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NUMERICAL METHOD FOR SIMULATING THE MODEL OF USING INSECTICIDE FOR THE OPTIMAL CONTROL OF MOSQUITOES FOR THE ERADICATION OF MALARIA

[a] **Ojo Olamiposi Aduroja**
[b*] **Samuel Adamu (https://orcid.org/0000-0003-1690-4269)**

[a] Department of Mathematics, University of Ilesa, Ilesa, Osun State, Nigeria
[b] Department of Mathematics, Nigerian Army University Biu, Borno State, Nigeria

***Corresponding Author:** malgwisa@gmail.com (Adamu, S.)

Abstract: In order to simulate the mathematical model of eliminating malaria by controlling the population of mosquitoes with insecticide and the insecticide's residual effects, the study developed a four-points hybrid block algorithm. The convergence and stability qualities of the block method are established. The block approach is applied after the variable control problems are generated using Pontryagin's principle. The forward-backward sweep methods of the block method are applied. The method is then implemented using a computer code using MATLAB R2018a mathematical software. According to the findings of this study, the simulated result from this approach displayed a significantly lower number of mosquitoes while lessening the negative effects of the insecticide, which in turn will reduce the high rate of malaria spreading.

Keywords: Block method, Insecticide, Malaria, Mosquito, Optimal control, Pontryagin's principle

1. Introduction

Malaria remains a significant public health challenge, particularly in the tropical and subtropical regions. The World Health Organization (2021) reported over 200 million cases annually, underscoring the need for effective control strategies. Eliminating mosquitoes is necessary to prevent malaria, a mosquito-borne disease that is one of the main causes of death worldwide, particularly in Africa (Araujo et al., 2022). Application of insecticides to control mosquito populations has gained attention (Bashir et al., 2020; Araujo et al., 2022).

Optimal control theory provides a mathematical framework for determining the best strategies to minimize or maximize a particular outcome, such as disease transmission (Adamu et al., 2024; Aduroja et al., 2024). Several studies have applied these principles to the management of mosquito populations through insecticide use. Araujo et al. (2022) developed a dynamic model integrating optimal control theory to optimize insecticide spraying schedules. Their findings indicated that strategically timed applications significantly reduced mosquito populations while minimizing costs and environmental impact. The model emphasized the importance of understanding mosquito life cycles and behavior to enhance the efficacy of insecticide application.

Similarly, He et al. (2023) explored the use of optimal control strategies in Integrated Vector Management (IVM). Their research highlighted that combining insecticide treatments with environmental management could yield better outcomes in reducing malaria incidence. By utilizing a control strategy that adjusts the intensity of insecticide application based on mosquito density, the researchers demonstrated a substantial reduction in malaria transmission rates.

The economic implications of insecticide application are crucial for public health decision making. Recent studies have assessed the cost effectiveness of optimal control strategies. Kweka et al. (2021) conducted a cost-benefit analysis of various insecticide application methods, including indoor residual spraying (IRS) and larviciding. They found that optimal application timing and dosage significantly improved cost-effectiveness, making these strategies more appealing for national malaria control programs. Tusting et al. (2023) used a mathematical model to evaluate the economic impact of optimal insecticide use in rural settings. Their results indicated that targeted insecticide applications could reduce malaria transmission at a lower cost compared to blanket spraying approaches, thus providing a more sustainable solution for resource-limited settings.

Ordinary and partial differential equations form the foundation of many mathematical models used by researchers to describe a wide range of phenomena in the social sciences, technology, engineering, and sciences (Adamu et al., 2019). First order ordinary differential equations of Initial Value situations (IVPs) arise in many physical situations in our world today (Orakwelua et al., 2023; Olaiya et al., 2022).

A few mathematical models of malaria transmission with and without a seasonal element were examined by (Fatmawati et al. 2021). A numerical simulation of the model and the provision of controls in the form of pesticide, treatment, and prevention all work together to reduce the number of infectious mosquitoes and exposed and infectious human population. In order to prevent mosquito populations from going extinct, (Araujo et al. 2022) investigated theoretical and numerical optimum control problems utilizing mobile devices that disperse pesticide.

