Common Fixed Point Theorems for Weakly Compatible Mapping by (CLR) Property in Dislocated Metric Spaces

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Abstract: In this paper, we employ the concepts of commutativity, weak compatibility, and common limit in the range (CLR) property to derive some common fixed point theorems for six mappings satisfying integral type contractive conditions in complete dislocated metric spaces. Our findings improve and extends some previous results found in the literature.

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1.Introduction

The concept of dislocated metric (d-metric) was introduced by Hitzler and Seda in [8,9] which is very useful in Logic Programming Semantics. With the passage of time many papers have been published concerning fixed point and common fixed point theorems satisfying certain contractive conditions in dislocated metric space (see [14]–[22]).

Branciari [5] introduced the notion of contraction of integral type and proved first fixed point theorem for this class of mapping. Further results on this class of mappings were obtained by Rhoades [21], Aliouche[2], Djoudi and Merghadi [7] and many others.

Sessa [25], initiated the tradition of improving commutativity conditions in metrical common fixed point theorems. While doing so Sessa [24] introduced the notion of weak commutativity. Motivated by Sessa [25], Jungck [11] defined the concept of compatibility of two mappings, which includes weakly commuting mappings as a proper subclass. Jungck and Rhoades [12] introduced the notion of weakly compatible (coincidentally commuting) mappings and showed that compatible mappings are weakly compatible but not conversely. Many interesting fixed point theorems for weakly compatible maps satisfying contractive type conditions have been obtained by various authors. The concept of compatible mappings was frequently used to show the existence of common fixed points. However, the study of the existence of common fixed points for noncompatible mappings is also very interesting. Amari and Moutawakil [1] gave a notion (E.A) which generalizes the concept of noncompatible mappings in metric spaces. The concept of E.A. property allows to replace the completeness requirement of the space with a more natural condition of closeness of the range. Sintunavarat and Kumam [25] introduce the CLR property which is so called "common limit in the range" for two self-mappings.

In this article, we have established some common fixed point results of integral type contractive conditions using the concept of weak compatible mappings, commutativity with (CLR) property in dislocated metric (d-metric) space. Our obtained results generalize some well-known results of the literature.

2. Preliminary Notes

We begin by recalling some basic concepts of the theory of dislocated metric (d-metric) spaces.

Throughout this work R^+ represent the set of non-negative real numbers. Now, we collect some known definitions and results from the literature which are helpful in the proof of our results.

Definition.2.1: Let X be a nonempty set. Suppose that a mapping $d: X \times X \to R^+$ satisfies:

(i) $\int_0^{d(x,x)} \emptyset(t)\, dt = 0 \;\forall\; x \in X$

(ii) $\int_0^{d(x,y)} \emptyset(t)\, dt = \int_0^{d(y,x)} \emptyset(t) dt = 0 \Rightarrow x = y$

(iii) $\int_0^{d(x,y)} \varnothing(t)\,dt = \int_0^{d(y,x)} \varnothing(t)\,dt$

(iv) $\int_0^{d(x,y)} \varnothing(t)\,dt \leq \int_0^{d(x,z)} \varnothing(t)\,dt + \int_0^{d(z,y)} \varnothing(t)\,dt$ for all $x, y, z \in X$,

where $\varnothing : R^+ \to R^+$ is a Lebesgue integrable mapping which is summable on each compact subset of R^+, non-negative and such that for any $s > 0$, $\int_0^{s} \varnothing(t)\,dt > 0$.The mapping *d* is called a metric on *X* and (*X*, *d*) is called a metric space. If *d* satisfies the conditions from(ii)- (iv), then *d* is said to be dislocated metric (OR) shortly (*d*-metric) on *X* and the pair (*X*, *d*) is called dislocated metric space. If *d* satisfies only (ii) and (iv), then *d* is called dislocated quasi-metric (OR) shortly (*dq*-metric) on *X* and the pair (*X*, *d*) is called dislocated quasi-metric space.

Every metric space is dislocated metric and dislocated quasi metric space but the converse is not true. Also, every dislocated metric space is dislocated quasi-metric space but the converse is not necessarily true.

Remark 2.1. The above definition change to usual definition of metric space if $\varnothing(t) = I$

Definition 2.2 A sequence $\{x_n\}$ in a *d*-metric space (X, d) is called a Cauchy sequence if for given $\epsilon > 0$, there exists $n_0 \in N$ such that for all $m.n \geq n_0$,we have $d(x_m, x_n) < \epsilon$.

The following simple but important results can be seen in [8].

Definition 2.3 A sequence in *d*-metric space converges if there exists $x \in X$ such that $d(x_n, x) \to 0$ as $n \to \infty$.

Definition 2.4 A d-metric space (X, d) is called complete if every Cauchy sequence is convergent.

Definition 2.5 Let (X, d) be a d-metric space. A map $T: X \to X$ is called contraction if there exists a number λ with $0 \leq \lambda < 1$ such that $d(Tx, Ty) \leq \lambda d(x, y)$.

As per Branciari [5], it is straightforward to demonstrate the theorem for dislocated metric spaces.

Theorem 2.1. Let(X, d) be a complete metric space for α ∈ (0, 1). Let $T: X \to X$ be a mapping such that for all $x, y \in X$ satisfying

$$\int_0^{d(Tx,Ty)} \varnothing(t)dt \leq \alpha \int_0^{d(x,y)} \varnothing(t)\,dt,$$

where $\varnothing : R^+ \to R^+$ is a Lebesgue integrable mapping which is summable on each compact subset of R^+, non-negative and such that for any $s > 0$, $\int_0^{s} \varnothing(t)\,dt > 0$.

Definition 2.6 Let *S* and *T* be two self mappings on a set *X*. Mappings *S* and *T* are said to be commuting if $STx = TSx \; \forall x \in X$.

