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Some Theorems on Multiplicative Metric Spaces

Priyanka Rani[*1], Dr G. V. V. Jagannadha Rao[2]

[1]Research Scholar, [2]Assistant Professor

Department of Mathematics, Kalinga University, Raipur-492101, Chhattisgarh, India

[*]Email: priyankagrewalg@gmail.com

Abstract The aim of this paper is to discuss the fixed point theorems under some contractive conditions in multiplicative spaces. We also give their appropriate examples.

Keywords Multiplicative metric space, fixed point theorem and contraction mapping.

Introduction

Banach contraction principle plays an important role in nonlinear analysis. It provides a unique technique for solving many problems in mathematical sciences and engineering. Various authors have generalized Banach contraction principle in different spaces.

Singxi et al. [18] studied some common fixed point theorems for different mappings in 2-metric space. Branciari [8] introduced the concept of generalized metric space by imposing a general inequality condition in place of usual triangular in metric spaces. Gu et al. [9] proved some common fixed point theorems related to weak commutative mappings on a complete metric spaces. Moreover, Muatafa and Sims [13,14] studied many results on the class of generalized metric spaces.

Agarwal et al. [5] discussed some fixed point results on partially ordered metric spaces. For more details interested reader can refer to [4-6, 15-19].

In 2008, Bashirov et al. [7] come across with the problem that the set of positive real numbers is not complete in usual metric space. To solve this problem, they introduced the concept of multiplicative metric space.

Definition 1.1 [7] Let X be a nonempty set. A multiplicative metric space is a mapping $d^*: X \times X \to R^+$ satisfying the following conditions:

(i) $d^*(x, y) \geq 1$ for all x, y ∈ X and $d^*(x, y) = 1$ if and only if x = y;

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(ii) $d^*(x,y) = d^*(y,x)$ for all x, y ∈ X;

(iii) $d^*(x,y) \leq d^*(x,z).d^*(z,y)$ for all x, y, z ∈ X (multiplicative triangle inequality).

Then the mapping d^* together with X, that is, (X, d^*) is a multiplicative metric space.

Example 1.2 [7] Let (X, d) be a metric space, then the metric d_a defined on X as follows is multiplicative metric, $d_a(x,y) = a^{d(x,y)}$ where a > 1 is a real number. For discrete metric d the corresponding mapping d_a called discrete multiplicative metric is defined as:

$$d_a(x,y) = a^{d(x,y)} = \begin{cases} 1 \;\; if \; x = y \\ a \;\; if \; x \neq y. \end{cases}$$

Example 1.3 [7] Let R_+^n be the collection of all $n-$ tuples of positive real numbers. Let $d^*: R_+^n \times R_+^n \to R$ be defined as:

$$d^*(x,y) = \left|\frac{x_1}{y_1}\right|^* . \left|\frac{x_2}{y_2}\right|^* \dots \left|\frac{x_n}{y_n}\right|^*,$$

where $x = (x_{1,}x_{2\,,\dots,}x_{n,}), y = (y_{1,}y_{2,\dots,}y_n) \in R_+^n \; and \; |\,.\,|^*: R_+ \to R_+$ is defined by

$$|a|^* = \begin{cases} a \; if \; a \geq 1, \\ \frac{1}{a} \; if \; a \; < 1. \end{cases}$$

Then it is obvious that all the conditions of a multiplicative metric are satisfied. Therefore $(R_+^n,\; d^*)$ is a multiplicative metric space.

Example 1.4 [7] Let $d: R \times R \to [1,\infty)$ be defined as $d(x,y) = a^{|x-y|} \; where \; x, y \; \in R$ and a > 1.Then $d(x,y)$ is multiplicative metric.

Remark 1.5 Neither every multiplicative metric is metric nor every metric is multiplicative metric. The d^* mapping defined above does not satisfy the triangular inequality which implies that it is multiplicative metric but not a metric. Let $d^*\left(\frac{1}{3},\frac{1}{2}\right) + d^*\left(\frac{1}{2},3\right) = \frac{3}{2} + 6 = 7.5 < 9 = d^*\left(\frac{1}{3},3\right)$. On the other side the usual metric on R is not multiplicative metric as it does not satisfy multiplicative triangular inequality. As $d(2,3).d(3,6) = 3 < 4 = d(2,6)$.

In 2012, Ozavsar and Cevikel [16] introduced the concept of multiplicative contraction mappings and proved some fixed point theorem of such mappings in multiplicative metric space.

