f-Primary Ideals in Semigroups

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Abstract: Right now, the terms left *f*-Primary Ideal, right *f*-Primary Idealand *f*- primary ideals are presented. It is Shown that An ideal U in a semigroup S fulfills the condition that If G, H are two ideals of S with the end goal that $f(G)\, f(H) \subseteq U$ and $f(H) \nsubseteq U$ then $f(G) \subseteq r_f(U)$iff $f(q), f(r) \subseteq S$, $<f(q)><f(r)> \subseteq U$ and $f(r) \nsubseteq U$ then $f(q) \subseteq r_f(U)$ in like manner it is exhibited that An ideal U out of a semigroup S fulfills condition If G, H are two ideals of S such that $f(G)\, f(H) \subseteq U$ and $f(G) \nsubseteq U$ then $f(H) \subseteq r_f(U)$ iff $f(q), f(r) \subseteq S, <f(q)><f(r)> \subseteq U$ and $f(q) \nsubseteq U \Rightarrow f(r) \subseteq r_f(U)$. By utilizing the meanings of left - *f*- primary and right *f*- primary ideals a couple of conditions are illustrated It is shown that J is a restrictive maximal ideal in Son the off chance that$r_f(U) = J$ for some ideal U in S at that point J will be a *f*- primary ideal and J^n is *f*-primary ideal for some n $\in N$ it is explained that if S is quasi-commutative then an ideal U of S is left *f* - primary iff right *f* -primary.

Keywords: Left *f*- primary Ideal,Right *f*-Primary ideal, f-primary ideal.

1. INTRODUCTION

The idea of a semigroup is basic and assumes an enormous function in the advancement of Mathematics. The hypothesis of *semigroups* is like group and ring theory. "*f*-Semi prime ideals in *Semigroups"* and "*f- prime radical in semi groups"* was developed by *T.Radha Rani* and *A.Gangadhara Rao*[1][2] "*The algebraic theory of semigroups"* was developed by *Clifford* and *Preston* [6], [7]; *Petrich* [8] "*Structure and ideal theory of semi groups"* was presented by *Anjaneyulu.A* [3] "*A generalization of prime ideals in semi groups"* was presented by *Hyekyung Kim* [4] "*generalization of prime ideals in rings"* was introduced by *Murata.K*, *Kurata.Y* and *Murabayashi.H* [9] "*prime and maximal ideals in semi groups"* was presented by *Scwartz.S* [5].

2. PRELIMINARIES

2.1 ***Definition***: (S,.) be a non-void set. If '.' Is binary operation on S and it holds associative then S is defined as a Semigroup.
2.2*Note*: Throughout this paper S will indicate a semigroup.
2.3*Definition:*If $qr=rq$ to all $q,r\in$ S then S is called as "*commutative"*
2.4*Definition:*S is supposed as"Quasi commutative" if uv =v^nu for some n $\in$ N
where $u,v\in S$.
2.5*Definition:**If $qs = s\ \forall\ s \in S$ then the component q in S is called as "left identity" of S.*
2.6*Definition*: If $sq = s \forall s\in S$ then the component q in S is called as "*right identity"* of S.
2.7 ***Definition***: A component q in S is both left and right identity in S then it is called as "*identity"*.
2.8 ***Definition***: Let $Q \neq \emptyset$ is a set in S. Q is entitled as "*left ideal*" in S when $SQ\subseteq Q$.
2.9 ***Definition***: Let $Q \neq \emptyset$ is a set in S. Q is entitled as "*right ideal*" in S when $QS\subseteq Q$.
2.10 ***Definition***: A subset Q in S *is* both left and right ideal in S then it is known as "*ideal*" in S.
2.11 ***Definition***: The intersection of each one of the ideals in S carrying a non-void set P is known as the "*ideal generated by P"*. It is signified as $<P>$.
2.12 ***Definition***: Some ideal Q of S is called as "*principal ideal"* given Q is an ideal created by single component set. On the off chance that an ideal Q is generated by q, at that point Q is indicated as $<q>$ or J[q]
2.13 ***Definition***: Some ideal Q of S is called as "*completely prime ideal"* given $u,v\in Q$,
$uv\in Q$, either $u \in Q$ or $v \in Q$.
2.14 ***Definition***: Some ideal D in S is known as "*prime ideal"* when Q, R be ideals of S,
$QR\subseteq D$ infers either $Q\subseteq D$ or $R\subseteq D$.
2.15 ***Definition***: Let P be some ideal of S, then the intersection of each one of the prime ideals carrying P is said to be "*prime radical"*or just "*radical of P*" and it is meant by $\sqrt{P}$ or *radP*.
2.16 ***Definition***: Let P is some ideal in S, then the intersection of each one of the completely prime ideals carrying P is entitled as "*complete prime radical* "or "*complete radical*" of P and it is meant by *c.rad P*.
2.17 ***Note***: Throughout this paper S be a semigroup and f is a function from S into Ideals of S to such an extent that,