In alignment with Sessa [25], we present the concept of weakly commuting mapping in the following manner.

Definition 2.7. Let *S* and *T* be mappings of a dislocated metric space (X, d) into itself. Then (S, T) is said to be **weakly commuting** pair if

$$d(STx, TSx) \leq d(Tx, Sx) \text{ for all } x \in X.$$

Obviously, a commuting pair is weakly commuting but its converse need not be true as is evident from the following example.

Example 2.1. Consider the set $X = [0,1]$ with the usual metric $d(x, y) = \|x - y\|$. Let $Sx = \frac{x}{2}$ and $Tx = \frac{x}{2+x}$, for every $x \in X$. Then, for all $x \in X$

$STx = \frac{x}{4+2x}$ and $TSx = \frac{x}{4+x}$

Hence, $ST \neq TS$. Thus, S and T do not commute.

Again,
$$d(STx, TSx) = \left\|\frac{x}{4+2x} - \frac{x}{4+x}\right\|$$
$$= \frac{x^2}{(4+x)(4+2x)}$$
$$\leq \frac{x^2}{4+2x} = \frac{x}{2} - \frac{x}{2+x} = d(Sx, Tx)$$

and thus, S and T commute weakly.

Obviously, the class of weakly commuting is wider and includes commuting mappings as subclass.

In accordance with Jungck [11], we present the concept of compatibility as follows.

Definition 2.8. Two self mappings S and T of a complete dislocated metric space (X,d) are compatible if and only if $\lim_{n\to\infty}\int_0^{d(STx_n,TSx_n)}\varphi(t)\,dt=0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}Tx_n=t$ for some $t\in X$.

In accordance with Jungck-Rhoades [10], we present the concept of compatibility as follows.

Definition 2.9. Let S and T be self maps of a set X. If $w=fx=gx$ for some x in X, then x is called a coincidence point of f and g, and w is called a point of coincidence of f and g.

Definition 2.10. [12] Let A and S be mappings on d-metric space (X,d), then A and S are said to be weakly compatible mappings if they commute at their coincident points such that $Ax=Sx$ implies $ASx=SAx$.The point $x\in X$ is called coincident point of A and S. It is easy to see that compatible mapping commute at their coincidence points.

Lemma 2.1. ([13]): Let A and B be weakly compatible self maps of a set X. If A and B have a unique point of coincidence $w=Ax=Bx$, then w is the unique common fixed point of A and B.

Example 2.2.

Let X = [0, 3] be equipped with the usual d- metric space $d(x,y)=|x-y|$.

Define S, T: [0, 3] → [0, 3] by

$$Sx=\begin{cases}x, & x\in[0,1)\\ 3, & x\in[1,3]\end{cases}\quad \text{and}\quad Tx=\begin{cases}3-x, & x\in[0,1)\\ 3, & x\in[1,3]\end{cases}$$

Then for any $x\in[1,3]$, $STx=TSx$, showing that S and T are weakly compatible maps on [0, 3].

Proposition 2.1. Let S and T be compatible mappings from a d-metric space (X,d) into itself. Suppose that

$$\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}Tx_n=x \text{ for some } x\in X.$$

if S is continuous then $\lim_{n\to\infty}TSx_n=Sx$.

Theorem 2.2. Let (X,d) be a complete d-metric space and let $T:X\to X$ be a contraction mapping, then T has a unique fixed point.

Definition 2.11. The function $\emptyset: R^+\to R^+$ is called sub additive integrable function if and only if for all a, $b\in R^+$

$$\int_0^{a+b}\emptyset(t)\,dt\le\int_0^{a}\emptyset(t)\,dt+\int_0^{b}\emptyset(t)\,dt$$

Example 2.3. Let $d(x,y)=|x-y|$ and $\varphi(t)=\frac{1}{1+t}$ for all $t>0$. Then for all $a,b\in R^+$,

$$\int_0^{a+b}\frac{dt}{t+1}=ln(a+b+1),\ \int_0^{a}\frac{dt}{t+1}=ln(a+1),\ \int_0^{b}\frac{dt}{t+1}=ln(b+1),$$

Since $ab\ge 0$, then $a+b+1\le a+b+1+ab=(a+1)(b+1)$.

Therefore, $ln(a+b+1)\le ln(a+1)(b+1)=ln(a+1)+ln(b+1)$ This shows that φ is an example of sub additive integrable function.

Definition 2.12: Let S and T be two self mappings of a d-metric space (X,d). We say that S and T satisfy the property *(E.A)* if there exist a sequence $\{x_n\}$ such that

$\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}Tx_n=u$, for some $x\in X$.

Definition 2.13: Let S and T be two self mappings defined on a d-metric space (X,d). We say that the mappings S and T satisfy the common limit in the range of S (CLR_S) property if there exists a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}Tx_n=Sx$, for some $x\in X$.

3.Main Results

Theorem 3.1 Let (X,d) be a complete dislocated metric space. Let $A,B,S,T,P,Q: X \to X$ satisfying the following conditions

(i) $AB(X) \subseteq Q(X)$ and $ST(X) \subseteq P(X)$ …(3.1)

(ii) The pairs (AB,P) or (ST,Q) are weakly compatible. -----(3.2)