Definition 1.6 [16] Let (X, d^*) be a multiplicative metric space. If $a \in X$ and r > 1 then a subset

$$B_r(a) = B(a;r) = \{x \in X: d^*(a,x) < r\},$$

of X is called multiplicative open ball centered at a with radius r. Relatively one can defined multiplicative closed ball as

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$\overline{B_r}\,(a) = \bar{B}\,(a; r) = \{x \in X: d^*(a, x) \leq r\}$.

Definition 1.7 [16] Let A be any subset of multiplicative metric space (X, d^*). A point $x \in X$ is called limit point of A if and only if $\left(A \cap B_\varepsilon(x)\right) - \{x\} \neq \varphi$ for every $\varepsilon > 1$.

Definition 1.8 [16] Let (X, d^*) and (Y, ρ^*) be given multiplicative metric spaces and a ∈ X. A function f: $(X, d^*) \rightarrow (Y, \rho^*)$ is said to multiplicative continuous at a, if for given $\varepsilon > 1$, there exists a $\delta > 1$ such that $d^*(x, a) < \delta \Rightarrow d^*(f(x), f(a)) < \varepsilon$ or equivalently, $f\left(B(a; \delta)\right) \subset$ B$(f(a); \varepsilon)$. Where $B(a; \delta)$ and B$(f(a); \varepsilon)$ are open balls in $(X, d^*)$and (Y, ρ^*) respectively. The function f is said to be continuous on X if it is continuous at each point of X.

Definition 1.9 [16] A sequence $\{x_n\}$ in a multiplicative metric space (X, d^*) is said to be multiplicative convergent to a point x ∈ X if for a given $\varepsilon > 1$ there exits a positive integer n_0 such that $d^*(x_n, x) < \varepsilon$ for all $n \geq n_0$ or equivalently, if for every multiplicative open ball $B_\varepsilon(x)$ there exists a positive integer n_0 such that $n \geq n_0 \Rightarrow x_n \in B_\varepsilon(x)$ then the sequence $\{x_n\}$ is said to be multiplicative convergent to a point x ∈ X denoted by $x_n \rightarrow x$ (n → ∞).

Definition 1.10 [16] A sequence $\{x_n\}$ in a multiplicative metric space (X, d^*) is said to be multiplicative Cauchy sequence if for all $\varepsilon > 1$ there exits a positive integer n_0 such that

$$d^*(x_n, x_m) < \varepsilon \; for\ all\ n, m \geq n_0.$$

Definition 1.11 [16] A multiplicative metric space (X, d^*) is said to be complete if every multiplicative Cauchy sequence in X converges in X.

Definition 1.12 [16] Let (X, d^*) be a multiplicative metric space. A mapping f: X → X is called multiplicative contraction if there exists a real constant λ ∈ [0, 1) such that

$$d^*\left(f(x_1), f(x_2)\right) \leq d^*(x_1, x_2)^\lambda \; for\ all\ x, y \in X.$$

Theorem 1.13 [16] In a multiplicative metric space every multiplicative convergent sequence is multiplicative Cauchy sequence.

Theorem 1.14 [16] Let (X, d^*) be a multiplicative metric space and let $f: X \rightarrow X$ be a multiplicative contraction. If (X, d^*) is complete, then f has a unique fixed point.

Theorem 1.15 [16] Let (X, d^*) be a complete metric space. Suppose the mapping $f: X \rightarrow X$ satisfies the contraction condition

$$d^*(fx, fy) \leq \left(d(fx, y).\, d(fy, x)\right)^\lambda \; for\ all\ x, y \in X,$$

where $\lambda \in \left[0, \frac{1}{2}\right)$ is a constant. Then f has a unique fixed point in X. And for any $x \in X$, iterative sequence $(f^n x)$ converges to the fixed point.

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In 2014, Xiaoju et al. [10] discussed the unique common fixed point of two pairs of weak commutative mappings on a complete multiplicative metric space.

Definition 1.16 [10] Suppose that S, T are two self-mappings of a multiplicative metric space (X, d^*). Then S, T are called commutative mappings if it holds that for all x ∈ X, STx = TSx.

Definition 1.17 [10] Suppose that S, T are two self-mappings of a multiplicative metric space (X, d^*). Then S, T are called weak commutative mappings if it holds that for all x ∈ X, d(STx,TSx) ≤ d(Sx,Tx).

Remark 1.18 [10] Commutative mappings must be weak commutative mappings, but the converse is not true.