(i) $q\in f(S)$ infers $f(q)\subseteq f(S)$,
(ii) $Q\subseteq S\Rightarrow f(Q)$ is an ideal in S(*by [1] Ref [4]*)
2.18 *Note*: Let A be any ideal of S. Then the ideal $\bigcup_{a\in A} f(a)$ is denoted by f(A).
Clearly $A\subseteq f(A)$ and $f(A)\subseteq f(B)$ if $A\subseteq B$. (*by proposition 1.1in Ref [4]*)
2.19 *Theorem*: If $f(a)$ is an Ideal in S then $f(A) = \bigcup_{a\in A} f(a)$ is an Ideal
2.20 *Definition*: Let U be someideal of $S.U$ is called as "*f-prime ideal*" if G, H be two ideals of S. $f(G)\, f(H)\subseteq U$ implies either $f(G)\subseteq U$ or $f(H)\subseteq U$.
here $f(G) = \bigcup_{g\in G} \boldsymbol{f(g)}$ ***and*** $f(H) = \bigcup_{h\in H} \boldsymbol{f(h)}$
2.21 *Definition*: Let Q be some ideal in S and q, r be two components in S. Q is defined as "*completely f-prime ideal*" if $f(u), f(v)\subseteq S, f(u).\, f(v)\subseteq Q\Rightarrow$ either $u\in Q$ or $v\in Q$.
2.22 *Theorem*: Every completely *f*-prime ideal of a semigroup is *f*-prime.
2.23 *Theorem*: If S is globally idempotent semigroup then every maximal ideal M of S is a *f*-prime ideal of S.

3. RESULTS AND DISCUSSION

3.1 *Definition*: A Subset Q of S is called a *p*-system $\Leftrightarrow \langle q\rangle\langle r\rangle \cap Q \neq \emptyset$ for any q, r in Q.
3.2 *Definition*: A Subset Q of S is called a *sp*-system $\Leftrightarrow \langle q\rangle^2 \cap Q \neq \emptyset$ for any q in Q.
3.3 *Note*: Every *p*-system is an *sp*-system, but converse need not be true.
3.4 *Example*: Let S = {u, v, w, x} be the semigroup with the following multiplication table

.	***u***	***v***	***w***	***x***
u	***u***	***u***	***u***	***u***
v	***u***	***v***	***u***	***u***
w	***u***	***u***	***w***	***u***
x	***u***	***u***	***u***	***x***