(iii) The pairs (AB,P) satisfy (CLR_{AB}) or (ST,Q) satisfy (CLR_{ST}) property,

(iv) $$\int_0^{d(ABx,STy)} \emptyset(t)dt \le \alpha \int_0^{d(Px,ABx)} \emptyset(t)dt + \beta \int_0^{d(Qy,STy)} \emptyset(t)dt + \gamma \int_0^{d(Px,Qy)} \emptyset(t)dt$$
$$+\mu \int_0^{d(Px,STy)} \emptyset(t)dt + \delta \int_0^{d(ABx,Qy)} \emptyset(t)dt, \quad .....(3.3)$$

for all $(x,y) \in X \times X$ and $\alpha,\beta,\gamma,\delta \ge 0$ with $0 \le \alpha+\beta+\gamma+\mu+\delta < 1$, $\alpha+2\beta+2\gamma+2\mu+\delta \ne 1$and $2\alpha+2\beta+\gamma+\mu+\delta \ne 1$,where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$$\int_0^{\epsilon} \emptyset(t)dt > 0 \text{ for all } \epsilon > 0. \quad …(3.4)$$

If any one of *AB*, *ST*, *P* and *Q* is continuous for all $x,y \in X$, then *AB*, *ST*, *P* and *Q* have a unique common fixed point in *X*. Furthermore, if the pairs *(A, B), (A, P), (B, P), (S, T), (S, Q) and (T, Q)* are commuting mappings then *A, B, S, T, P* and *Q* have a unique common fixed point in *X*.

Proof.

Assume that the pair (AB,P) satisfy (CLR_{AB}) property, so there exists a sequence $\{x_n\} \in X$ Such that

$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Px_n = ABx \quad …(3.5)$$

For some $u \in X$. Since $AB(X) \subseteq Q(X)$, so there exists a sequence $\{y_n\} \in X$ such that

$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Qy_n = ABx. \quad …(3.6)$$

We claim that $\lim_{n\to\infty} STx_n = ABx$.From condition (3.3), we have

$$\int_0^{d(ABx_n,STy_n)} \emptyset(t)dt \le \alpha \int_0^{d(Px_n,ABx_n)} \emptyset(t)dt + \beta \int_0^{d(Qy_n,STy_n)} \emptyset(t)dt + \gamma \int_0^{d(Px_n,Qy_n)} \emptyset(t)dt$$
$$+\mu \int_0^{d(Px_n,STy_n)} \emptyset(t)dt + \delta \int_0^{d(ABx_n,Qy_n)} \emptyset(t)dt \quad -----(3.7)$$

Taking limit as $n \to \infty$, we get

$$\operatorname{Lim}_{n\to\infty} \int_0^{d(ABx_n,STy_n)} \emptyset(t)dt \le \alpha \lim_{n\to\infty} \int_0^{d(Px_n,ABx_n)} \emptyset(t)dt + \beta \lim_{n\to\infty} \int_0^{d(Qy_n,STy_n)} \emptyset(t)dt$$
$$+\gamma \int_0^{d(Px_n,Qy_n)} \emptyset(t)dt + \mu \lim_{n\to\infty} \int_0^{d(Px_n,STy_n)} \emptyset(t)dt$$
$$+\delta \lim_{n\to\infty} \int_0^{d(ABx_n,Qy_n)} \emptyset(t)dt \quad ----(3.8)$$

Since

$$\operatorname{Lim}_{n\to\infty} d(Px_n,Qy_n) = \lim_{n\to\infty} d(Px_n,ABx_n) = \lim_{n\to\infty} d(ABx_n,Qy_n) = 0$$
$$\lim_{n\to\infty} d(ABx_n,STy_n) = \lim_{n\to\infty} d(ABx,STy_n) = \lim_{n\to\infty} d(STy_n,Qy_n)$$

Hence

$$\lim_{n\to\infty} \int_0^{d(ABx,STy_n)} \emptyset(t)dt \le (\gamma+\mu) \lim_{n\to\infty} \int_0^{d(ABx,STy_n)} \emptyset(t)dt,$$

which is a contradiction, since $(\gamma+\mu) \ne 1$.

Therefore, $\lim_{n\to\infty} \int_0^{d(ABx,STy_n)} \emptyset(t)dt = 0 \Rightarrow \lim_{n\to\infty} d(ABx,STy_n) = 0 \Rightarrow \lim_{n\to\infty} STy_n = ABx$.

Since $ST(X) \subseteq P(X)$, so there exists a sequence $x_n \in X$ such that $STx_n = Px_n$. Hence, we have

$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} STy_n = \lim_{n\to\infty} Qy_n = ABx$

Assume $AB(X) \subseteq Q(X)$, then there exists $v \in X$ such that $ABx = Qv$.

We claim that $STv = Qv$. Now from condition (3.3), we have

$$\int_0^{d(ABx_n,STv)} \emptyset(t)\,dt \leq \alpha \int_0^{d(Px_n,ABx_n)} \emptyset(t)\,dt + \beta \int_0^{d(Qv,STv)} \emptyset(t)\,dt + \gamma \int_0^{d(Px_n,Qv)} \emptyset(t)\,dt + \mu \int_0^{d(Px_n,STv)} \emptyset(t)\,dt + \delta \int_0^{d(ABx_n,Qv)} \emptyset(t)\,dt \qquad \text{-------(3.9)}$$

Since

$\lim_{n\to\infty} d(Px_n, STv) = d(ABx, STv) = d(Qv, STv)$

$\lim_{n\to\infty} d(Px_n, Qv) = \lim_{n\to\infty} d(Px_n, ABx_n) = \lim_{n\to\infty} d(Qv, ABx_n) = 0$

So, taking limit as $n \to \infty$ in (3.9), we conclude that

$$\int_0^{d(Qv,STv)} \emptyset(t)\,dt \leq (\beta + \mu) \int_0^{d(Qv,STv)} \emptyset(t)\,dt, \qquad \dots (3.10)$$

which is a contradiction, since $(\beta + \mu) \neq 1$.

Hence, $\int_0^{d(Qv,STv)} \emptyset(t)\,dt = 0 \Rightarrow d(Qv, STv) = 0 \Rightarrow Qv = STv$.

This proves that v is the coincidence point of maps Q and ST.