Theorem 1.19 [10] Let S, T, A and B be self-mappings of a complete multiplicative metric space X. They satisfy the following conditions:

(i) SX ⊂ BX, TX ⊂ AX;
(ii) A and S are weak commutative, B and T also are weak commutative;
(iii) One of S, T, A and B is continuous;
(iv) $d^*(Sx, Ty) \leq \{max\{d^*(Ax, By), d^*(Ax, Sx), d^*(By, Ty), d^*(Sx, By), d^*(Ax, Ty)\}\}^{\lambda}$, $\lambda \in \left(0, \frac{1}{2}\right), for\ all\ x, y \in X.$

Then S, T, A and B have a unique common fixed point.

Theorem 1.20 [10] Let S, T, A and B be self-mappings of a complete multiplicative metric space satisfying the following conditions:

(i) SX ⊂ BX, TX ⊂ AX;
(ii) A and S are commutative mappings, B and T also are commutative mappings;
(iii) One of S, T, A and B is continuous;
(iv) $d^*(S^p x, T^q y) \leq \{max\{d^*(Ax, By), d^*(Ax, S^p x), d^*(By, T^q y), d^*(S^p x, By), d^*(Ax, T^q y)\}\}^{\lambda}$, $\lambda \in \left(0, \frac{1}{2}\right), for\ all\ x, y \in X, p, q \in Z^+.$

Then S, T, A and B have a unique common fixed point.

In the same year, Yamaod and Sintunavarat introduced the new types of contraction mappings in the sense of a multiplicative space. They proved a fixed point theorem involving a cyclic mapping and also generalized Banach-contraction, Kannan-contraction and Chatterjea-contraction mappings in multiplicative metric spaces.

Definition 1.21 [20] Let (X, d^*) be a multiplicative metric space. A self-mapping f is said to be multiplicative Banach-contraction if

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$$d^*(fx, fy) \le d^*(x, y)^\lambda, for\ all\ x, y \in X, where\ \lambda \in [0,1).$$

Definition 1.22 [20] Let (X, d^*) be a multiplicative metric space. A self-mapping f is said to be multiplicative Kannan-contraction if

$$d^*(fx, fy) \le \left(d^*(fx, x).d^*(fy, y)\right)^\lambda, for\ all\ x, y \in X, where\ \lambda \in [0,\tfrac{1}{2}).$$

Definition 1.23 [20] Let (X, d^*) be a multiplicative metric space. A self-mapping f is said to be a multiplicative Chatterjea-contraction if

$$d^*(fx, fy) \le \left(d^*(fx, y).d^*(fy, x)\right)^\lambda, for\ all\ x, y \in X, where\ \lambda \in [0,\tfrac{1}{2}).$$

Definition 1.24 [20] Let X be a nonempty set, f be a self-mapping on X, and α,β : X → [0,∞) be two mappings. We say that f is a cyclic (α, β)-admissible mapping if

$$x \in X, \alpha(x) \ge 1 \ \Rightarrow\ \beta(fx) \ge 1$$

and $x \in X, \beta(x) \ge 1 \ \Rightarrow\ \alpha(fx) \ge 1.$

Definition 1.25 [20] Let (X, d^*) be a multiplicative metric space and let α,β : X → [0,∞) be two mappings. The mapping f: X → X is said to be a multiplicative (α,β) -Banach-contraction if

$$\alpha(x)\beta(y).d^*(fx, fy) \le d^*(x, y)^\lambda, for\ all\ x, y \in X, where\ \lambda \in [0,1).$$

Theorem 1.26 [20] Let (X, d^*) be a complete multiplicative metric space and f : X → X be a multiplicative (α, β)-Banach-contraction mapping. Suppose that the following conditions hold:

(1) there exists x_0 ∈ X such that α(x_0) ≥ 1 and β(x_0) ≥ 1;
(2) f is a cyclic (α, β)-admissible mapping;
(3) one of the following conditions holds:
 (3.1) f is continuous;
 (3.2) if {x_n} is a sequence in X such that $x_n \to x_*$ ∈ X as n → ∞ and β(x_n) ≥ 1 for all n ∈ N, then β(x) ≥ 1.

Then f has a fixed point. Furthermore, if α(x) ≥ 1 and β(x) ≥ 1 for all fixed point x ∈ X, then f has a unique fixed point.

