**Fuzzy Mean e-Open and e-Closed Sets**

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**Abstract**

The notions of fuzzy mean $e$-open and $e$-closed sets is established. Moreover, some comparative study of these with other fuzzy mappings are investigated. Finally, we extend fuzzy mean $e$-open to fuzzy para $e$-open sets in fuzzy topology.

**Keywords and phrases:** Fuzzy minimal $e$-open, fuzzy mean $e$-open, fuzzy $e$-para open.

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1. **Introduction**

Fuzzy sets were established by Zadeh [10] and the perception of fuzzy topology instigated by Chang [2] in 1968. The ideas of fuzzy minimal (resp. maximal open) [3] sets explored in [3]. Subsequently the concepts of fuzzy mean open set investigated by Swaminathan [9]. On combining fuzzy mean open [9] and fuzzy paraopen open [4] sets, we extend the perception of fuzzy mean open (resp. closed) sets and from which we investigate some results.

The following terminologies “fuzzy $e$-open (resp. closed), fuzzy $e$-open (resp. closed), fuzzy minimal $e$-open (resp. maximal), fuzzy minimal $e$-closed set, fuzzy $e$-paraclosed (resp. paraclosed) and fuzzy $e$-connected topological space respectively abbreviated as Fe-O, Fe-C, FMe-O, FMe-C, FMe-O, FMAe-O, FMAe-C, Fe-PO, Fe-PC and Fe-CTS. Entire paper $F$ stands for fuzzy topology $(F, \tau)$.

2. **Preliminaries**

**Definition 2.1.** A fuzzy subset $\beta \in \wp$ is said to be fuzzy regular open [1] if $\beta = \text{Int} \{ \text{Cl}(\beta) \}$

The union of all fuzzy regular open sets contained in fuzzy subset $\beta \in \wp$ is $F e$-interior of $\beta$. If $\beta = \text{Int}\delta(\beta)$ then fuzzy subset $\beta$ is called $F e$-O [8] such that its complement is called $F e$-C (i.e., $\bar{\beta} = \text{Cl}\delta(\beta)$).

**Definition 2.2.** [5] A proper nonzero $F e$-O set $\beta \in \wp$ is called (i) FMI $e$-O if only $F e$-O sets contained in $\beta$. are $\bar{\beta}$ and 0 (ii) FMA $e$-O if only $F e$-O sets containing $\beta$ are 1 and $\bar{\beta}$.

**Definition 2.3.** A FO set $\mu \in \wp$ is said to be a FPO [4] set if it is neither FMIO nor FMAO set.

3. **Fuzzy $e$-Paraopen and $e$-Paraclosed Sets**

**Definition 3.1.** A $F e$-O set $\zeta \subset \wp$ which is neither FMI $e$-O nor FMA $e$-O set is said to be $F e$-PO set.

**Definition 3.2.** A $F e$-C set $\alpha \subset \wp$ is said to be a $F e$-PC set iff its complement $1 - \alpha$ is $F e$-PO set.

**Remark 3.1.** The converse of the statement: Every $F e$-PO set (resp. $F e$-PC) is a FO set (resp. FC set). Need not to be true proven by following example.

**Example 3.2.**

**Remark 3.3.** Union (resp. intersection) of $F e$-PO (resp. $F e$-PC) sets need not be $F e$-PO (resp. $F e$-PC) set.

**Theorem 3.4.** Let $\wp$ be a FTS and $\alpha$ be a nonempty proper $F e$-PO subset of $\wp$, then $\exists \mu$ FMI $e$-O set $\zeta$ with $\zeta < \alpha$.

**Proof.** Clearly $\zeta < \alpha$ as per the FMI $e$-O set definition.

**Theorem 3.5.** Let $\alpha$ be a nonempty proper $F e$-PO subset of a FTS $\wp$, then $\exists \psi$ FMA $e$-O set with $\alpha < \psi$.