Suppose {u,v} and {v, w, x} are two subsets of S.
Clearly {u, v} is a *p*-system and {v, w, x} is a *sp* – system but not a *p*-system.
3.5 *Definition*: For any $f\in F$ a subset Q of S is called an f – system if and only if it contains a *p*-system Q^* such that $Q^*\cap f(q) \neq \emptyset$ for each q in Q.
3.6 *Definition*: For any $f\in F$ a subset Q of S is called an *sf* – system if and only if it contains a *sp*-system Q^* such that $Q^*\cap f(q) \neq \emptyset$ for each q in Q.
3.7 *Definition*: Let G be an ideal of S then f-*rad* G= {$x/Q\cap G \neq \emptyset$for each *f*-system Q containing x} will be called the *f*-radical of A and is denoted by $r_f(A)$.
3.8 *Theorem*: Let G be an ideal of S. Then *f- rad G* is the intersection of all *f*-prime ideals of S containing G
Proof: Let G be an Ideal of S.
Assume that $\mathscr{L}$= the intersection of all *f*-prime ideals of S containing G.
Now we show that $\mathscr{L}= r_f(G)$
Suppose if possible $r_f(G) \nsubseteq \mathscr{L}$.
$\Rightarrow$there exists a *f*-prime ideal P contained in $r_f(G)$ and not contained in $\mathscr{L}$.
Since P contained in $r_f(G) \Rightarrow P\cap G \neq\emptyset$
$\Rightarrow P\nsubseteq \mathscr{L}$ implies $P^c\subseteq\mathscr{L}\Rightarrow P^c \cap G \neq\emptyset$.
Which is a contradiction.so, our supposition is wrong.
Therefore $r_f(G)\subseteq \mathscr{L}$ --------(1)
Suppose if possible $\mathscr{L} \nsubseteq r_f(G)$
$\Rightarrow$there exists a *f*-prime ideal P contained in $\mathscr{L}$ and not contained in $r_f(G)$.
Since $P\subseteq\mathscr{L}\Rightarrow P\cap G \neq\emptyset$.
Now $P\nsubseteq r_f(G)\Rightarrow P^c\subseteq r_f(G)$
Since $r_f(G)$= {$x/Q\cap G \neq \emptyset$for each *f*-system Q containing x}
So, P^c is a *f*-system and $P^c\cap G \neq \emptyset$
It contradicts our assumption.
Therefore $\mathscr{L}\subseteq r_f(G)$ --------(2)
From (1) and (2) $\mathscr{L}=r_f(G)$
i.e., $r_f(G)$is the intersection of all *f*-prime ideals of S containing G.
3.9 *Theorem*: If P is a *f*-prime ideal of a semigroup S, then $r_f(P)^n = P$ for all $n\in N$.
3.10 *Theorem*: In a semigroup S with identity there is a unique maximal ideal M such that $r_f(M)^n = M$ for all $n\in N$. (*by Ref [2]*)