Therefore, $Qv = STv = ABx = w$ (Say).

Since the pair (ST, Q) is weakly compatible, so $STQv = QSTv \Rightarrow STw = Qw$.

Since $ST(X) \subseteq P(X)$, there exists a point $u \in X$ such that $STv = Pu$. We show that

$Pu = ABu = w$

From condition (3.3), we have

$$\begin{aligned}\int_0^{d(ABu,STv)} \emptyset(t)\,dt &\leq \alpha \int_0^{d(Pu,ABu)} \emptyset(t)\,dt + \beta \int_0^{d(Qv,STv)} \emptyset(t)\,dt + \gamma \int_0^{d(Pu,Qv)} \emptyset(t)\,dt \\ &\quad + \mu \int_0^{d(Pu,STv)} \emptyset(t)\,dt + \delta \int_0^{d(ABu,Qv)} \emptyset(t)\,dt \\ &= \alpha \int_0^{d(STv,ABu)} \emptyset(t)\,dt + \beta \int_0^{d(STv,STv)} \emptyset(t)\,dt + \gamma \int_0^{d(STv,STv)} \emptyset(t)\,dt \\ &\quad + \mu \int_0^{d(STv,STv)} \emptyset(t)\,dt + \delta \int_0^{d(ABu,STv))} \emptyset(t)\,dt \\ &= (\alpha + 2\beta + 2\gamma + 2\mu + \delta) \int_0^{d(ABu,STv)} \emptyset(t)\,dt, \qquad \dots\dots (3.11)\end{aligned}$$

which is a contradiction, since $\alpha + 2\beta + 2\gamma + 2\mu + \delta \neq 1$

Therefore, $\int_0^{d(ABu,STv)} \emptyset(t)\,dt = 0 \Rightarrow d(ABu, STv) = 0 \Rightarrow ABu = STv$.

$\therefore \quad ABu = STv = Pu = w$.

This proves that u is the coincidence point of the maps AB and P.

Since the pair (AB, P) are weakly compatible so, $ABPu = PABu \Rightarrow ABw = Pw$.

We claim that $ABw = w$.

From condition (3.3), we have

$$\begin{aligned}\int_0^{d(ABw,w)} \emptyset(t)\,dt = \int_0^{d(ABw,STv)} \emptyset(t)\,dt &\leq \alpha \int_0^{d(Pw,ABw)} \emptyset(t)\,dt + \beta \int_0^{d(Qv,STv)} \emptyset(t)\,dt + \gamma \int_0^{d(Pw,Qv)} \emptyset(t)\,dt \\ &\quad + \mu \int_0^{d(Pw,STv)} \emptyset(t)\,dt + \delta \int_0^{d(ABw,Qv)} \emptyset(t)\,dt\end{aligned}$$

$$= \alpha \int_0^{d(ABw,ABw)} \phi(t)\,dt + \beta \int_0^{d(STv,STv)} \phi(t)\,dt + \gamma \int_0^{d(ABw,STv)} \phi(t)\,dt$$
$$+\mu \int_0^{d(ABw,STv)} \phi(t)\,dt + \delta \int_0^{d(ABw,STv)} \phi(t)\,dt$$
$$= (2\alpha + 2\beta + \gamma + \mu + \delta) \int_0^{d(ABw,STv)} \phi(t)\,dt, \qquad \text{----(3.12)}$$

which is a contradiction, since $2\alpha + 2\beta + \gamma + \mu + \delta \neq 1$.

Hence, $\int_0^{d(ABw,STv)} \phi(t)\,dt = 0 \Rightarrow d(ABw, w) = 0 \Rightarrow ABw = w$.

Similarly, we obtain $STw = w$.

$\therefore \quad ABw = STw = Qw = Pw = w.$

Hence, w is a common fixed point of four mappings AB, ST, P and Q.

Uniqueness:

Let $z(\neq w)$ be other common fixed point of the mappings AB, ST, P and Q, then by the condition (3.3)

$$\int_0^{d(w,z)} \phi(t)\,dt = \int_0^{d(ABw,STz)} \phi(t)\,dt$$
$$\leq \alpha \int_0^{d(Pw,ABw)} \phi(t)\,dt + \beta \int_0^{d(Qz,STz)} \phi(t)\,dt + \gamma \int_0^{d(Pw,Qz)} \phi(t)\,dt$$
$$+\mu \int_0^{d(Pw,STz)} \phi(t)\,dt + \delta \int_0^{d(ABw,Qz)} \phi(t)\,dt$$
$$= \alpha \int_0^{d(w,w)} \phi(t)\,dt + \beta \int_0^{d(z,z)} \phi(t)\,dt + \gamma \int_0^{d(w,z)} \phi(t)\,dt$$
$$+\mu \int_0^{d(w,z)} \phi(t)\,dt + \int_0^{d(w,z)} \phi(t)\,dt$$
$$= (2\alpha + 2\beta + \gamma + \mu + \delta) \int_0^{d(w,z)} \phi(t)\,dt,$$

which is a contradiction, since $2\alpha + 2\beta + \gamma + \mu + \delta \neq 1$.

Hence, $\int_0^{d(w,z)} \phi(t)\,dt = 0 \Rightarrow d(w,z) = 0 \Rightarrow w = z$.

This establishes the uniqueness of the common fixed point of mappings AB, ST, P and Q. Finally, we prove that w is also a common fixed point of *A*, *B*, *S*, *T*, *P* and *Q*.

Let both the pairs (AB, P) and (ST, Q) have a unique common fixed point u.