**Proof.** Clearly $\alpha < \psi$ as per the FMA $e$-O set definition.
Theorem 3.6. Suppose that \( p \) is a FTS, then

(i) \( \zeta \land \zeta = 0 \) or \( \zeta < \zeta \) for any \( F \)-\( e \)-PO \( \zeta \) and a FMI \( e \)-O set \( \zeta \). (ii) \( \zeta \lor \lambda = 1 \) or \( \zeta < \lambda \) for any \( F \)-\( e \)-PO \( \zeta \) and a FMA \( e \)-O set \( \lambda \). (iii) Intersection of \( F \)-\( e \)-PO sets is either \( F \)-\( e \)-PO or FMI \( e \)-O set.

Proof. (i) For any \( F \)-\( e \)-PO set \( \zeta \) and a FMI \( e \)-O open set \( \zeta \) in \( p \). Then \( \zeta \land \zeta = 0 \) or \( \zeta \land \zeta \neq 0 \). If \( \zeta \land \zeta = 0 \), then proof could be over. Assume \( \zeta \land \zeta 
eq 0 \). Then we write \( \zeta \land \zeta \) is a FO set and \( \zeta \land \zeta < \zeta \). Hence \( \zeta < \zeta \).

(ii) For any \( F \)-\( e \)-PO set \( \zeta \) and a FMA \( e \)-O set \( \xi \) in \( p \). Then \( \zeta \lor \xi = 1 \) or \( \zeta \lor \xi \neq 1 \). If \( \zeta \lor \xi = 1 \), then proof could be over. Assume \( \zeta \lor \gamma \neq 1 \). Clearly, \( \zeta \lor \gamma \) is a FO set and \( \gamma < \zeta \lor \gamma \). Hence \( \gamma \) is a FMA \( e \)-O set, \( \zeta \lor \gamma = \gamma \) implies \( \zeta < \gamma \).

(iii) Let \( \zeta \) and \( \xi \) be a \( F \)-\( e \)-PO sets in \( p \). If \( \zeta \land \xi \) is a \( F \)-\( e \)-PO set, then proof could be over. Suppose \( \zeta \land \xi \) is not a \( F \)-\( e \)-PO set. By definition, \( \zeta \land \xi \) is a FMI \( e \)-O or FMA \( e \)-O set. If \( \zeta \land \xi \) is a FMI \( e \)-O set, then proof could be over. Suppose \( \zeta \land \xi \) is a \( F \)-\( e \)-O set. Now \( \zeta \land \xi < \zeta \) and \( \zeta \land \xi < \xi \) contradicting the fact that \( \zeta \) and \( \xi \) are \( F \)-\( e \)-PO sets. Hence, \( \zeta \land \xi \) is not a FMA \( e \)-O set. (i.e.) \( \zeta \land \xi \) is a FMI \( e \)-O set.

Theorem 3.7. A subset \( \lambda \) of a FTS \( p \) is \( F \)-\( e \)-PC iff it is neither \( FMI \) \( e \)-C nor \( FMI \) \( e \)-C set.

Proof. The complement of FMI \( e \)-O set and FMA \( e \)-O set are FMA \( e \)-C set and FMI \( e \)-C set respectively.

Theorem 3.8. Let \( \lambda \) be a nonempty \( F \)-\( e \)-PC subset of a FTS \( p \). Then \( \exists \) a FMI \( e \)-C set \( \psi \) with \( \psi < \lambda \).

Proof. Clearly by FMI \( e \)-C set definition, it follows that \( \psi < \lambda \).

Theorem 3.9. Suppose that \( \lambda \) is a nonempty \( F \)-\( e \)-PC subset of FTS \( p \) then \( \exists \) a FMI \( e \)-C set \( \kappa \) such that \( \lambda < \kappa \).

Proof. Clearly by FMA \( e \)-C set definition, it follows that \( \lambda < \kappa \).