A thorough examination of an optimal control problem is conducted, and the ideal trajectory is derived utilizing optimality conditions. Forward Backward Sweep (FBS) is an iterative technique (Garret, 2015; Adamu, 2023). FBS solves the state "forward" in time first, then solves the adjoint "backward" in time to approximate the control function. The indirect method is the process of determining the best outcome by resolving the required conditions. With the vital benefit of not requiring an earlier assessment of the infimal cost function, Pontryagin's Maximum (or Minimum) Principle (PMP) is a potent technique for computing optimal controls (Garret, 2015; Lenhart & Workman, 2007; Adamu et al., 2023; Aduroja et al., 2024).

Orakwelua et al. (2023) applied the forward-backward sweep approach utilizing the traditional Runge-Kutta method to limit the mosquito population in the ponds and marshes. This was done by adopting the best management strategies. Garret (2015) also used FBS in conjunction with the Classical Runge-Kutta Method (CRKM) to solve the Optimal Control Problem (OCP). As a result, CRKM emerged as the preferred option for utilizing FBS to solve optimal control problems. Davaeifar and Rashidinia (2017) state that the primary advantages of using FBPs in the collocation method's construction are its validity and reliability. It follows that a method with higher approximation accuracy will be obtained when FBPs are used in conjunction with the hybrid point. Adamu et al. (2024) develop an iterative method for simulating the insecticide and mosquito model.

This study simulates the ideal control strategy for insecticide and mosquito population growth using a four points block algorithm. The impact of the insecticide used to kill mosquitoes and their number is demonstrated.

2. Methodology

2.1 Development of the block method

Consider Polynomials approximate solution of the form

$$y(x) = \sum_{n=0}^{k} a_n x^n, \tag{1}$$

where $x \in [a,b],\ a_n \in \mathrm{R}$ are the unknown parameters to be determined.

The first and second derivative of (1) gives

$$y'(x) = \sum_{n=1}^{k} n a_n x^{n-1},\ y''(x) = \sum_{n=2}^{k} n(n-1) a_n x^{n-2}, \tag{2}$$

where $k = s + r - 1.$ s is the number of interpolation points and r is the number of collocation points. Interpolating (1) at x_{n+k} where $k = 0,1,2,3,\ldots,s$ and collocating (2) at x_{n+k} where $k = 0,1,2,3,\ldots,r$ gives a system of nonlinear equation of the form

$$U = AX, \tag{3}$$

where

$$A = [a_0,\ a_1,\ \ldots,\ a_k]$$

$$U = \left[y_n,\ y_{n+v_1},\ \ldots,\ y_{n+v_m},\ y'_n,\ y'_{n+v_1}\ldots,\ y'_{n+v_m},\ y''_n,\ y''_{n+v_1}\ldots,\ y''_{n+v_m}\right]^T$$

Imposing the following conditions on (1)

$$
\begin{aligned}
y(x_{n+v_i}) &= y_{n+v_i}, \quad i = 0,1,..m \\
y'(x_{n+v_i}) &= f_{n+v_i}, \quad i = 0,1,..,m \\
y''(x_{n+v_i}) &= g_{n+v_i}, \quad i = 0,1,..,m
\end{aligned}
$$

$$
X = \begin{bmatrix}
1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^k \\
0 & 1 & 2x_n & 3x_n^2 & \cdots & kx_n^{k-1} \\
0 & 1 & 2x_{n+v_1} & 3x_{n+v_1}^2 & \ldots & kx_{n+v_1}^{k-1} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 1 & 2x_{n+v_m} & 3x_{n+v_m}^2 & \ldots & kx_{n+v_m}^{k-1} \\
0 & 0 & 2 & 6x_n & \ldots & k(k-1)x_n^{k-2} \\
0 & 0 & 2 & 6x_{n+v_1} & \ldots & k(k-1)x_{n+v_1}^{k-2} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 2 & 6x_{n+v_m} & \ldots & k(k-1)x_{n+v_m}^{k-2}
\end{bmatrix}
$$