Theorem 1.27 [20] Let (X, d^*) be a complete multiplicative metric space and f : X → X be a multiplicative (α, β)-Kannan-contraction mapping. Suppose that the following conditions hold:

(1) there exists x_0 ∈ X such that α(x_0) ≥ 1 and β(x_0) ≥ 1;
(2) f is a cyclic (α, β)-admissible mapping;
(3) one of the following conditions holds:
 (3.1) f is continuous;

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(3.2) if $\{x_n\}$ is a sequence in X such that $x_n \to x_* \in X$ as $n \to \infty$ and $\beta(x_n) \geq 1$ for all $n \in N$, then $\beta(x) \geq 1$.

Then f has a fixed point. Furthermore, if $\alpha(x) \geq 1$ and $\beta(x) \geq 1$ for all fixed point $x \in X$, then f has a unique fixed point.

Theorem 1.28 [20] Let (X, d^*) be a complete multiplicative metric space and $f : X \to X$ be a multiplicative (α, β)-Chatterjea-contraction mapping. Suppose that the following conditions hold:

(1) there exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$ and $\beta(x_0) \geq 1$;
(2) f is a cyclic (α, β)-admissible mapping;
(3) one of the following conditions holds:
   (3.1) f is continuous;
   (3.2) if $\{x_n\}$ is a sequence in X such that $x_n \to x_* \in X$ as $n \to \infty$ and $\beta(x_n) \geq 1$ for all $n \in N$, then $\beta(x) \geq 1$.

Then f has a fixed point. Furthermore, if $\alpha(x) \geq 1$ and $\beta(x) \geq 1$ for all fixed point $x \in X$, then f has a unique fixed point.

In 2015, Abbas et al. [1] proved common fixed point result for quasi-weak commutative mappings on a closed ball in the framework of multiplicative metric spaces.

Theorem 1.29 [1] Let S, T, f, and g be self-maps of a complete multiplicative metric space X and (f, S) and (g, T) weakly commutative with $SX \subset g\,X$, $TX \subset fX$, and one of S, T, f, and g is continuous. If $Sx_0 = y_0$ for some given point x_0 in X and there exists $\lambda \in (0, 1/2)$ with $h = \lambda/(1-\lambda)$ such that

$$d\,(Sx, Ty) \leq (M\,(x, y))\; for\ any\ x, y \in (y_0, r),$$

holds, where $M(x, y) = \max\{d(fx, gy), d(fx, Sx), d(gy, Ty), d(Sx, g\,y), d(fx, Ty)\}$. Then there exists a unique common fixed point of f, T, S, and g in $B(y_0, r)$ provided that $d(y_0, Tx_1) \leq r^{(1-h)}$ for some x_1 in X.

Example 1.30 [21] Let X = R and d: $R^2 \to R^+$ be a multiplicative metric defined by $(x, y) = e^{|x-y|}$. Note that (R, d^*) is a complete multiplicative metric space. Define mappings f, g, S and T: R → R by

$$f\,(x) = 2x,\ S\,(x) = x,\ T\,(x) = \frac{1}{2}x,\ g\,(x) = 3x.$$

In 2015, Kang et al. [11] introduced the notions of compatible mappings and its variants in multiplicative metric spaces. To prove the main result, firstly they gave the definitions of compatible mappings and also proved few propositions to support their result.

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Theorem 1.31 [11] Let S, T, A and B be mappings of a complete multiplicative metric space (X, d^*) into itself satisfying the following conditions:

(C1) SX ⊂ BX, T X ⊂ AX;

(C2) $d^*(Sx, Ty) \le [\max\{d^*(Ax, By), d^*(Ax, Sx), d^*(By, Ty), d^*(Sx, By), d^*(Ax, Ty)\}]^{\lambda}$ for all x, y ∈ X, where $\lambda \in (0, 1/2)$.

(C3) one of the mappings S, T, A and B is continuous.

Assume that the pairs (A, S) and (B, T) are weakly commuting. Then S, T, A and B have a unique common fixed point.

Theorem 1.32 [11] Let S, T, A and B be mappings of a complete multiplicative metric space (X, d^*) into itself satisfying (C1)-(C3). Assume that the pairs (A, S) and (B, T) are compatible. Then S, T, A and B have a unique common fixed point.

In the same year 2015, Mongkolkeha and Sintunavarat [12] introduced the concept of multiplicative proximal contraction mapping in multiplicative metric space.