Theorem 3.10. Suppose that \( p \) is a FTS then

(i) \( \kappa \land \eta = 0 \) or \( \eta < \kappa \) for any \( F \)-\( e \)-PC set \( \kappa \) and FMI \( e \)-C set \( \eta \). (ii) \( \kappa \lor \xi = 1 \) or \( \kappa < \xi \) for any \( F \)-\( e \)-PC set \( \kappa \) and FMA \( e \)-C set \( \xi \).

(iii) Intersection of \( F \)-\( e \)-PC sets is either \( F \)-\( e \)-PC or FMI \( e \)-C set.

Proof. (i) Suppose that \( \kappa \) is a \( F \)-\( e \)-PC and \( \eta \) is a FMI \( e \)-C set in \( p \). Then \( (1 - \kappa) \) is \( F \)-\( e \)-PO and \( (1 - \eta) \) is FMA \( e \)-O set in \( p \). Then by Theorem 3.6 (ii) \( (1 - \kappa) \lor (1 - \eta) \neq 1 \) or \( (1 - \kappa) < (1 - \eta) \) implying \( 1 - (\kappa \land \eta) \) or \( \eta < \kappa \). Hence, \( \kappa \land \eta = 0 \) or \( \eta < \kappa \).

(ii) Suppose that \( \kappa \) is a \( F \)-\( e \)-PC and \( \zeta \) is a FMA \( e \)-C set in \( p \). Then \( (1 - \kappa) \) is \( F \)-\( e \)-PO and \( (1 - \zeta) \) is FMI \( e \)-O set in \( p \). Then by Theorem 3.6 (ii) \( (1 - \kappa) \lor (1 - \zeta) = 0 \) or \( 1 - \zeta < 1 - \kappa \) implying \( 1 - (\kappa \lor \zeta) = 0 \) or \( \kappa < \zeta \). Hence, \( \kappa \lor \zeta = 1 \) or \( \kappa < \zeta \).

(iii) Suppose that \( \kappa \) and \( \xi \) is a \( F \)-\( e \)-PC set in \( p \). If \( \kappa \land \xi \) is a \( F \)-\( e \)-PC set, then proof could be over. Suppose \( \kappa \land \xi \) is not a \( F \)-\( e \)-PC set. Then clearly, \( \kappa \land \xi \) is a FMI \( e \)-C or FMA \( e \)-C set. Suppose \( \kappa \land \xi \) is a FMI \( e \)-C set, then proof could be over. Suppose \( \kappa \land \xi \) is a FMA \( e \)-C set. Now \( \kappa < \kappa \land \xi \) and \( \xi < \kappa \land \xi \) a contradiction for \( \kappa \) and \( \xi \) are \( F \)-\( e \)-PC sets. Hence, \( \kappa \land \xi \) is not a FMA \( e \)-C set. (i.e.) \( \kappa \land \xi \) is a FMI \( e \)-C set.

4. Fuzzy Mean -Open and -Closed Sets

Definition 4.1. A \( F \)-\( e \)-O set \( \psi \subset p \) is said to be a FME \( e \)-O set if \( \exists \omega_1, \omega_1(\neq \psi) \) two distinct proper \( F \)-\( e \)-O sets such that \( \omega_1 < \psi < \omega_2 \).

Remark 4.1. It could be understood from the succeeding example that the union and intersection of \( FME \) \( e \)-O need not be \( FME \) \( e \)-O sets.

