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INVESTIGATING THE BEHAVIOR OF DYNAMIC SYSTEMS BASED ON LINEAR DIFFERENTIAL EQUATIONS

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ABSTRACT.
Linear differential equations are fundamental tools in the analysis and modeling of dynamical systems that are used in many physical and engineering phenomena. This paper examines the structure, properties, and applications of these linear equations in the analysis of mass-spring systems, RLC circuits, and heat transfer processes. By presenting mathematical models and analyzing the time responses of these systems, it is shown how linear equations can model complex behaviors in a simple, predictable, and robust manner. The results of this research show that linear differential equations, despite their simplicity, are effective tools for the analysis of physical systems, and are used to more accurately model more complex systems over time.

KEY WORDS: Circuit, dynamic analysis, Linear differential equations, modeling, RLC.

1.INTRODUCTION
Finding a common language to describe physical and engineering phenomena has always been one of the main goals of scientists. In the meantime, mathematics has played a key role in creating this language, and differential equations, especially their linear type, have become one of the most fundamental tools for analyzing the behavior of systems. Ordinary linear differential equations, with their simple structure, are able to describe the behavior of many natural systems. From the motion of an oscillating body to the discharge of a capacitor in an electrical circuit, everything can be analyzed with these equations.

The research method in this paper deals with the modeling of electrical circuits using the laws of Kharshoff KVL and circuits RLC. The research design is quantitative-analytical, in that first the theoretical foundations and basic principles are reviewed and then these concepts are examined in the form of mathematical models. For modeling electrical circuits using differential equations, these equations are used to analyze voltages and currents to display the dynamic behavior of circuits. is quantitative analytical. First, the theoretical and basic concepts of linear differential equations are introduced, then, using reliable sources and mathematical analysis of applied examples, the role of these equations in describing physical phenomena is examined. Diagrams and graphs are also drawn using numerical software to display the results.

- How do these equations model physical phenomena?
- To what extent can the results obtained from linear equations be trusted?
- What are the differences between linear and nonlinear modeling?

This research design aims to analyze the structure, solution methods and applications of ordinary linear differential equations in physics and engineering. In order to explain the concepts, three main practical examples have been selected, which are accompanied by diagrams and mathematical analysis.

The theoretical framework of this article consists of the main sections of introduction, Literature review, Literature review, discussion, conclusion and list of references.

2. LITERATURE REVIEW

About 300 years, differential equations and the connection between functions derivatives and transformations have been established. Thus, its history inevitably begins with the English scientist Isaac Newton's discovery of the derivative between (1642-1772). Moreover, Gottfried Leibniz, a German scientist studied differential equations, particularly first-order differential equations in the years (1646-1716). The differential equation of Bernoulli was introduced by Jacob in 1674, but it was not verified by him until it is verified in 1705 by Euler. The boundary problem with the initial boundary in linear differential equations was proposed by Sturm-Liouville, Jacques Francois Sturm and Joseph Liouville are the names of the classical Sturm-Liouville theory and its applications, which was put out between (1855-1803) and in (1809-1882) in the second order, the theory of linear differential equations was developed in 1969. The Green's function is a method for solving differential equations with boundary conditions that was described by Russian physicist Naimark in his book Linear Differential Functions. The boundary conditions are strictly regular and defined, according to the theories of Mikhailov and Kesselman [4] since there is a positive integer like this, the asymptotic equation's eigenvalues are straightforward and distinct. δ, for both eigenvalues of the function [3], which are farther apart from one another by a larger distance δ. It is also concluded from the works [1, 2, 3, 7, 8] that in the space, the system of eigenfunctions and associated functions forms a basis Res. Many pure mathematicians have been working on finding Green's function for linear differential equations in the last few years. One of the scientist is Kanguzhin, a scientist from Kazakhstan who published an article titled "considering Green's function for second-order linear differential equations" in 2019 [4, 6, 8].

Numerous studies have examined differential equations. Reference books such as Boyce and DePrima [8] and Zell [9] have explained the basic concepts and methods for solving linear equations in detail. In more recent studies, Strogatz [4] and Khalil [6] have analyzed the differences and limitations of linear versus nonlinear modeling. Also, studies by Hirsch and Smil [5] have shown that in many cases, linear analysis is the starting point for understanding complex systems. This study attempts to examine these issues from a critical and integrated perspective.