The definition of multiplicative proximal contraction is as follows:

Definition 1.33 [12] Let A and B be nonempty subsets of a multiplicative metric space (X, d^*). A mapping T: A → B is called a multiplicative proximal contraction if there exists α ∈ [0, 1) satisfying the following condition:

$$\left.\begin{array}{l} d^*(u,Tx) = d^*(A,B) \\ d^*(v,Ty) = d^*(A,B) \end{array}\right\} \Rightarrow \quad d^*(u,v) \le d^*(x,y)^{\alpha},$$

for all u, v, x, y ∈ A.

Theorem 1.34 [12] Let (X, d^*) be a complete multiplicative metric space and A, B be nonempty closed subsets of X such that A_0 and B_0 are nonempty and B is approximatively compact with respect to A. Suppose that T : A → B and g : A → A satisfy the following conditions:

(a) T is a multiplicative proximal contraction;

(b) $T(A_0) \subseteq B_0$;

(c) g is an isometry;

(d) $A_0 \subseteq g(A_0)$.

Then there exists a unique point $x^* \in A$ such that

$$d^*(gx^*, Tx^*) = d^*(A, B).$$

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Moreover, for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$ defined by

$$d^*(gx_{n+1}, T x_n) = d^*(A, B)$$

converges to the element x^* .

In 2016, Abdou [3] proved common fixed points for weakly compatible mappings satisfying the generalized contractiveness and the (CLR)-property.

Definition 1.35 [3] The self-mappings f and g of a set X are said to be:

(1) commutative or commuting on X if fgx = gfx for all $x \in X$;

(2) weakly commutative or weakly commuting on X if $d(fgx, gfx) \leq d(fx, gx)$ for all $x \in X$;

(3) compatible on X if $\lim_{n\to\infty} d(fgx_n, gfx_n) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$;

(4) weakly compatible on X if fx = gx for all $x \in X$ implies fgx = gfx, that is, $d(fx, gx) = 1 \Rightarrow d(fgx, gfx) = 1$.

Theorem 1.36 [3] Let (X, d^*) be a multiplicative space. Let S, T, A, B: X → X be single-valued mappings such that $S(X) \subset B(X)$, $T(X) \subset A(X)$ and there exists $\lambda \in (0, 1/2)$ such that

$$d^{*^p}(Sx, Ty) \leq \left[\phi\left(max\left\{d^{*^p}(Ax, By), \frac{d^{*^p}(Ax,Sx)d^{*^p}(By,Ty)}{1+d^{*^p}(Ax,By)}, \frac{d^{*^p}(Ax,Ty)d^{*^p}(By,Ax)}{1+d^{*^p}(Ax,By)}\right\}\right)\right]^\lambda$$

for all $x, y \in X$ and $p \geq 1$, where $\phi : [0, \infty) \to [0, \infty)$ is a monotone increasing function such that $\phi(0) = 0$ and $\phi(t) < t$ for all $t > 0$.

Suppose that one of the following conditions is satisfied:

(a) either A or S is continuous, the pair (S, A) is compatible and the pair (T, B) is weakly compatible;

(b) either B or T is continuous, the pair (T, B) is compatible and the pair (S, A) is weakly compatible.

Then S, T, A and B have a unique common fixed point in X.

In 2016, Agarwal et al. [6] proved that all the presented fixed point results in the setting of multiplicative metric spaces can be derived from the corresponding existing results in the context of standard metric spaces.

Theorem 1.37 [6] Let X be a non-empty set. A mapping $d^*: X \times X \to [0, \infty)$ is said to be a multiplicative metric. Then the mapping $d: X \times X \to [0, \infty)$ with $d(x, y) = \ln(d^*(x, y))$ forms a metric.

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Theorem 1.38 [6] Let (X, d*) be a complete multiplicative metric space and f: X → X. Suppose that

$$\psi\left(d^*(fx, fy)\right) \leq \frac{\psi\left(M^f_{d^*}(x,y)\right)}{\phi\left(M^f_{d^*}(x,y)\right)}$$

for any x, y ∈ X, where

$$M^f_{d^*}(x,y) = \left\{d^*(x,y), d^*(fx,x), d^*(y,fy), \left(d^*(fx,y)d^*(x,fy)\right)^{\frac{1}{2}}\right\}$$

and $\psi : [1,\infty) \to [1,\infty)$ is continuous, non-decreasing, $\psi^{-1}(\{1\}) = \{1\}$, and $\phi : [1,\infty) \to [1,\infty)$ is lower semi-continuous and $\phi^{-1}(\{1\}) = \{1\}$. Then f has a unique fixed point.

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