Example 4.2. Let \( p = \{ x, y, z, w \} \). Then fuzzy sets

\[
\begin{align*}
\omega_1 &= \{(0.5, x), (0.4, y), (0.4, z), (0.5, w)\} ; \\
\omega_2 &= \{(0.5, x), (0.4, y), (0.6, z), (0.5, w)\} ; \\
\omega_3 &= \{(0.5, x), (0.6, y), (0.4, z), (0.5, w)\}
\end{align*}
\]
and $\omega_4 = \{(0.5, x), (0.6, y), (0.6, z), (0.5, w)\}$ of the fuzzy topology $\tau = \{0, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$. Hence $\omega_2$ and $\omega_3$ are FME $e$ - $O$ sets but their union $\omega_2 \vee \omega_3 = \omega_4$ and intersection $\omega_2 \cap \omega_3 = \omega_1$ are not FME $e$ - $O$ sets.

**Definition 4.2.** A F $e$ - $C$ set $v \subset \wp$ is said to be a FME $e$ - $C$ set if two F $e$ - $C$ sets $\xi_1 \neq \xi_2(\neq v)$ such that $\xi_1 \cup \xi_2 \subset v$.

**Definition 4.3.** A F $e$ - $O$ set $\xi \subset \wp$ which is neither FMI $e$ - $O$ nor FMA $e$ - $O$ set is said to be F $e$ - $PO$ set where its complement is known to be F $e$ - PC set.

**Theorem 4.3.** A F $e$ - $O$ set of a fits is a FME $e$ - $O$ set iff its complement is a FME $e$ - $C$ set.

**Proof.** By deploying definition 4.1 for any FME $e$ - $O$ set $\psi$ in $\wp$ we have $\omega_1 < \psi < \omega_2$.

Conversely, let $1 - \psi$ be a FME $e$ - $C$ set for any FME $e$ - $O$ set $\psi$ in $\wp$. By definition 4.2, F $e$ - $C$ sets $\xi_1 \neq 0$, $1 - \psi$ and $\xi_2 \neq 1$, $1 - \psi$ such that $\xi_1 < 1 - \psi < \xi_2$ implying that $1 - \xi_2 < \psi < 1 - \xi_1$. As $\xi_2 \neq 0$, $\psi$ and $1 - \xi_1 \neq \psi$, $1$; $\psi$ is a FME $e$ - $O$ set.

**Theorem 4.4.** A proper F $e$ - $PO$ set is a FME $e$ - $O$ set and vice-versa.

**Proof.** The proof of necessary part is obvious by theorem 1.7 [9].

Conversely, let $\psi$ be a proper FME $e$ - $O$ set in $\wp$. Then two F $e$ - $O$ sets $\xi_1 \neq \xi_2$ such that $\xi_1 < \psi < \xi_2$. Clearly $\psi$ is neither FMI $e$ - $O$ nor FMA $e$ - $O$ set as $\xi_1 \neq 0$, $\psi$ and $\xi_2 \neq \psi$, $1$. As $\psi \neq 0$, $1$, $\psi$ is a proper F $e$ - $PO$ set.

**Theorem 4.5.** A proper F $e$ - $PC$ set is a FME $e$ - $C$ set and vice-versa.

**Proof.** The proof of necessary part is obvious by theorem 1.10 [9].

Conversely, let $9$ be a proper FME $e$ - $C$ set in $\wp$. Then two F $e$ - $C$ sets $\psi_1 \neq \psi_2 \neq 9$ such that $\psi_1 < \psi < \psi_2$. Clearly $9$ is neither a FMI $e$ - $C$ nor a FMA $e$ - $C$ set as $\psi_1 \neq 0$, $9$ and $\psi_2 \neq 1$, $9$. As $9 \neq 0$, $1$, $9$ is a proper F $e$ - $PC$ set.

**Theorem 4.6.** ([5]) Let $\wp$ be a fits.

(i) If $\xi$ is a FMI $e$ - $O$ and $\xi$ is a F $e$ - $O$ sets in $\wp$, then $\xi \wedge \xi = 0$ or $\xi \neq \xi$. (ii) If $\xi$ and $\kappa$ are FMI $e$ - $O$ sets, then $\xi \wedge \kappa = 0$ or $\xi = \kappa$.