3.11 ***Definition***: Let Q be some ideal in $S.Q$ is defined as "*left f-primary ideal*" if
(i) If U, V are two ideals in S with $f(U)f(V)\subseteq Q$ and $f(V)\nsubseteq Q$ then $f(U)\subseteq r_f(Q)$
(ii) $r_f(Q)$ is f-prime ideal.
3.12 ***Definition***: Let Q be some ideal in $S.Q$ is defined as "*right f-primary ideal*" if
(i) If U, V are two ideals in S with $f(U)f(V)\subseteq Q$ and $f(U)\nsubseteq Q$ then $f(V)\subseteq r_f(Q)$
(ii) $r_f(Q)$ is f-prime ideal
3.13 ***Definition***: Q is both left and right f-primary ideal implies Q is "*f - primary ideal.*"
3.14 ***Theorem***: Some ideal Q in S satisfies condition (i) of 3.1 iff $f(g), f(h)\subseteq S$, $<f(g)><f(h)>\subseteq Q$ and $h\notin Q$ then $g\in r_f(Q)$.
Proof: Let Q be some ideal in S.
Suppose that Q satisfies the condition (i) of 3.1.
i.e., If G, H are two ideals in S with $f(G)f(H)\subseteq Q$ and $f(H)\nsubseteq Q$ then $f(G)\subseteq r_f(Q)$
Let $g,h\in S \Rightarrow f(g), f(h)\subseteq S \Rightarrow <f(g)><f(h)>\subseteq Q$ and $h\notin Q$
$\Rightarrow f(h)\nsubseteq Q$. While $f(h)\nsubseteq Q$, $<f(h)>\nsubseteq Q$.
From the supposition $<f(g)><f(h)>\subseteq Q$ and $<f(h)>\nsubseteq Q \Rightarrow <f(g)>\subseteq r_f(Q)$
Therefore $f(g)\subseteq r_f(Q) \Rightarrow g\in r_f(Q)$.
If we observe the other side, $f(g), f(h)\subseteq S$, $<f(g)><f(h)>\subseteq Q$ and $h\notin Q$ then $g\in r_f(Q)$
Let $f(G)$, $f(H)$ be two ideals of S with $f(G)f(H)\subseteq Q$ and $f(H)\nsubseteq Q$.
Suppose if possible, $f(G)\nsubseteq r_f(Q)$. Then there exists $g\in f(G)$ with $g\notin r_f(Q)$.
Since $f(H)\nsubseteq Q$, let $h\in f(H)$ implies that $h\notin Q$.
Now $<f(g)><f(h)>\subseteq f(G)f(H)\subseteq Q$ and $h\notin Q \Rightarrow g\in r_f(Q)$. It is a contradiction.
Therefore $f(G)\subseteq r_f(Q)$. Therefore, Q satisfies the condition (i) of 3.1.
3.15 ***Theorem***: Some ideal Q in S satisfies condition (i) of 3.2 iff $f(g), f(h)\subseteq S$, $<f(g)><f(h)>\subseteq Q$ and $g\notin Q$ then $h\in r_f(Q)$.
Proof: Let Q be some ideal in S.
Suppose that Q satisfies the condition (i) of 3.2.
i.e If G, H are two ideals in S with $f(G)f(H)\subseteq Q$ and $f(G)\nsubseteq Q$ then $f(H)\subseteq r_f(Q)$
Let $g, h\in S \Rightarrow f(g), f(h)\subseteq S \Rightarrow <f(g)><f(h)>\subseteq Q$ and $g\notin Q$
$\Rightarrow f(g)\nsubseteq Q$. While $f(g)\nsubseteq Q$, $<f(g)>\nsubseteq Q$.
From the supposition $<f(g)><f(h)>\subseteq Q$ and $<f(g)>\nsubseteq Q \Rightarrow <f(h)>\subseteq r_f(Q)$