where $v_i \in (x_0, x_{n+m}),\ i = 1,2,...,m.$ Solving (3) for the unknown parameters and substituting into (1) gives the continuous method

$$
\begin{aligned}
y_{n+t} = \sum_{j=0}^{r} \alpha_j(t) y_{n+j} + h\left[\sum_{j=0}^{s} \beta_j(t) f_{n+j} + \sum_{v_i} \beta_{v_i}(t) f_{n+v_i}\right] \\
+h^2\left[\sum_{j=2}^{\tau} \sigma_j(t) g_{n+\tau} + \sum_{v_i} \sigma_{v_i}(t) g_{n+v_i}\right]
\end{aligned} \tag{4}
$$

where $\alpha_j(t),\ \beta_j(t),$ and $\sigma_j(t)$ are polynomial of degree $r+s+\tau-1$ and $t = \dfrac{x - x_{n+v_i}}{h}$.

Evaluating equation (4) at point $y_{n+\xi}$ $\left(\xi = \frac{1}{3}, 1, \frac{4}{3}, 2\right)$ gives the discrete schemes which is written in block form as

$$
A^{(1)}Y_{m+1} = A^{(0)}Y_m + hB^{(0)}F_m + hB^{(1)}F_{m+1} + h^2\gamma^{(1)}G_{m+\phi},\ \phi = \frac{1}{3}, 1, \frac{4}{3}, 2 \tag{5}
$$

where

$$
Y_{m+1} = \left[y_{n+\frac{2}{3}}\ y_{n+1}\ y_{n+\frac{4}{3}}\ y_{n+2}\right]^T, Y_m = \left[y_{n-1}\ y_{n-2}\ y_{n-3}\ y_n\right]^T,
$$

$$
F_m = \left[f_{n-1}\ f_{n-2}\ f_{n-3}\ f_n\right]^T,\ F_{m+1} = \left[f_{n+\frac{2}{3}}\ f_{n+1}\ f_{n+\frac{4}{3}}\ f_{n+2}\right]^T,\quad G_{m+1} = \left[g_{n+\frac{1}{3}}\ g_{n+1}\ g_{n+\frac{4}{3}}\ g_{n+2}\right]^T,
$$

$$A^{(1)}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\ A^{(0)}=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix},B^{(0)}=\begin{bmatrix}0&0&0&\frac{16597}{162000}\\0&0&0&\frac{193}{400}\\0&0&0&\frac{1612}{2025}\\0&0&0&\frac{178}{125}\end{bmatrix},$$

$$B^{(1)}=\begin{bmatrix}\frac{37403}{162000}&\frac{101}{6480}&-\frac{73}{12960}&\frac{47}{162000}\\\frac{207}{400}&\frac{79}{240}&\frac{9}{160}&-\frac{1}{1200}\\\frac{1088}{2025}&\frac{64}{81}&-\frac{34}{405}&-\frac{8}{2025}\\\frac{72}{125}&\frac{8}{15}&\frac{9}{20}&\frac{134}{375}\end{bmatrix},\text{ and }\gamma^{(1)}=\begin{bmatrix}-\frac{487}{16200}&0&0&0\\0&-\frac{3}{40}&0&0\\0&0&\frac{16}{405}&0\\0&0&0&-\frac{1}{25}\end{bmatrix}$$

2.2 Stability Properties

2.2.1 Order and error constant of the method

Evaluating each row of (4) in a Taylor series about x_n gives

$$L[y(x);h]=y_{n+t}-\sum_{j=0}^{r}\alpha_j(t)y_{n+j}-h\left[\sum_{j=0}^{s}\beta_j(t)f_{n+j}+\sum_{v_i}\beta_{v_i}(t)f_{n+v_i}\right]$$
$$-h^2\left[\sum_{j=0}^{\tau}\sigma_j(t)g_{n+j}+\sum_{v_i}\sigma_{v_i}(t)g_{n+v_i}\right] \tag{6}$$
$$h^{p+n}\neq 0$$

where n is the order of the differential equation. Therefore, the order of the block method (5) is $p=[6,6,6,6]^T$ with $\text{Error Constant}=\left[-\frac{617}{220449600}h^7,\ \frac{1}{2721600}h^7,\frac{8}{1148175}h^7,\frac{2}{42525}h^7\right]^T$

