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AN INEQUALITY IN GEOMETRIC FUNCTION THEORY FOR CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS

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ABSTRACT: We introduce some classes of analytic functions, its subclasses and obtain sharp upper bounds of the functional $|a_3 - \mu a_2^2|$ for the analytic function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n, |z| < 1$ belonging to these classes and subclasses.

KEYWORDS: Univalent functions, Starlike functions, Close to convex functions and bounded functions.

MATHEMATICS SUBJECT CLASSIFICATION: 30C50

1. Introduction : Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the unit disc $\mathbb{E} = \{z: |z| < 1|\}$. Let $\mathcal{S}$ be the class of functions of the form (1.1), which are analytic univalent in $\mathbb{E}$.

In 1916, Bieber Bach ([3]) proved that $|a_2| \leq 2$ for the functions $f(z) \in \mathcal{S}$. In 1923, Löwner [5] proved that $|a_3| \leq 3$ for the functions $f(z) \in \mathcal{S}$..

With the known estimates $|a_2| \leq 2$ and $|a_3| \leq 3$, it was natural to seek some relation between a_3 and ${a_2}^2$ for the class $\mathcal{S}$, Fekete and Szegö [6] used Löwner's method to prove the following well known result for the class $\mathcal{S}$.

Let $f(z) \in \mathcal{S}$, then

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$$|a_3 - \mu a_2^2| \leq \begin{cases} 3-4\mu, if\ \mu \leq 0; \\ 1+2\exp\left(\frac{-2\mu}{1-\mu}\right), if\ 0 \leq \mu \leq 1; \\ 4\mu - 3, if \mu \geq 1. \end{cases}$$

(1.2)

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes $\boldsymbol{S}$.

Let us define some subclasses of $\boldsymbol{S}$.

We denote by S*, the class of univalent starlike functions

$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \boldsymbol{\mathcal{A}}$ and satisfying the condition

$$Re\left(\frac{zg(z)}{g(z)}\right) > 0, z \in \mathbb{E}.$$

(1.3)

We denote by $\mathcal{K}$, the class of univalent convex functions $h(z) = z + \sum_{n=2}^{\infty} c_n z^n, z \in \boldsymbol{\mathcal{A}}$ and satisfying the condition

$$Re\frac{((zh'(z))}{h'(z)} > 0, z \in \mathbb{E}.$$

(1.4)

A function $f(z) \in \boldsymbol{\mathcal{A}}$ is said to be close to convex if there exists $g(z) \in S^*$ such that

$$Re\left(\frac{zf\prime(z)}{g(z)}\right) > 0, z \in \mathbb{E}.$$

(1.5)

The class of close to convex functions is denoted by C and was introduced by Kaplan and it was shown by him that all close to convex functions are univalent.

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$$S^*(A,B)=\left\{f(z)\in\boldsymbol{\mathcal{A}};\frac{zf'(z)}{f(z)}\prec\frac{1+Az}{1+Bz},-1\le B<A\le 1,z\in\mathbb{E}\right\}$$

(1.6)

$$\mathcal{K}(A,B)=\left\{f(z)\in\boldsymbol{\mathcal{A}};\frac{\left(zf'(z)\right)'}{f'(z)}\prec\frac{1+Az}{1+Bz},-1\le B<A\le 1,z\in\mathbb{E}\right\}$$

(1.7)

It is obvious that $S^*(A,B)$ is a subclass of S^* and $\mathcal{K}(A,B)$ is a subclass of $\mathcal{K}$.

We introduce a new subclass as $\left\{f(z)\in\boldsymbol{\mathcal{A}};(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta}+\alpha\left(\frac{\left(zf'(z)\right)'}{f'(z)}\right)^{1-\beta}\prec\frac{1+z}{1-z};z\in\mathbb{E}\right\}$ and we will denote this class as $S^*(f,f',\alpha,\beta)$.