Therefore $f(h)\subseteq r_f(Q) \Rightarrow h\in r_f(Q)$.
If we observe the other side, $f(g), f(h)\subseteq S$, $<f(g)><f(h)>\subseteq Q$ and $g\notin Q$ then $h\in r_f(Q)$
Let $f(G)$, $f(H)$ be two ideals of S with $f(G)f(H)\subseteq Q$ and $f(G)\nsubseteq Q$.
Suppose if possible, $f(H)\nsubseteq r_f(Q)$. Then there exists $h\in f(H)$ with $h\notin r_f(Q)$.
Since $f(G)\nsubseteq Q$, let $g\in f(G)$ implies that $g\notin Q$.
Now $<f(g)><f(h)>\subseteq f(G)f(H)\subseteq Q$ and $g\notin Q \Rightarrow h\in r_f(Q)$. It is a contradiction.
Therefore $f(H)\subseteq r_f(Q)$. Therefore, Q satisfies the condition (i) of 3.2.
3.16 ***Theorem***: If U is an ideal in S and S is Commutative in that case the given conditions are comparable.
1) U is a *f-primary ideal*.
2) $f(Q), f(R)$ are two ideals in $S, f(Q)f(R)\subseteq U$ and $f(R)\nsubseteq U$ then $f(Q)\subseteq r_f(U)$.
3) $f(q), f(r)\subseteq S, f(q)f(r)\subseteq U, r\notin U$ then $q\in r_f(U)$
Proof: *(1)* $\Rightarrow$ *(2)*: Assume (1) i.e., U is a *f-primary ideal*.
$\Rightarrow U$ is a left *f-primary* ideal. So, by 3.1,
We have $f(Q), f(R)$ are two ideals of S, $f(Q)f(R)\subseteq U$, $f(R)\nsubseteq U \Rightarrow f(Q)\subseteq r_f(U)$.
(2) $\Rightarrow$ (3): Suppose that $f(Q), f(R)$ are two ideals of S, $f(Q)f(R)\subseteq U$, $f(R)\nsubseteq U$
$\Rightarrow f(Q)\subseteq r_f(U)$.
Let $f(q), f(r)\subseteq S$, $f(q)f(r)\subseteq U$, $r\notin U$. $f(q)f(r)\subseteq U$.
Since $f(q)f(r)\subseteq U \Rightarrow <f(q)f(r)>\subseteq U \Rightarrow <f(q)><f(r)>\subseteq U$.
Also, $r\notin U \Rightarrow <f(r)>\nsubseteq U$. Now $<f(q)><f(r)>\subseteq U$ and $<f(r)>\nsubseteq U$.
Therefore, by assumption $<f(q)>\subseteq r_f(U) \Rightarrow q\in r_f(U)$
(3) $\Rightarrow$ (1): Suppose that $f(q), f(r)\subseteq S$, $f(q)f(r)\subseteq U$, $r\notin U$ then $q\in r_f(U)$.
Let $f(Q), f(R)$ be two ideals of S with $f(Q)f(R)\subseteq U$ and $f(R)\nsubseteq U$.
If $f(R)\nsubseteq U \Rightarrow$ there exists $r\in f(R)$ with $r\notin U$. Suppose if possible, $f(Q)\nsubseteq r_f(U)$.
Then there exists $q\in f(Q)$ such that $q\notin r_f(U)$. Now $qr\in f(Q)f(R)\subseteq U$.
Therefore $qr\in U$ and $r\notin U$, $q\notin r_f(U)$. It is a contradiction. Therefore $f(Q)\subseteq r_f(U)$.
Assume $f(q), f(r)\subseteq S$ and $f(q)f(r)\subseteq r_f(U)$. Suppose that $q\notin r_f(U)$
Now $f(q)f(r)\subseteq r_f(U) \Rightarrow (f(q)f(r))^m\subseteq U \Rightarrow f(q)^m f(r)^m\subseteq U$.
Since $f(r)\nsubseteq r_f(U)$, $f(r)^m\nsubseteq r_f(U)$.