2.2.2 Zero Stability of the Block Method

The block method (5) is zero stable since the roots $z_s, s=1,2,3,..n$ of the first characteristics polynomial $\rho(z)$ is defined by

$$\rho(\lambda)=\det\left[A^{(1)}\lambda-A^{(0)}\right]=0 \tag{7}$$
$$\lambda^4-\lambda^3=\lambda^3(\lambda-1)=0$$

Solving for λ we have $\lambda=[0,0,0,1]$. Hence the block method (5) is zero stable.

2.2.3 Consistency

Since the method (5) has order $p=6>1$, therefore, the method is consistent.

2.2.4 Convergence

The method (5) is convergent since they are consistent and zero-stable.

2.2.5 Region of Stability

This is achieved by substituting the test equation $y' = \lambda y$ in the method and the result coded using MATLAB 8.5 to give

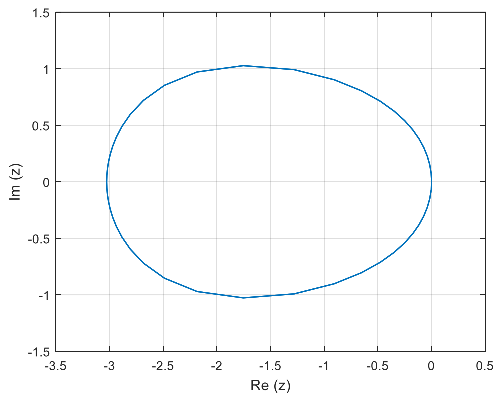

Figure 1: Region of Absolute Stability

3. Numerical Examples

The insecticide and mosquito model's parameter values, numerical analysis, and simulation are presented in the section below. It also illustrates how the mosquito population react to the application of insecticides in a yard.

Problem: Supposed the population concentration of mosquitoes at time t is given by $x(t)$, and the population over a fixed period of time wished to be reduced. Assuming x has a carrying capacity M and growth rate r. An application of the substance which is known to decrease the rate of change of $x(t)$, by decreasing the proportional rate to the amount of $u(t)$ and $x(t)$. Assumed the amount of the substance to be added at time t is $u(t)$. Then the concentration of mosquitoes is taken to be $x(t)$ and the insecticide known to kill it taken to be $u(t)$. Hence, the optimal control problem for 4 days' regimen is

$$\min_u J(x,u) = \int_0^4 Ax(t) + u(t)^2 \, dt$$

subject to

$$x'(t) = r(M - x(t)) - u(t)x(t), \; x(0) = x_0.$$

where the population size at the initial point is given as $x_0 > 0$. Here, term $u(t)x(t)$ pulls down the growth rate of the mosquito. Both mosquito and insecticide have negative effects on individuals around them, so both need to be minimized. Little amount is acceptable for both, there is then need to penalize for amounts too large, so quadratic terms for both will be analysed.

The coefficient A is the weight parameter, balancing the relative importance of the two terms in the objective functional (Adamu *et al.,* 2024).

Solution: The optimality system of the problem is developed by first constructing the Hamiltonian

$$H(t,x,u,\lambda) = f(t,x,u) + \lambda g(t,x,u),$$

$$H(t,x,u,\lambda) = Ax + u^2 + \lambda r(M - x) - \lambda xu.$$

The optimality condition is

$$0 = \frac{\partial H}{\partial u} = 2u - \lambda x \Rightarrow u^* = \frac{\lambda x}{2}.$$

The adjoint equation is

$$\lambda'(t) = -\frac{\partial H}{\partial x} = -A + \lambda r + \lambda u$$

$$= -A + \lambda r + \frac{1}{5}\lambda^2 x$$

$$x'(t) = Mr - x(r+u),\ x(0) = x_0.$$

$$\lambda'(t) = -A + \lambda r + \frac{1}{5}\lambda^2 x, \lambda(T) = 0.$$

Using the optimality system, the numerical code is generated, written in MATLAB R2018a. The results are shown in Figures.