3. ELEMENTARY BASIC

A linear ordinary differential equation is an equation in which the unknown function and its derivatives appear with constant coefficients or as a function of the independent variable, and neither reaches a power or another nonlinear function.

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_2 y'' + a_1 y' + a_0 y = f(x) \tag{1}$$

Equation (1) represents higher-order linear equations with constant coefficients the solution of the second relation, which expresses the general solution of an inhomogeneous differential equation of order (1), can be written as follows:

$$y(\mathrm{x}) = C_1 y_1 + C_2 y_2 + \dots + C_n y_n + y_H(\mathrm{x}) \tag{2}$$

The general form of second-order equations with constant coefficients is:

$$y'' + a_1 y' + a_0 y = f(\mathrm{x}) \tag{3}$$

The solution to the third equation can be written as follows:

$$y(\mathrm{x}) = C_1 y_1 + C_2 y_2 + y_H(\mathrm{x}) \tag{4}$$

We have examples have been selected, which are accompanied by diagrams and mathematical analysis.

$$m\,X'' + k\,X' = 0 \tag{5}$$

Equation (5) describes a simple oscillator that has application in spring, mass, and pendulum systems, and solution to the above equation can be written as follows:

$$\mathrm{x}(t) = A\cos\left(\omega t\right) + B\sin\left(\omega t\right), \qquad \omega = \sqrt{\frac{k}{m}} \tag{6}$$

Figure 1

Figure 1 shows the motion of the oscillator.

The second-order linear differential equation with constant coefficients is presented as follows Its area of application is in electronic circuits and filters, and its motion is called an oscillatory damper.

$$\mathrm{L}\frac{d^2 f}{dt^2} + R\frac{df}{dt} + \frac{1}{C} f = 0 \tag{7}$$

This is a second-order linear differential equation with constant coefficients and is commonly known as an RLC circuit in electrical engineering.

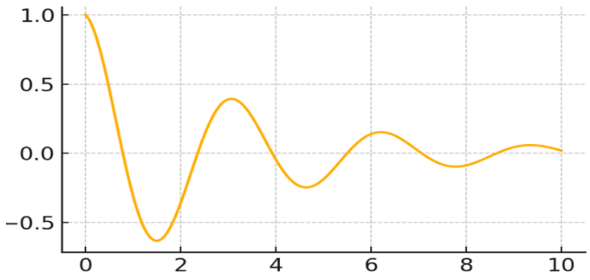

Figure 2

Figure 2 shows the movements of an electric circuit RLC, To solve it, we use the characteristic equation method.

We assume:

$$f(\mathrm{t}) = e^{\lambda t}$$

Substituting into equation (7) results in:

$$\mathrm{L}\lambda^2 + R\lambda + \frac{1}{C} = 0 \qquad (8)$$

Equation (8) expresses a characteristic equation whose roots are obtained from the following relation:

$$\lambda = \frac{-R \pm \sqrt{R^2 - 4L\frac{1}{C}}}{2L} \qquad (9)$$

To better understand circuit RLC, consider the following:

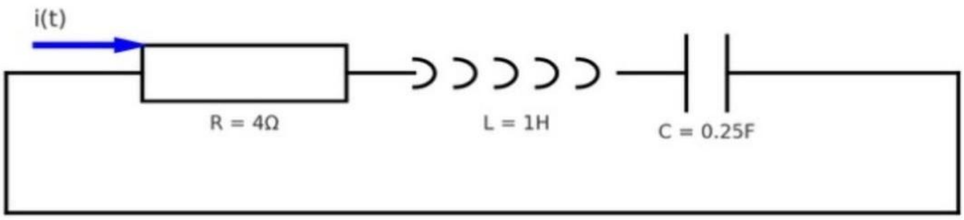

Figure 3

In Figure 3, the $R = 4\Omega$, $L = 1H$, $C = 0.25F$ are considered in the voltage-free circuit. Considering Kirchhoff's law and second-order linear differential equations, we obtain the current of the entire circuit as follows:

$$\mathrm{L}\frac{d^2 f}{dt^2} + R\frac{df}{dt} + \frac{1}{C}f = 0$$

$$\frac{d^2 f}{dt^2} + 4\frac{df}{dt} + \frac{1}{0.25}f = 0$$

$$\frac{d^2 f}{dt^2} + 4\frac{df}{dt} + 4f = 0$$

As a result, its general solution in terms of time is:

$$f(t) = (\mathrm{A} + Bt)\,\mathrm{e}^{-2t}$$

If we consider the boundary condition of the Cauchy problem, its particular solution is:

$$f(t) = (1 + 2t)\,\mathrm{e}^{-2t}$$

It should also be noted that the diagram below shows the time response of circuit RLC.