Now $f(q)^m f(r)^m \subseteq U$, $f(r)^m \not\subseteq r_f(U) \Rightarrow f(q)^m \subseteq r_f(U) \Rightarrow f(q) \subseteq r_f(r_f(U)) = r_f(U)$.
So, $r_f(U)$a completely *f*-prime ideal $\Rightarrow r_f(U)$ is a *f*-prime ideal.
Thus, *U* is *left f-primary ideal*. likewise, *U* is *right f-primary ideal*.
From now *U* is *f-primary ideal*.
3.17 *Note*: *In a random semigroup a left f-primary ideal is not certainly a right f-primary ideal.*
3.18 *Example*: Assume $S = \{u,v,w\}$ be the semigroup under multiplication given in the following table.

.	*u*	*v*	*w*
u	*u*	*u*	*u*
v	*u*	*u*	*u*
w	*u*	*v*	*w*

Now consider the ideal $<u> = S^1uS^1 = \{u\}$. Let $xy \in <u>$, $y \notin <u> \Rightarrow x^n \in <u>$ for some natural number *n*. Since $vw \in <u>$, $w \notin <u> \Rightarrow v \in <u>$. Therefore $<u>$ is left *f-primary*. If $v \notin <u>$, then $w^n \notin <u>$ for any natural number *n*. Therefore $<u>$ is not right *f-primary*.
3.19 *Theorem*: Each ideal *U* in *S* be *left f-primary* iff each ideal *U* meets with the condition (i) of 3.1.
***Proof*:** Suppose each ideal *U* in *S* is left *f-primary,*
now obviously each ideal satisfies condition (i) of 3.1.
on the other hand, assume that each ideal in *S* meets with the condition (i) of 3.1.
assume that *U* be any ideal in *S*. choose $<f(q)><f(r)> \subseteq r_f(U)$.
If $r \notin r_f(U)$ then by our assumption $q \in r_f(r_f(\underline{U})) = r_f(U)$
Thus $r_f(U)$ is a *f*-prime ideal. So, *U* is left *f - primary*.
3.20 *Theorem*: Each ideal *U* in *S* is right *f-primary* iff each ideal *U* meets with the condition (i) of 3.2.
Proof: Suppose each ideal *U* in *S* is left *f-primary,*
Now obviously each ideal meet with the condition (i) of 3.2.
on the other hand, assume that each ideal in *S* meets with the condition (i) of 3.2.
Assume that *U* be any ideal in *S*. choose $<f(q)><f(r)> \subseteq r_f(U)$
If $q \notin r_f(U)$ then by our assumption $r \in r_f(r_f(\underline{U})) = r_f(U)$
Thus $r_f(U)$ is a *f-prime* ideal. So, *U* is right *f - primary*.
3.21 *Definition*:If each ideal in *S* is *leftf - primary* ideal then *S* is known as *"left f-primary.*"
3.22 *Definition*: If each ideal in *S* is *right f - primary* ideal then *S* is known as *"right f-primary.*"
3.23 *Definition*: If each ideal in *S* is *f - primary* ideal then *S* is known as *"f-primary*".
3.24 *Theorem*: If *S* has identity and assume that *J* is maximal ideal in *S* and *J* is unique.
If$r_f(U) = J$ for any ideal *U* in *S*, then *U* is a *f- primary* ideal.
***Proof*:** Assume that $<f(q)><f(r)> \subseteq U$ and $r \notin r_f(U)$.
If $q \notin r_f(U)$= then $<f(q)> \not\subseteq r_f(U) = J$.
We know that $J = \bigcup_{Q \subseteq S} Q$, *Q* is an ideal in *S*
So, $<f(q)> = S$ implies $<f(r)> = <f(q)><f(r)> \subseteq U$.
$\Rightarrow <f(r)> \subseteq U \Rightarrow r \in U$ It is a contradiction.
Thus$q \in r_f(U)$. Clearly $r_f(U)= J$ is a *f*-prime ideal.
Hence, *U* is *left f - primary*. Likewise, *U* is *right f – primary* implies *U* is a *f - primary ideal.*
3.25 *Note*: when *S* does not follow the identity condition consequently theorem 3.24 is notcorrect, even *S* contains a maximal ideal with uniqueness. Consider the *example 3.18*, $\sqrt{<u>} = J$here $J = \{u,v\}$ be a unique maximal ideal. But $<u>$ will not be a *f - primary* ideal.
3.26 *Theorem*: If *S* has identity and assume that *J* is maximal ideal in *S* and *J* is unique then $\forall\, n \in N\, J^n$ is a *f - primary ideal* of *S*.
Proof: Now *J* be the individual *f* - prime ideal having J^n, we have $r_f(J)^n = J$
So, by the theorem 3.22, $J^n$is *f - primary ideal*.
3.27 *Note*: If *S* does not follow the identity condition consequently the theorem 3.26 is not correct. Consider the *example 3.18*, $J = \{u,w\}$be a unique maximal ideal, but $J^2 = \{u\}$ will not be a *f-primary* ideal.
3.28 *Theorem*: *S* be quasi commutative then an ideal *Q* in *S* is *left f-primary iff right f-primary*.
***Proof*:** Assume *Q* be a left *f-primary* ideal. Consider $f(q)\, f(r) \subseteq Q$ and $q \notin Q$.
Meanwhile *S* be a quasi-commutative, so $q.r = r^n q$ for some natural number *n*.
So, $r^n q \in Q$ and $q \notin Q$. while *Q* be left *f*-primary, implies $r^n \in r_f(Q)$
and given that$r_f(Q)$ be a *f*-prime ideal, $r \in r_f(Q)$ Thus, *Q* be a right *f-primary* ideal.
Likewise, we can prove that if *Q* is a *right f- primary ideal* implies *Q* is a *left f-primary ideal*.
3.29 *Corollary*: Let *S* be quasi commutative, and *Q* is an ideal in *S* then the following are equivalent.
1) *Qis f - primary*.
2) *Q* is left *f - primary*.
3) *Q* is right *f - primary***.**

***Proof*:** By theorem 3.28, we have *S* be quasi commutative then an ideal *Q* in *S* is *left f-primary iff right f-primary*. So, the proof of the theorem is clear.

4. CONCLUSION

In Mathematics, study of semigroups becomes an object of the exercise for several researchers. here, we tried to study the hypotheses of *f*-primary ideals in semigroups and their characterizations.

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