First considering parameters $M = 10,\ r = 0.3,\ x_0 = 1,\ A = 1$

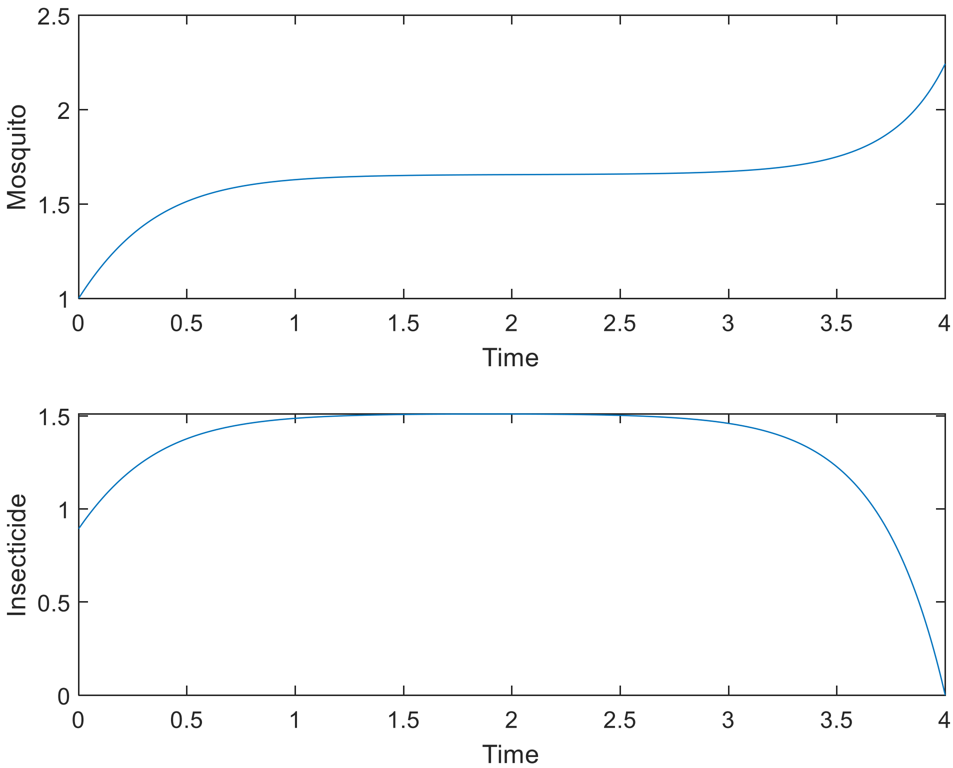

Figure 2: The optimal Mosquito population and Insecticide

The goal is to reduce the number of mosquitoes and lessen the impact of the insecticides on nearby people. In Figure 2, the mosquito population is growing when the carrying capacity M=10 and the weight parameter A=1, but it levels out and becomes constant, and when the insecticide is introduced and kept at a constant level; which kept the insecticide and mosquito population at parallel. The mosquito population starts to rise again on day three as the insecticide starts to fade, with rapid growth at the start and finish.