We will deal with two subclasses of $S^*(f,f',\alpha,\beta)$ defined as follows in our next paper:

$$S^*(f,f',\alpha,\beta,A,B)=\left\{f(z)\in\boldsymbol{\mathcal{A}};(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta}+\alpha\left(\frac{\left(zf'(z)\right)'}{f'(z)}\right)^{1-\beta}\prec\frac{1+Az}{1+Bz};z\in\mathbb{E}\right\}\quad(1.8)$$

$$S^*(f,f',\alpha,\beta,\delta)=\left\{f(z)\in\boldsymbol{\mathcal{A}};(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta}+\alpha\left(\frac{\left(zf'(z)\right)'}{f'(z)}\right)^{1-\beta}\prec\left(\frac{1+z}{1-z}\right)^{\delta};z\in\mathbb{E}\right\}\quad(1.9)$$

Symbol $\prec$ stands for subordination, which we define as follows:

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Principle of Subordination: Let $f(z)$ and $F(z)$ be two functions analytic in $\mathbb{E}$. Then $f(z)$ is called subordinate to F(z) in $\mathbb{E}$ if there exists a function $w(z)$ analytic in $\mathbb{E}$ satisfying the conditions $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = F(w(z))$; $z \in \mathbb{E}$ and we write $f(z) \prec F(z)$.

By $\mathcal{U}$, we denote the class of analytic bounded functions of the form $w(z) = \sum_{n=1}^{\infty} d_n z^n, w(0) = 0, |w(z)| < 1$.

(1.10)

It is known that $|d_1| \leq 1, |d_2| \leq 1 - |d_1|^2$.

(1.11)

2. **PRELIMINARY LEMMAS:** For $0 < c < 1$, we write $w(z) = \left(\frac{c+z}{1+cz}\right)$ so that

$$\frac{1+w(z)}{1-w(z)} = 1 + 2cz + 2z^2 + \cdots.$$

(2.1)

3. MAIN RESULTS

THEOREM 3.1: Let $f(z) \in S^*(f, f', \alpha, \beta)$., then

$$|a_3 - \mu a_2^2| \leq$$
$$\begin{cases} \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} - 4\mu\right], if \mu \leq \frac{8\alpha+3\beta+4\alpha^2-\beta^2-3\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}; \\ \frac{1}{3\alpha+\beta-4\alpha\beta} if \frac{8\alpha+3\beta+4\alpha^2-\beta^2-3\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)} \leq \mu \leq \frac{8\alpha+3\beta+8\alpha^2+\beta^2-24\alpha^2\beta-6\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}; \\ \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[4\mu - \frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}\right], if\ \mu \geq \frac{8\alpha+3\beta+8\alpha^2+\beta^2-24\alpha^2\beta-6\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)} \end{cases}$$

The results are sharp.

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Proof: By definition of $S^*(f, f', \alpha, \beta)$, we have

$$(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta}+\alpha\left(\frac{\left(zf'(z)\right)'}{f'(z)}\right)^{1-\beta} = \frac{1+w(z)}{1-w(z)}; w(z) \in \mathcal{U}.$$

(3.4)

Expanding the series (3.4), we get

$$(1-\alpha)\left\{1+\beta a_2 z+(2\beta a_3+\frac{\beta(\beta-3)}{2}a_2^2)z^2+---\right\}+\alpha\{1+2(1-\beta)a_2 z+$$
$$2(1-\beta)(3a_3-(\beta+2)a_2^2)z^2+---\}=(1+2c_1 z+2(c_2+{c_1}^2)z^2+---).$$

(3.5)

Identifying terms in (3.5), we get

$$a_2=\frac{2}{(1-\alpha)\beta+2\alpha(1-\beta)}\ c_1$$

(3.6)

$$a_3=\frac{1}{3\alpha+\beta-4\alpha\beta}\ c_2+\ \frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\ {c_1}^2.$$

(3.7)

From (3.6) and (3.7), we obtain

$$a_3-\mu a_2^2=\frac{1}{3\alpha+\beta-4\alpha\beta}c_2+\left[\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}-\right.$$
$$\left.\frac{4}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu\right]c_1^2. \qquad (3.8)$$

Taking absolute value, (3.8) can be rewritten as

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$|a_3-\mu a_2^2| \leq \frac{1}{3\alpha+\beta-4\alpha\beta}|c_2| + \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left|\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} - 4\mu\right| |c_1^2|.$ (3.9)

Using (1.9) in (3.9), we get

$|a_3-\mu a_2^2| \leq \frac{1}{3\alpha+\beta-4\alpha\beta}(1-|c_1|^2) + \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left|\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} - 4\mu\right| |c_1^2$