Then,

$$Aw = A(ABw) = A(BAw) = AB(Aw), Aw = A(Pw) = P(Aw)$$
$$\text{and } Bw = B(ABw) = B\big(A(Bw)\big) = BA(Bw) = AB(Bw), Bw = B(Pw) = P(Bw),$$

which implies that (AB, P) has common fixed points which are Aw and Bw. We get thereby $Aw = w = Bw = Pw = ABw$.

Similarly, using the commutativity of $(S,T), (S,Q)$ and (T,Q), $Sw = w = Tw = Qw = STw$ can be shown.

Hence $Aw = Bw = Sw = Tw = Pw = Qw = w$.

Consequently, w is a unique common fixed point of *A*, *B*, *S*, *T*, *P* and *Q*.

If we put *AB* = *A*, *ST* = *B* in Theorem (3.1), we get the following, which generalize the result of Panthi and Kumari [19] in dislocated metric spaces.

Corollary 3.1. Let (X, d) be a complete dislocated metric space. Let $A, B, P, Q : X \to X$ satisfying the following conditions

(i) $A(X) \subseteq Q(X)$ and $B(X) \subseteq P(X)$

(ii) The pairs (A, P) and (B, Q) are weakly compatible.

(iii) The pairs (A, P) satisfy (CLR_A) or (B, Q) satisfy (CLR_B) property

(iv) $$\int_0^{d(Ax,By)} \phi(t)dt \leq \alpha \int_0^{d(Px,Ax)} \phi(t)\,dt + \beta \int_0^{d(Qy,By)} \phi(t)\,dt + \gamma \int_0^{d(Px,Qy)} \phi(t)dt$$
$$+ \mu \int_0^{d(Px,By)} \phi(t)\,dt + \delta \int_0^{d(Ax,Qy)} \phi(t)dt,$$

for all $(x,y) \in X \times X$ and $\alpha, \beta, \gamma, \delta \geq 0$ with $0 \leq \alpha + \beta + \gamma + \mu + \delta < 1$, $\alpha + 2\beta + 2\gamma + 2\mu + \delta \neq 1$ and $2\alpha + 2\beta + \gamma + \mu + \delta \neq 1$, where $\emptyset : R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)\,dt > 0$ for all $\epsilon > 0$,

then the mappings *A*, *B*, *P* and *Q* have a unique common fixed point in *X*.

If we put *A*=*B* in the above Corollary 3.1, we can obtain the following corollary easily.

Corollary 3.2. Let (X,d) be a complete dislocated metric space. Let $A, P, Q : X \to X$ satisfying the following conditions

(i) $A(X) \subseteq Q(X)$ and $A(X) \subseteq P(X)$

(ii) The pairs (A,P) and (A,Q) are weakly compatible.

(iii) The pairs (A,P) or (A,Q) satisfy (CLR_A) property

(iv) $$\int_0^{d(Ax,Ay)} \emptyset(t)dt \leq \alpha \int_0^{d(Px,Ax)} \emptyset(t)\,dt + \beta \int_0^{d(Qy,Ay)} \emptyset(t)\,dt + \gamma \int_0^{d(Px,Qy)} \emptyset(t)\,dt + \mu \int_0^{d(Px,Ay)} \emptyset(t)\,dt + \delta \int_0^{d(Ax,Qy)} \emptyset(t)\,dt,$$

for all $(x,y) \in X \times X$ and $\alpha, \beta, \gamma, \delta \geq 0$ with $0 \leq \alpha + \beta + \gamma + \mu + \delta < 1$, $\alpha + 2\beta + 2\gamma + 2\mu + \delta \neq 1$ and $2\alpha + 2\beta + \gamma + \mu + \delta \neq 1$, where $\emptyset : R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)\,dt > 0$ for all $\epsilon > 0$,

then the mappings *A*, *P* and *Q* have a unique common fixed point in *X*.

If we put *P*=*Q* in the above Corollary 3.1, we can obtain the following corollaries easily.

Corollary 3.3. Let (X,d) be a complete dislocated metric space. Let $A, B, P : X \to X$ satisfying the following conditions

(i) $A(X) \subseteq P(X)$ and $B(X) \subseteq P(X)$

(ii) The pairs (A,P) and (B,P) are weakly compatible,

(iii) The pairs (A,P) satisfy (CLR_A) or (B,P) satisfy (CLR_B) property,

(iv) $$\int_0^{d(Ax,By)} \emptyset(t)dt \leq \alpha \int_0^{d(Px,Ax)} \emptyset(t)\,dt + \beta \int_0^{d(Py,By)} \emptyset(t)dt + \gamma \int_0^{d(Px,Py)} \emptyset(t)dt + \mu \int_0^{d(Px,By)} \emptyset(t)\,dt + \delta \int_0^{d(Ax,Py)} \emptyset(t)\,dt$$

for all $(x,y) \in X \times X$ and $\alpha, \beta, \gamma, \delta \geq 0$ with $0 \leq \alpha + \beta + \gamma + \mu + \delta < 1$, $\alpha + 2\beta + 2\gamma + 2\mu + \delta \neq 1$ and $2\alpha + 2\beta + \gamma + \mu + \delta \neq 1$, where $\emptyset : R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)\,dt > 0$ for all $\epsilon > 0$,

then the mappings *A*, *B* and *I* have a unique common fixed point in *X*.

If we put *A*=*B* and *P*=*Q* in the above Corollary 3.1, we can obtain the following corollaries easily.

Corollary 3.4. Let (X,d) be a complete dislocated metric space. Let $A, P : X \to X$ satisfying the following conditions

(i) $A(X) \subseteq P(X)$

(ii) The pair (A,P) is weakly compatible.

