the dynamic response of system behavior and resource loss patterns under typical parameter structures. Overall, the key characteristics of involution revealed by the model are as follows:

Strategic complementarity is the fundamental driving force behind the involution mechanism. The effort levels between participants have a mutually amplifying effect, forming a positive feedback loop.

- Involution equilibria are non-Pareto optimal. The game structure leads to a decrease in individual utility and low social resource allocation efficiency.
- Dynamic evolutionary paths carry the risk of trap lock-in. In repeated interactions, the system may stabilize in a highinput, low-efficiency state.
- The competition incentive strength β and the discount factor δ are crucial parameters driving the amplification of effort levels.
- Supermodular games provide a tool for parameter sensitivity analysis. Through comparative static analysis, key regulatory parameters and institutional intervention levers can be identified.

This model systematically explains the involution phenomenon in various fields such as education, the workplace, and business, with good theoretical extensibility and practical adaptability. It points out that involution is not caused by individual laziness or greed but is an inevitable result of the game structure. The solution to involution does not require ”everyone to stop striving” but to avoid falling into the institutional trap of ”everyone having to strive.” Therefore, the path to solving involution must begin at the institutional level, optimizing resource allocation rules and reconstructing incentive mechanisms, so as to break out of the ”competition-based inefficiency” collective trap and move toward a ”coordinated and effective” social equilibrium.

VI-B RESEARCH OUTLOOK

Future research can expand in the following directions: considering incomplete information scenarios regarding β or others' effort levels among participants; introducing cost structure or objective function differences to explore local and global equilibria in hierarchical games; combining real-world institutional environments for institutional simulations to test the feasibility of optimal mechanisms; studying the stability and disturbance response of involution equilibria from a dynamic systems perspective; and the ultimate goal is to organically integrate micro-incentive mechanisms, game structures, and macro-institutional optimization to provide theoretical support and empirical tools for solving the involution dilemma.

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