Varying the weight parameter A to A=1.5

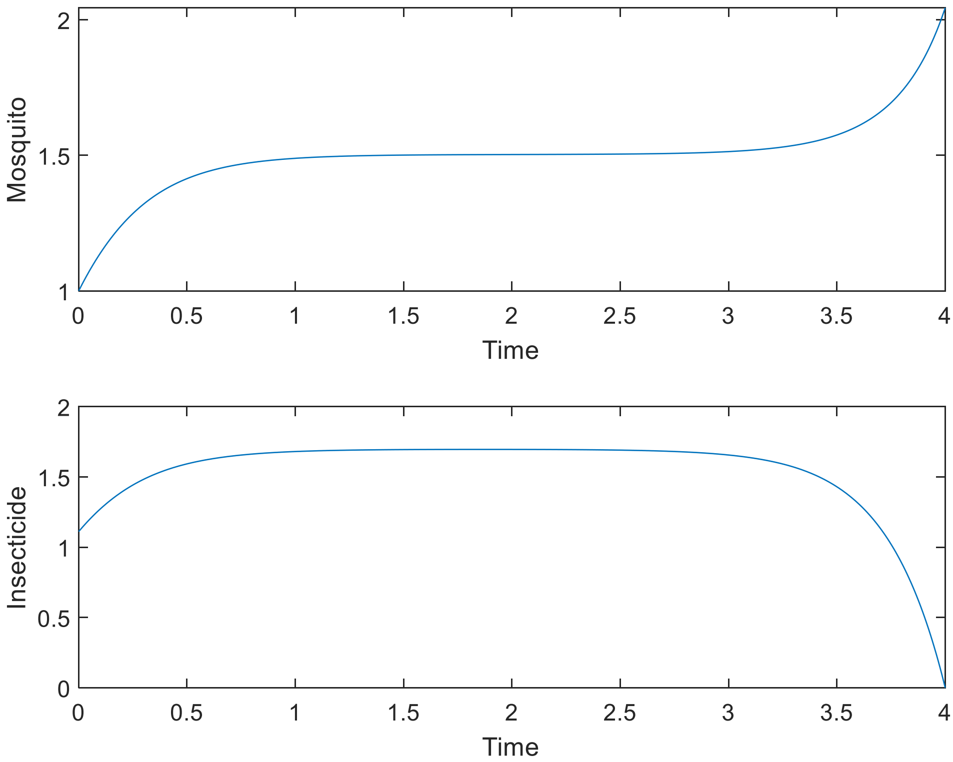

Figure 3: The optimal Mosquito population and Insecticide

In Figure 3, the amount of insecticide applied is marginally higher when carrying capacity is kept at M=10 and the weight parameter is changed to A=1.5. For a longer time, it is evident that state and control are in balance. Starting at its maximum, the control gradually decreases before briefly becoming steady before dropping to zero. Up to day three, the state growth rate decreased and stabilized. However, the mosquito population grows quickly after the conclusion of the interval, when the insecticide's effects are no longer detrimental to the insects.

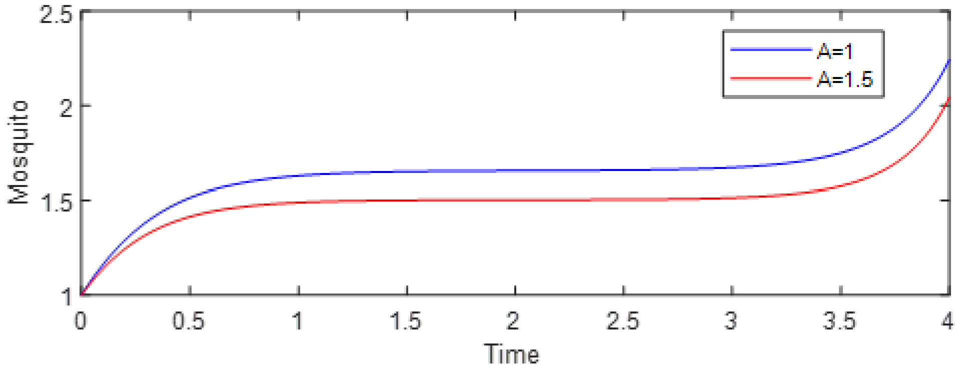

Figure 4: The Mosquito population growth rate when A is varied

From Figure 4 show the combined mosquitoes population growth rate when the weight parameter is varied from 1 to 1.5. It shows that, the higher the weight parameter, the slower the growth of the mosquito population.