$= \frac{1}{3\alpha+\beta-4\alpha\beta} + \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\left|\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} - 4\mu\right| - \frac{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{3\alpha+\beta-4\alpha\beta}\right]|c_1|^2.$

(3.10)

Case I: $\mu \leq \frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$. (3.10) can be rewritten as

$|a_3-\mu a_2^2 \leq \frac{1}{3\alpha+\beta-4\alpha\beta} + \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\frac{8\alpha+3\beta+4\alpha^2-\beta^2-3\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} - 4\mu\right]|c_1|^2.$

(3.11)

Subcase I (a): $\mu \leq \frac{8\alpha+3\beta+4\alpha^2-\beta^2-3\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$. Using (1.9), (3.11) becomes

$|a_3-\mu a_2^2 \leq \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} - 4\mu\right].$

(3.12)

Subcase I (b): $\mu \geq \frac{8\alpha+3\beta+4\alpha^2-\beta^2-3\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$. We obtain from (3.11)

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$|a_3 - \mu a_2^2 \leq \frac{1}{3\alpha+\beta-4\alpha\beta}$.

(3.13)

Case II: $\mu \ \geq \frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$

Preceding as in case I, we get

$|a_3 - \mu a_2^2 \leq \frac{1}{3\alpha+\beta-4\alpha\beta} + \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\Big[4\mu -$

$\frac{8\alpha+3\beta+8\alpha^2+\beta^2-24\alpha^2\beta-6\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}\Big]|c_1|^2$. (3.14)

Subcase II (a): $\mu \ \leq \frac{8\alpha+3\beta+8\alpha^2+\beta^2-24\alpha^2\beta-6\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$

(3.14) takes the form $|a_3 - \mu a_2^2 \leq \frac{1}{3\alpha+\beta-4\alpha\beta}$

(3.15)

Combining subcase I (b) and subcase II (a), we obtain

$|a_3 - \mu a_2^2| \leq \frac{1}{3\alpha+\beta-4\alpha\beta}\, if\, \frac{8\alpha+3\beta+4\alpha^2-\beta^2-3\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)} \leq \mu \leq$

$\frac{8\alpha+3\beta+8\alpha^2+\beta^2-24\alpha^2\beta-6\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$ (3.16)

Subcase II (b): $\mu \ \geq \frac{8\alpha+3\beta+8\alpha^2+\beta^2-24\alpha^2\beta-6\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$

Preceding as in subcase I (a), we get

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$$|a_3-\mu a_2^2|\le \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[4\mu-\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}\right].$$

(3.17)

Combining (3.12), (3.16) and (3.17), the theorem is proved.

Extremal function for (3.1) and (3.3) is defined by

$f_1(z)=(1+az)^b$

Where $a=\frac{\{(2\alpha+\beta-3\alpha\beta)^2-(1-\alpha)\beta(\beta-3)+4\alpha(1-\beta)(\beta+2)\}a_2^2-4(3\alpha+\beta-4\alpha\beta)a_3}{(2\alpha+\beta-3\alpha\beta)a_2}$

And $b=\frac{(2\alpha+\beta-3\alpha\beta)^2a_2^2}{\{(2\alpha+\beta-3\alpha\beta)^2-(1-\alpha)\beta(\beta-3)+4\alpha(1-\beta)(\beta+2)\}a_2^2-4(3\alpha+\beta-4\alpha\beta)a_3}$

Extremal function for (3.2) is defined by $f_2(z)=z(1+Bz^2)^{\frac{A-B}{2B}}$.

Corollary 3.2: Putting $\alpha=1,\beta=0$ in the theorem, we get

$$|a_3-\mu a_2^2|\le\begin{cases}1-\mu, if\mu\le 1;\\ \frac{1}{3} if 1\le\mu\le\frac{4}{3};\\ \mu-1, if\ \mu\ge\frac{4}{3}\end{cases}$$

These estimates were derived by Keogh and Merkes [12] and are results for the class of univalent convex functions.

Corollary 3.3: Putting $\alpha=0,\beta=1$ in the theorem, we get

$$|a_3-\mu a_2^2|\le\begin{cases}3-4\mu, if\mu\le\frac{1}{2};\\ 1 if\frac{1}{2}\le\mu\le 1;\\ 4\mu-3, if\ \mu\ge 1\end{cases}$$

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These estimates were derived by Keogh and Merkes [12] and are results for the class of univalent starlike functions.

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