(iii) The pairs (A,P) satisfy $((CLR_A)$ property

(iv) $$\int_0^{d(Ax,Ay)} \emptyset(t)dt \leq \alpha \int_0^{d(Px,Ax)} \emptyset(t)\,dt + \beta \int_0^{d(Py,Ay)} \emptyset(t)\,dt + \gamma \int_0^{d(Px,Py)} \emptyset(t)\,dt + \mu \int_0^{d(Px,Ay)} \emptyset(t)\,dt + \delta \int_0^{d(Ax,Py)} \emptyset(t)\,dt$$

for all $(x,y) \in X \times X$ and $\alpha, \beta, \gamma, \delta \geq 0$ with $0 \leq \alpha + \beta + \gamma + \mu + \delta < 1$, $\alpha + 2\beta + 2\gamma + 2\mu + \delta \neq 1$ and

$2\alpha + 2\beta + \gamma + \mu + \delta \neq 1$, where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)\,dt > 0$ for all $\epsilon > 0$,

then the mappings *A* and *I* have a unique common fixed point in *X*.

Now, we establish the following theorem for six mappings involving Ciric's[6] type contractive condition using the concept of weak compatible mappings , commutativity with (CLR) property in complete dislocated metric spaces.

Theorem 3.2 Let (X, d) be a complete dislocated metric space. Let $A, B, S, T, I, J: X \to X$ satisfying the following conditions

(i) $AB(X) \subseteq J(X)$ and $ST(X) \subseteq I(X)$, … (3.13)

(ii) The pairs (AB, I) and (ST, J) are weakly compatible,

(iii) The pairs (AB, I) satisfy (CLR_{AB}) or (ST, J) satisfy (CLR_{ST}) property,

(iv) $\int_0^{d(ABx, STy)} \emptyset(t)\,dt \leq k \int_0^{M(x,y)} \emptyset(t)\,dt, k \in \left[0, \frac{1}{2}\right)$ … (3.14)

for all $(x, y) \in X \times X$ where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$$\int_0^{\epsilon} \emptyset(t)\,dt > 0 \text{ for all } \epsilon > 0, \quad \dots (3.15)$$

$$M(x, y) = \max\{d(Ix, Jy), d(Ix, ABx), d(Jy, STy), d(Ix, STy), d(Jy, ABx)\}. \quad \dots (3.16)$$

Then the mappings *AB, ST, I* and *J* have a unique common fixed point in *X*. Furthermore, if the pairs *(A, B), (A, I), (B, I), (S, T), (S, J)* and *(T, J)* are commuting mappings then *A, B, S, T, I* and *J* have a unique common fixed point in X.

Proof:

Assume that the pair (AB, I) satisfy (CLR_{AB}) property, so there exists a sequence $\{x_n\} \in X$ Such that

$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Ix_n = ABx \quad \dots (3.17)$$

For some $x \in X$. Since $A(X) \subseteq S(X)$, so there exists a sequence $\{y_n\} \in X$ such that

$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Jy_n = ABx. \quad \dots (3.18)$$

We claim that $\lim_{n\to\infty} STx_n = ABx$

From condition (14), we have

$$\int_0^{d(ABx_n, STy_n)} \emptyset(t)\,dt \leq k \int_0^{M(x_n,y_n)} \emptyset(t)\,dt, \quad \dots (3.19)$$

where

$$M(x_n, y_n) = \max\{d(Ix_n, Jy_n), d(Ix_n, ABx_n), d(Jy_n, STy_n), d(Ix_n, STy_n), d(Jy_n, ABx_n)\}$$

Taking limit as $n \to \infty$, we get

$$\lim_{n\to\infty} \int_0^{d(ABx_n, STy_n)} \emptyset(t)\,dt \leq k \lim_{n\to\infty} \int_0^{M(x_n,y_n)} \emptyset(t)\,dt \quad \dots (3.20)$$

Since

$$\operatorname{Lim}_{n\to\infty} d(Ix_n, Jy_n) = \lim_{n\to\infty} d(Ix_n, ABx_n) = \lim_{n\to\infty} d(Jy_n, ABx_n) = 0$$

$$\lim_{n\to\infty} d(ABx_n, STy_n) = \lim_{n\to\infty} d(ABx, STy_n) = \lim_{n\to\infty} d(STy_n, Jy_n)$$

Hence, we have

$$\lim_{n\to\infty} \int_0^{d(ABx, STy_n)} \emptyset(t)\,dt \leq 2k \lim_{n\to\infty} \int_0^{d(ABx, STy_n)} \emptyset(t)\,dt,$$

which is a contradiction, since $k \in \left[0, \frac{1}{2}\right)$.

Therefore, $\lim_{n\to\infty} d(ABx, STy_n) = 0 \Rightarrow \lim_{n\to\infty} STy_n = ABx$.

Now, we have

$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Ix_n = \lim_{n\to\infty} STy_n = \lim_{n\to\infty} Jy_n = ABx$$

Assume $AB(X) \subseteq J(X)$, then there exists $v \in X$ such that $ABx = Jv$.

We claim that $STv = Jv$.

Now from condition (14), we have

$$\int_0^{d(ABx_n, STv)} \phi(t)\,dt \le k\int_0^{M(x_n,v)} \phi(t)\,dt, \quad \ldots(3.21)$$

where

$M(x_n, v) = max\{d(Ix_n, Jv), d(Ix_n, ABx_n), d(Jv, STv), d(Ix_n, STv), d(Jv, ABx_n)\}$,

Since $\lim_{n\to\infty} d(Ix_n, STv) = d(ABx, STv) = d(Jv, STv)$

$$\lim_{n\to\infty} d(Ix_n, Jv) = \lim_{n\to\infty} d(Ix_n, ABx_n) = \lim_{n\to\infty} d(Jv, ABx_n) = 0$$

So, taking limit as $n \to \infty$ in (3.21), we conclude that

$$\int_0^{d(Jv,STv)} \phi(t)\,dt \le 2k\int_0^{d(Jv,STv)} \phi(t)\,dt, \quad \ldots(3.22)$$

which is a contradiction since $k \in \left[0, \frac{1}{2}\right)$.