Varying the parameter r to r=0.4

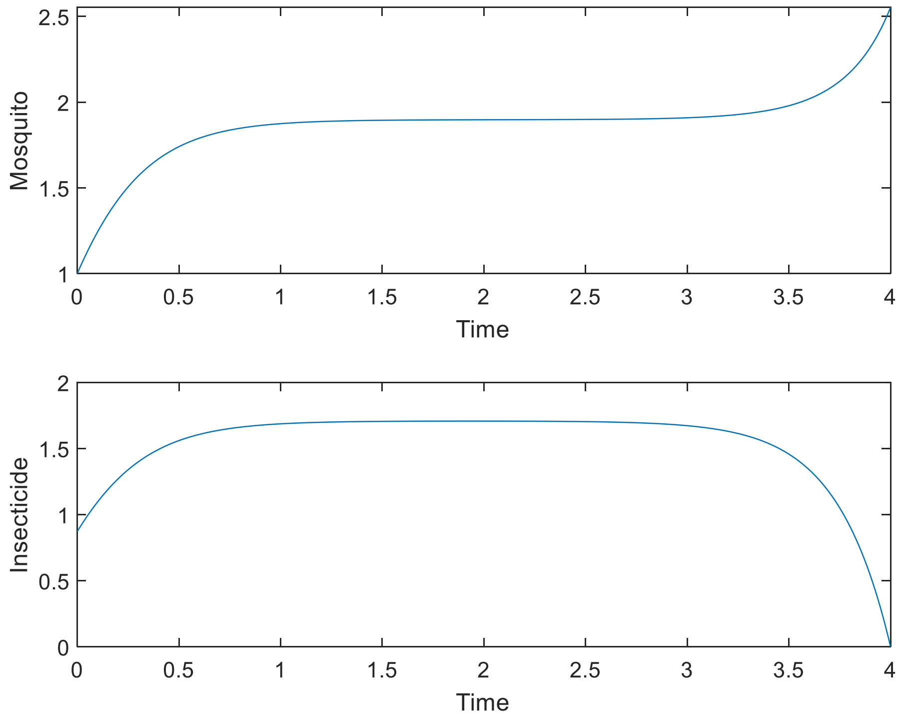

Figure 5: The optimal Mosquito population and Insecticide

A significantly higher amount of pesticide is needed in Figure 5 when the carrying capacity is kept at M=10 and the growth rate parameter is changed once more to r=0.4. Long-term equilibrium between the insecticide and mosquito population has been noted. The insecticide is at its peak at first, then it starts to decline a little, then it stabilizes, and finally it drops to zero. Up until day 2.5, the state growth rate likewise decreased and stabilized. However, the mosquito population grows quickly after the conclusion of the interval, when the insecticide's effects are no longer detrimental to the insects.

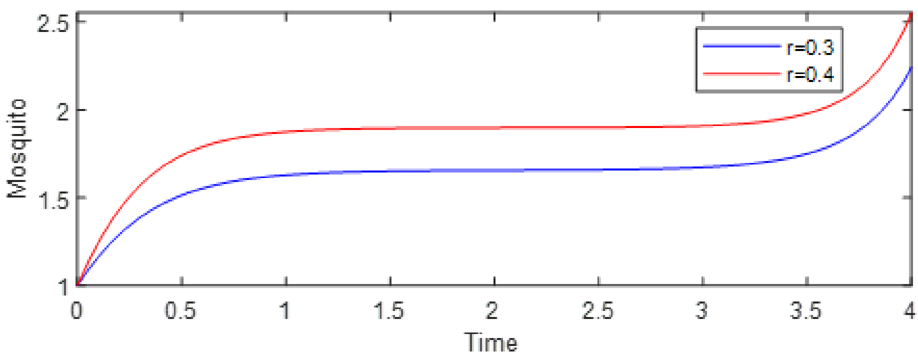

Figure 6: The Mosquito population growth rate when r is varied

From Figure 6 show the combined mosquitoes population growth rate when the growth rate parameter is varied from 0.3 to 0.4. It shows that, the lower the growth rate parameter, the slower the growth of the mosquito population.

4. Conclusion

Thus, examining the result, it is evident that applying an insecticide that lasts for four days slightly lowers the mosquito population; nevertheless, as soon as the pesticide's impact wears off, the population starts to grow once more. Additionally, the mosquito population decreases as more insecticide is used. Starting a second regimen around day 3 is the best approach to keep up a four-day regimen.

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