**Theorem 4.7.** ([5]) Let $\wp$ be a fits.

(i) If $\xi$ is a FMA $e$ - $O$ and $\xi$ is a F $e$ - $O$ sets in $\wp$, then $\xi \lor \xi = 1$ or $\xi \neq \xi$. (ii) If $\xi$ and $\kappa$ are FMA $e$ - $O$ set, then $\xi \lor \kappa = 1$ or $\xi = \kappa$.

**Theorem 4.8.** If $\xi_1$ is a FMA $e$ - $O$ set and $\xi_2$ is a FMI $e$ - $O$ set of a fits $\wp$, then either $\xi_2 < \xi_1$ or $\wp$ is fuzzy $e$ - disconnected.

**Theorem 4.9.** Let a F $e$ - $CTS$ $\wp$ contain a FMA $e$ - $O$ set $\xi_2$, a FMI $e$ - $O$ set $\xi_1 \neq \xi_2$ and a proper F $e$ - $O$ set $\xi \neq \xi_1$, $\xi_2$. Then exactly one of the succeeding could be true on $\wp$:

(i) $\xi$ is a FME $e$ - $O$ set with $\xi_1 < \xi < \xi_2$.

(ii) $\xi_1 < 1 - \xi < \xi_2$.

(iii) $\xi_1 < \xi, \xi_1 \lor \xi = 1$ and $\xi_2 \land \xi \neq 0$.

(iv) $\xi < \xi_2, \xi_1 \land \xi_2 = 0$ and $\xi \lor \xi_2 \neq 1$.

**Proof.** By deploying theorem 4.8, a FMI $e$ - $O$ set $\xi_1 \neq \xi_2$ a FMA $e$ - $O$ set. This implies either $\xi_1 < \xi$ or $\xi_1 \land \xi = 0$ and $\xi < \xi_2$ or $\xi_2 \lor \xi = 1$. Hence the feasible combinations are (i) $\xi_1 < \xi < \xi_2$; (ii) $\xi_1 \land \xi = 0$; $\xi_2 \lor \xi = 1$; (iii) $\xi_1 < \xi < \xi_2$; $\xi_2 \lor \xi = 1$; (iv) $\xi_1 \land \xi = 0$ and $\xi \lor \xi_2$. Clearly $\xi_1 < 1 - \xi < \xi_2$ if (ii) is true. Also, $0 \neq \xi_1 < \xi_1 \land \xi$ as $\xi_1 < \xi_2$ and (iii) is true. Again $\xi_1 \lor \xi \neq \xi_2 \neq 1$ as $\xi_1 < \xi_2$ and (iv) is true.

Case (I): As (i) and (ii) are true, then $\xi_1 < \xi < (1 - \xi) < \xi_2$ and $\xi_1 < \xi \land (1 - \xi) < \xi_2$. As $\xi_1 < \xi \lor (1 - \xi) < \xi_2 < 1 < \xi_2$ then $\xi_2 = 1$, an absurd result.

Similarly, for $\xi_1 < \xi \land (1 - \xi) < \xi_2$ we get $\xi_1 = 0$, an absurd result.
Case (II): As both (i), (iii) are true, then $\xi < \zeta$ and $\zeta \lor \xi = 1$ gives $\zeta = 1$, an absurd result.

Case (III): As both (i), (iv) are true, then $\zeta_1 < \xi$ and $\zeta_1 \land \xi = 0$ gives $\zeta_1 = 0$, an absurd result.

Case (IV): As both (ii), (iii) are true, then $\zeta_1 < 1 - \xi$ and $\zeta_1 < \xi$ gives $\zeta_1 = 0$, an absurd result.

Case (V): As both (ii), (iv) are true, then $1 - \xi < \zeta$ and $\xi < \zeta$ gives $\zeta = 1$, an absurd result.