Hence, $d(Jv, STv) = 0 \Rightarrow Jv = STv$.

This proves that v is the coincidence point of maps J and ST.

Therefore, $Jv = STv = ABx = w$ (Say).

Since the pair (ST, J) is weakly compatible, so $STJv = JSTv \Rightarrow STw = Jw$.

Since $ST(X) \subseteq I(X)$, there exists a point $u \in X$ such that $STv = Iu$. We show that

$$Iu = ABu = w$$

From condition (3.14), we have

$$\int_0^{d(ABu, STv)} \phi(t)\,dt \le k\int_0^{d(u,v)} \phi(t)\,dt,$$

where,

$$M(u, v) = max\{d(Iu, Jv), d(Iu, ABu), d(Jv, STv), d(Iu, STv), d(Jv, ABu)\}$$

$$= max\{d(STv, STv), d(STv, ABu), d(STv, STv), d(STv, STv), d(STv, ABu)\}$$

$$= max\{d(STv, STv), d(STv, ABu)\}$$

Hence we have

$$\int_0^{d(ABu, STv)} \phi(t)\,dt \le k\int_0^{max\{d(STv,STv), d(STv,ABu)\}} \phi(t)\,dt,$$

Since $d(STv, STv) \le 2d(STv, ABu)$

So if $max\{d(STv, STv), d(STv, ABu)\} = d(STv, STv)$ or $d(STv, ABu)$,

we get the contradiction since

. $$\int_0^{d(ABu,STv)} \phi(t)\,dt \le 2k\int_0^{d(ABu,STv)} \phi(t)\,dt$$

or $$\int_0^{d(ABu,STv)} \phi(t)\,dt \le k\int_0^{d(ABu,STv)} \phi(t)\,dt$$

Therefore, $d(ABu, STv) = 0 \Rightarrow ABu = STv$.

$\therefore \quad ABu = STv = Iu = w$.

This proves that u is the coincidence point of the maps AB and I.

Since the pair (AB, I) is weakly compatible so, $ABIu = IABu \Rightarrow ABw = Iw$.

We claim that $ABw = w$. From condition (3.14)

$\int_0^{d(ABw,w)} \emptyset(t)\,dt = \int_0^{d(ABw,STv)} \emptyset(t)dt \le k\int_0^{M(w,v)} \emptyset(t)dt,$

where

$M(w, v) = max\{d(Iw, Jv), d(Iw, ABw), d(Jv, STv), d(Iw, STv), d(Jv, ABw)\}$

$= \max\{d(ABw, w), d(ABw, ABw), d(ABw, w), d(ABw, w), d(ABw, ABw)\}$

$= max\{d(ABw, w), d(ABw, ABw)\}$

Hence $\int_0^{d(ABw,w)} \emptyset(t)\,dt \le k\int_0^{max\{d(ABw,w),d(ABw,ABw)\}} \emptyset(t)\,dt$

Since

$d(ABw, ABw) \le 2d(ABw, w)$

So if $max\{d(ABw, w), d(ABw, ABw)\} = (ABw, w)$ or $d(ABw, ABw)$

we get the contradiction,

Since $\int_0^{d(ABw,w)} \emptyset(t)dt \le 2k\int_0^{d(ABw,w)} \emptyset(t)\,dt$

or $\int_0^{d(ABw,w)} \emptyset(t)\,dt \le k\int_0^{d(ABw,w)} \emptyset(t)\,dt,$,

which is a contradiction since $k \in \left[0, \frac{1}{2}\right)$.

Hence, $d(ABw, w) = 0 \Rightarrow ABw = w$.

Therefore, $w = ABw = Iw$. Similarly, $STw = w = Jw$.

Hence, $w = ABw = STw = Iw = Jw$.

This represents that u is a common fixed point of the mappings AB, ST, I and J.

Uniqueness:

If possible, let $z(\neq u)$ be other common fixed point of the mappings, then by the condition (3.14)

$\int_0^{d(u,z)} \emptyset(t)\,dt = \int_0^{d(Au,Bz)} \emptyset(t)\,dt \le k\int_0^{M(u,z)} \emptyset(t)\,dt,$

where

$M(u, z) = max\{d(Iu, Jz), d(Iu, ABu), d(Jz, STz), d(Iu, STz), d(Jz, ABu)\}$

$= \max\{d(u, z), d(u, u), d(z, z), d(u, z), d(z, u)\}$

$= max\{d(u, z), d(u, u), d(z, z)\}$

Hence $\int_0^{d(u,z)} \emptyset(t)\,dt \le k\int_0^{max\{d(u,z),d(u,u),d(z,z)\}} \emptyset(t)dt$

Since

$d(u, u) \le 2d(u, z)$ and $d(z, z) \le 2d(z, u)$

So if $max\{d(u, z), d(u, u), d(z, z)\} = d(u, z)$ or $d(u, u)$ or $d(z, z)$ we get the contradiction,

Since $\int_0^{d(u,z)} \emptyset(t)\,dt \le 2k\int_0^{d(u,z)} \emptyset(t)dt$

or $\int_0^{d(u,z)} \emptyset(t)\,dt \le k\int_0^{d(u,z)} \emptyset(t)dt$,

which is a contradiction.

Hence, $d(u, z) = 0 \Rightarrow u = z$.

This establishes the uniqueness of the common fixed point of mappings AB, ST, I and J . Finally, we prove that u is also a common fixed point of A, B, S, T, I and J.

Let both the pairs (AB, I) and (ST, J) have a unique common fixed point u.