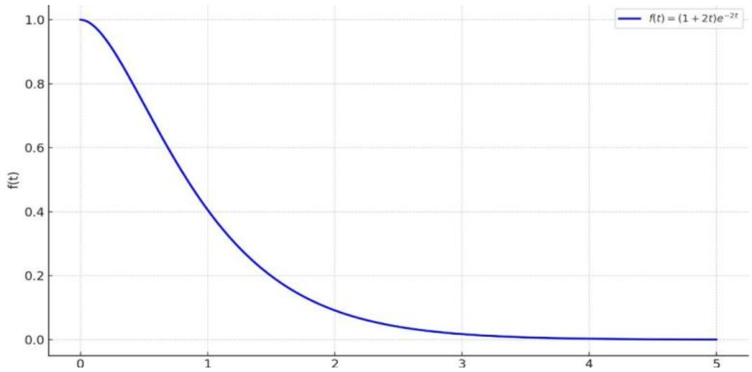

Figure 4

Similarly, Newtonian cooling can be written as a simple model to describe how a hot object cools in a cooler environment. According to this model, the rate of change of the object's temperature is proportional to the temperature difference between it and the environment. Applications in fields such as thermal engineering or the design of ventilation systems, in the medical field, the cooling of bodies after death, and in the food industry, the cooling of food, its differential equation is a linear first order, which is defined as follows:

$$\frac{dT}{dt} = -k(\mathrm{T} - \mathrm{T}_p) \tag{10}$$

In equation (10), T is the temperature in terms of time, T_p ambient temperature and k a positive constant that depends on the object and the conditions of the object. The solution of equation (10) with the Cauchy boundary condition is:

$$T(\mathrm{t}) = \mathrm{T}_p + (\mathrm{T}_0 - \mathrm{T}_p)\,\mathrm{e}^{-kt}, \qquad T(0) = T_p(0) \tag{11}$$

Equation (11) shows that the temperature of the object approaches the ambient temperature exponentially, and its graph is as follows:

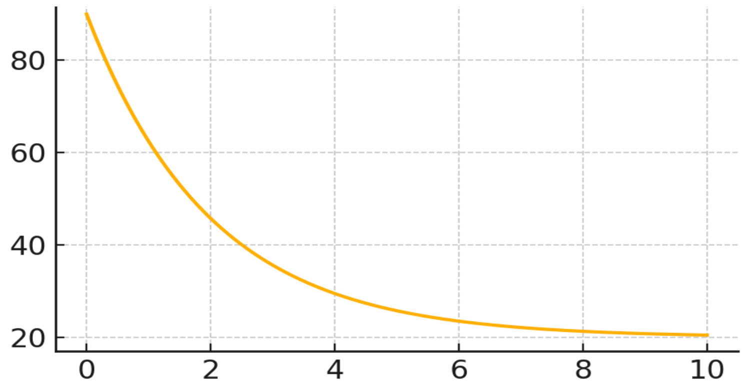

Figure 5

4. DISCUSSION

Despite the simplicity of linear differential equations, some researchers believe that these models are not suitable for complex systems. However, linear differential equations are often used as basic models or initial estimates, especially in the simple oscillatory behavior of electrical circuits and other dynamical systems, examples of which were mentioned. It should be said that nonlinear models are more consistent with nature, but their solution and analysis are more complex.

5. CONCLUSION

Linear differential equations, by providing a simple and systematic framework, allow for accurate analysis and prediction of the behavior of physical systems over time. These equations are particularly widely used in the analysis of mass-spring systems, RLC circuits, and heat transfer processes. Therefore, it is recommended to use these equations as basic models or initial estimates. As a result, a great deal of effort has been made to examine mass-spring systems, RLC circuits, and heat transfer processes over time with examples and to illustrate their behavior over time in diagrams. However, it should not be left unsaid that the limitations of these models become apparent when dealing with nonlinear and more complex systems. If greater accuracy is needed, one must move towards nonlinear models. Ultimately, linear differential equations remain an important part of human science that has played a fundamental role in engineering and scientific analysis.

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