Case (VI): As both (iii), (iv) are true, then $\zeta_1 < \xi < \zeta_2$, $\zeta_2 \lor \xi = 1$ and $\zeta_1 \land \xi = 0$. Clearly $\zeta_2 = 1$ as $\xi < \zeta_2$ and $\zeta_2 \lor \xi = 1$ a contradiction. Similarly, we get $\zeta_1 = 0$ as $\zeta_1 < \xi$ and $\zeta_1 \land \xi = 0$ a contradiction.

**Theorem 4.10.** Let a F e -CTS $\wp$ contain a FMA e -C set $u_2$, a FMI e -C set $u_1$ with $u_1 \neq u_2$ and a proper F e -C set $\beta \neq u_1, u_2$. Then any one of them could be true on $\wp$:

(i) $\beta$ is a FME e -C set such that $u_1 < \beta < u_2$.

(ii) $u_1 < 1 - \beta < u_2$.

(iii) $\beta < u_2$, $u_1 \land \beta = 0$ and $u_1 \lor \beta \neq 1$.

(iv) $u_1 < \beta$, $u_2 \lor \beta = 1$ and $u_2 \land \beta \neq 0$.

**Proof.** Let $\wp$ be a F e -CTS containing $1 - u_1$, a FMA e -O set; $1 - u_2$ a FMI e -O set and $1 - \beta$ a proper F e -O set such that $1 - u_1 \neq 1 - u_2$ and $1 - \beta \neq 1 - u_1, 1 - u_2$. By deploying Theorem 4.9, any one of them could be true:

(i) For any FME e -O set $1 - \beta$ we get $u_1 < \beta < u_2$ as $1 - u_2 < 1 - \beta < 1 - u_1$. Hence, $\beta$ is a FME e -C set.

(ii) Clearly, $u_1 < 1 - \beta < u_2$, as $1 - u_2 < 1 - (1 - \beta) < 1 - u_1$.

(iii) If $1 - u_2 < 1 - \beta$, $(1 - u_1) \lor (1 - \beta) = 1$ and $(1 - u_1) \land (1 - \beta) \neq 0$ then $\beta < u_2$; $u_1 \land \beta = 0$ and $u_1 \lor \beta \neq 1$.

(iv) If $1 - \beta < 1 - u_1$; $(1 - u_2) \land (1 - \beta) = 0$ and $(1 - u_2) \lor (1 - \beta) \neq 1$ then $u_1 < \beta$; $u_2 \lor \beta = 1$ and $u_2 \land \beta \neq 0$.

**Theorem 4.11.** Let two distinct FMA e -O and FME e -O sets in $\wp$. Then intersection of the two FMA e -O sets is nonzero.

**Proof.** By deploying theorem 4.7, $\kappa_1 \lor \kappa_2 = 1$ for any two distinct FMA e -O sets $\kappa_1$ and $\kappa_1$ in $\wp$. Let $\sigma$ be a FMA e -O set in a fts $\wp$ then $\sigma$ is neither FMA e -O nor FMI e -O such that, $\sigma \neq \kappa_1, \kappa_2$ and $\sigma \neq 1$. By Theorem 4.7, we get $\sigma \not\subseteq \kappa_1$ or $\sigma \supseteq \kappa_2$ or $\sigma \kappa_2 = 1$. The feasible combinations are (i) $\sigma \not\subseteq \kappa_1$ and $\sigma \not\subseteq \kappa_2$, (ii) $\sigma \supseteq \kappa_1$ and $\sigma \not\subseteq \kappa_2 = 1$, (iii) $\sigma \subseteq \kappa_2$ and $\sigma \not\subseteq \kappa_1 = 1$ and (iv) $\sigma \not\subseteq \kappa_1 = 1$ and $\sigma \not\subseteq \kappa_2 = 1$. Case (I): Obviously true.