Then,

$Au = A(ABu) = A(BAu) = AB(Au)$

$Az = A(Iu) = I(Au)$

$Bu = B(ABu) = B\big(A(Bu)\big) = BA(Bu) = AB(Bu)$

$Bu = B(Iu) = I(Bu)$

which implies that (AB, I) has common fixed points which are Au and Bu. We get thereby $Au = u = Bu = Iu = ABu$.

Similarly, using the commutativity of (S, T), (S, J) and (T, J), $Su = u = Tu = Ju = STu$ can be shown.

Hence
$Au = Bu = Su = Tu = Iu = Ju = u$
Consequently, u is a unique common fixed point of A, B, S, T, I and J.

If we put *AB* = *A*, *ST* = *B* in Theorem (3.2), we get the following, which generalize the result of Panthi and Subedi [20] in dislocated metric spaces.

Corollary 3.5. Let (X, d) be a complete dislocated metric space. Let $A, B, I, J: X \to X$ satisfying the following conditions

(i) $A(X) \subseteq J(X)$ and $B(X) \subseteq I(X)$,

(ii) The pairs (A, I) and (B, J) are weakly compatible,

(iii) The pairs (A, I) satisfy (CLR_A) or (B, J) satisfy (CLR_B) property,

(iv) $\int_0^{d(Ax,By)} \emptyset(t)dt \leq k \int_0^{M(x,y)} \emptyset(t)dt,\ k \in \left[0, \frac{1}{2}\right)$
for all $(x, y) \in X \times X$, where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that
$\int_0^{\epsilon} \emptyset(t)\,dt > 0$ for all $\epsilon > 0$,
$M(x, y) = max\{d(Ix, Jy), d(Ix, Ax), d(Jy, By), d(Ix, By), d(Jy, Ax)\}$,
then the mappings *A*, *B*, *I* and *J* have a unique common fixed point in *X*.

If we put *A*=*B* in the above Corollary 3.5, we can obtain the following corollary easily.

Corollary 3.6. Let (X, d) be a complete dislocated metric space. Let $A, I, J: X \to X$ satisfying the following conditions

(i) $A(X) \subseteq J(X)$ and $A(X) \subseteq I(X)$

(ii) The pairs (A, I) and (A, J) are weakly compatible.

(iii) The pairs (A, I) or (A, J) satisfy (CLR_A) property

(iv) $\int_0^{d(Ax,Ay)} \emptyset(t)dt \leq k \int_0^{M(x,y)} \emptyset(t)dt,\ k \in \left[0, \frac{1}{2}\right)$
for all $(x, y) \in X \times X$, where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that
$\int_0^{\epsilon} \emptyset(t)\,dt > 0$ for all $\epsilon > 0$,
$M(x, y) = max\{d(Ix, Jy), d(Ix, Ax), d(Jy, Ay), d(Ix, Ay), d(Jy, Ax)\}$,
then the mappings *A*, *I* and *J* have a unique common fixed point in *X*.

If we put *I*=*J* in the above Corollary 3.5, we can obtain the following corollaries easily.

Corollary 3.7. Let (X, d) be a complete dislocated metric space. Let $A, B, I: X \to X$ satisfying the following conditions

(i) $A(X) \subseteq I(X)$ and $B(X) \subseteq I(X)$

(ii) The pairs (A, I) and (B, I) are weakly compatible.

(iii) The pairs (A, I) satisfy (CLR_A) or (B, I) satisfy (CLR_B) property

(iv) $\int_0^{d(Ax,By)} \emptyset(t)dt \leq k \int_0^{M(x,y)} \emptyset(t)dt,\ k \in \left[0, \frac{1}{2}\right)$

for all $(x, y) \in X \times X$, where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)\,dt > 0$ for all $\epsilon > 0$,
$M(x, y) = max\{d(Ix, Iy), d(Ix, Ax), d(Iy, By), d(Ix, By), d(Iy, Ax)\}$,

then the mappings *A*, *B* and *I* have a unique common fixed point in *X*.

If we put $A=B$ and $I=J$ in the above Corollary 3.5, we can obtain the following corollaries easily.

Corollary 3.8. Let (X,d) be a complete dislocated metric space. Let $A, I: X \to X$ satisfying the following conditions

(i) $A(X) \subseteq I(X)$

(ii) The pair (A,I) is weakly compatible.

(iii) The pairs (A,I) satisfy (CLR_A) property

(iv) $\int_0^{d(Ax,Ay)} \emptyset(t)dt \leq k \int_0^{M(x,y)} \emptyset(t)dt, k \in \left[0,\frac{1}{2}\right)$

for all $(x,y) \in X \times X$, where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$,
$M(x,y) = max\{d(Ix,Iy), d(Ix,Ax), d(Iy,Ay), d(Ix,Ay), d(Iy,Ax)\}$,
then the mappings *A* and *I* have a unique common fixed point in *X*.

4 Conclusion

Fixed point theory is a rich, interesting, and exciting branch of mathematics. It is relatively young but fully developed area of research. Study of the existence of fixed points falls within several domains such as functional analysis, operator theory, general topology. Fixed points and fixed point theorems have always been a major theoretical tool in fields as widely apart as topology, mathematical economics, game theory, approximation theory and initial and boundary value problems in ordinary and partial differential equations. Moreover, recently, the usefulness of this concept for applications increased enormously by the development of accurate and efficient techniques for computing fixed points, making fixed point methods a major tool in the arsenal of mathematics. In recent years several definitions of conditions weaker than commutativity have appeared which facilitated significantly to extend the Jungck's theorem and several others. In this paper, we employ the concepts of commutativity, weak compatibility and common limit in the range (CLR) property to derive some common fixed point theorems for six mappings satisfying integral type contractive conditions in complete dislocated metric spaces. Our findings improve and extends some previous results found in the literature. In this way we can see that our result is superior to many other results.

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