Case (II): By assuming $\sigma \land \kappa_2 \neq 0$, we have to prove that $\kappa_1 \land \kappa_2 \neq 0$. As $\sigma \land \kappa_2 \neq 0$ and $\sigma \not\subseteq \kappa_1$, there exists $\chi_\sigma \in \kappa_1$ such that $\chi_\sigma \neq \chi_2$. Since $\sigma \lor \kappa_2 = 1$, $\chi_\sigma \in \kappa_2$. So, $\kappa_1 \land \kappa_2 \neq 0$.

Case (III): Similar to previous case.

Case (IV): $\sigma \lor \kappa_1 = 1$; $\sigma \lor \kappa_2 = 1$ imply that $\sigma \lor (\kappa_1 \land \kappa_2) = 1$ then $\sigma = 1$ if $\kappa_1 \land \kappa_2 = 0$. Again $\kappa_1 \land \kappa_2 \neq 0$ as $\sigma \neq 1$.

**Theorem 4.12.** Let two distinct FMI e -O and FME e -O sets in $\wp$. Then union of the two

FMI e -O sets is not equal to 1.

**Proof.** By deploying theorem 4.6, we have $\kappa_1 \lor \kappa_2 = 0$ for any two distinct FMI e -O sets $\kappa_1, \kappa_2$ in a fts $\wp$. Let $\sigma$ being a FME e -O set in $\wp$, then it is neither FMA e -O nor FMI e -O. Hence, $\sigma \neq \kappa_1, \kappa_2$ and $\sigma \neq 0, 1$. By theorem 4.6, we get $\kappa_1 \not\subseteq \sigma$ or $\sigma \not\subseteq \kappa_1 = 0$ and $\kappa_2 \not\subseteq \sigma$ or $\sigma \not\subseteq \kappa_2 = 0$. The possible combinations are (I) $\kappa_1 \not\subseteq \sigma$ and $\kappa_2 \not\subseteq \sigma$, (II) $\kappa_1 \not\subseteq \sigma$ and $\sigma \land \kappa_2 = 0$, (III) $\kappa_2 \not\subseteq \sigma$ and $\sigma \land \kappa_1 = 0$ and (IV) $\sigma \not\subseteq \kappa_1 = 0$ and $\sigma \not\subseteq \kappa_2 = 0$ as $\sigma \neq 1$.

Case (I): Obviously, if $\kappa_1 \not\subseteq \sigma$ and $\kappa_2 \not\subseteq \sigma$ then $\kappa_1 \lor \kappa_2 \neq 1$.

Case II: Suppose that $\sigma \lor \kappa_2 \neq 1$. Since $\kappa_1 \not\subseteq \sigma$, there exists $x_\sigma \in \sigma$ such that $x_\sigma \neq \kappa_1$.

As $\sigma \land \kappa_2 = 0$; clearly $x_\sigma \neq \kappa_2$. Hence, $x_\sigma \neq \kappa_1, \kappa_2$ imply that $\kappa_1 \lor \kappa_2 \neq 1$.

Case III: Similar to previous case.

Case IV: As $\sigma \land \kappa_1 = 0$; $\sigma \land \kappa_2 = 0$ imply that $\sigma \land (\kappa_1 \lor \kappa_2) = 0$ then $\sigma = 0$ if $\kappa_1 \lor \kappa_2 = 1$.  

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Clearly $\kappa_1 \lor \kappa_2 \neq 1$ as $\sigma \neq 0$.

"On combining theorems 4.11 and 4.12, we get theorems 4.13 and 4.14 and the proofs succeeded by theorems 4.11 and 4.12."

**Theorem 4.13.** Let $\kappa$ and $\sigma$ be distinct FMA $e$-C and FME $e$-C sets in a FTS respectively. Then the intersection of two FMA $e$-O sets is nonzero.

**Theorem 4.14.** Let $\zeta$ and $\xi$ be distinct FMI $e$-C and FME $e$-C sets in a FTS respectively. Then the union of two FMI $e$-C sets is not equal to 1.

**References**


