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Research Article

(1,2) *-D*-closed sets and (1,2) *-D**-closed sets in ideal bitopological spaces

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ABSTRACT. In this paper**,** we introduce the notion of (1,2)*-D*-closed sets and (1,2)*-D**-closed sets in ideal bitopological spaces. Additionally, we examine the characteristics of the novel concepts and contrast them with preexisting ones.

1. INTRODUCTION

In topological spaces, Levine proposed the notion of g-closed sets. Subsequent to these endeavors, contemporary mathematics expanded upon this notion and discovered numerous extensions of g-closed sets.

2. PRELIMINARIES

Definition 2.1

A subset S of a TPS X is called:

(i) semi-open if $S\subseteq cl(int(S))$;

(ii) α-open if $S\subseteq int(cl(int(S)))$;

(iii) β-open (semi-pre-open) if $S\subseteq cl(int(cl(S)))$;

(vi) regular open if $S= int(cl(S))$

The complements of the above-mentioned open sets are called their respective closed sets.

The semi-closure (resp. α-closure, semi-pre-closure, regular-closure) of a subset S of X, scl(S) (resp. α cl(S), spcl(S), rcl(S)) is defined to be the intersection of all semi-closed (resp. α -closed, semi-pre-closed, regular-closed) of X containingS. It is known that scl(S) (resp. αcl(S), spcl(S), rcl(S)) is semi-closed (resp. α -closed, semi-pre-closed, regular-closed).

Definition 2.2

A subset S of a TPS X is called

(i) g-closed set (briefly, g-cld) if $cl(S) \subseteq P$ whenever $S\subseteq P$ and P is open.

(ii) αgs-closed (briefly, αgs -cld) if $\alpha cl(S) \subseteq P$whenever $S\subseteq P$ and P is semi-open.

(iii) semi-generalized closed (briefly, sg-cld) if $scl(S) \subseteq P$ whenever $S\subseteq P$ and P is semi-open.

(iv) ψ-closed (briefly, ψ-cld) if $scl(S) \subseteq P$ whenever $S\subseteq P$ and P is sg-open.

(v) generalized semi-closed (briefly, gs-cld) if $scl(S)\subseteq P$ whenever $S\subseteq P$ and P is open.

(vi) α -generalized closed (briefly, α g-cld) if α $cl(S) \subseteq P$ whenever $S\subseteq P$ and P is open.

(vii) generalized semi-pre-closed(briefly, gsp-cld) if $spcl(S) \subseteq P$ whenever $S\subseteq P$ and P is open .

The complements of the above-mentioned closed sets are called their respective open sets.

Definition 2.3

The intersection of all sg-open subsets of X containing S is called the sg-kernel of S and denoted by sg-ker(S).

Definition 2.4

A subset S of X is called locally closed (briefly, lc) if S = U $\cap$ F, where U is open and F is closed in X.

Definition 2.5

A subset S of a space X is called:

(i) ĝ-cld (=ω-cld) if cl(S) $\subseteq$P whenever S$\subseteq$P and P is semi-open in X. The complement of ĝ-cldis called ĝ-open set;

(ii) $\ddot{g}$ -cld if cl(S) $\subseteq$Pwhenever S$\subseteq$P and P is sg-open in X.

The complement of $\ddot{g}$ -cld is called $\ddot{g}$ -open.

Definition 2.6

A subset S of a space X is called a g*s-cld set if scl(S) $\subseteq$P whenever S$\subseteq$P and P is gs-open in X. The complement of g*s-cld is called g*s-open.

Definition 2.10

A subset S of a space X is called:

(i) generalized locally closed (briefly, glc) if S = V$\cap$ F, where V is g-open and F is g-cld.

(ii) semi-generalized locally closed (briefly, sglc) if S = V$\cap$ F, where V is sg-open and F is sg-cld.

(iii) regular-generalized locally closed (briefly, rg-lc) if S = V$\cap$ F, where V is rg-open and F is rg-cld.

(iv) generalized locally semi-closed (briefly, glsc) if S = V$\cap$ F, where V is g-open and F is semi-cld.

(v) locally semi-closed (briefly, lsc) if S = V$\cap$ F, where V is open and F is semi-cld.

(vi) α -locally closed (briefly, α -lc) if S = V$\cap$ F, where V is α -open and F is α -cld.

(vii) ω -locally closed (briefly, ω -lc) if S = V$\cap$ F, where V is ω -open and F is ω -cld.

The class of all generalized locally closed (resp. generalized locally semi-closed, locally semi-closed, ω -locally closed) sets in X is denoted by GLC (X) (resp. $GLSC$ (X), LSC (X), ω -LC(X)).

Throughout this paper (X, τ_1, τ_2) or X will always denote bitopological spaces when A is a subset of $\tau_{1,2}$ -cl(A) and $\tau_{1,2}$ -int(A) denote the $\tau_{1,2}$ -closure set of A and $\tau_{1,2}$ -interior set of A respectively.

Definition 3.1

A subset A of X is called

(i) (1,2)*-D-closed (briefly, (1,2)*-D-cld) if (1,2)*-scl(A) $\subseteq$ $\tau_{1,2}$ -int U whenever A$\subseteq$U and U is (1,2)*-ω-open. The complement of (1,2)*-D-closed set is called (1,2)*-D-open.

(ii) (1,2)*-$\widehat{D}$-closed (briefly, (1,2)*-$\widehat{D}$-cld) if (1,2)*-spcl(A) ⊆ U whenever $A \subseteq U$ and U is (1,2)*-D-open. The complement of (1,2)*-$\widehat{D}$-closed set is called (1,2)*-$\widehat{D}$-open.

The class of all (1,2)*-$\widehat{D}$-cld in X is denoted by (1,2)*-$\widehat{D}$C.

3. (1,2)*-D-CLOSED SETSIN BITOPOLOGICAL SPACES**

Definition 3.1

A subset A of X is called

(iii) (1,2)*-D*-closed (briefly, (1,2)*-D*-cld) if (1,2)*-scl*(A) $\subseteq$ $\tau_{1,2}$ -int U whenever A$\subseteq$U and U is (1,2)*-ω-open. The complement of (1,2)*-D*-closed set is called (1,2)*-D*-open.

(iv) (1,2)*-D**-closed (briefly, (1,2)*-D**-cld) if (1,2)*-spcl(A) ⊆ U whenever $A \subseteq U$ and U is (1,2)*-D*-open. The complement of (1,2)*-D**-closed set is called (1,2)*- D** -open.

The class of all (1,2)*-D**-cld in X is denoted by (1,2)*-D**C.

Proposition 3.2

Each $\tau_{1,2}$ -closed (resp. (1,2)*-α-cld, (1,2)*-pre-cld, (1,2)*-semi-cld) is (1,2)*-D**-cld.

Proof

Let A be any $\tau_{1,2}$ -closed set. Let $A \subseteq U$ and U is (1,2)*-D*-open set in X. Then $\tau_{1,2}$ -cl(A) ⊆U. But (1,2)*-spcl(A) ⊆ $\tau_{1,2}$ -cl(A) ⊆U. Thus A is (1,2)*- D**-cld. The proof follows from the facts that(1,2)*-spcl(A) ⊆(1,2)*-scl(A) ⊆$\tau_{1,2}$-cl(A) and (1,2)*-spcl(A) ⊆(1,2)*-pcl(A) ⊆(1,2)*-αcl(A) ⊆$\tau_{1,2}$-cl(A).

Remark 3.3

The reverse of the above proposition need not be true.

Example 3.4

Let X={1, 2, 3, 4, 5} with $\tau_1 = \{\phi, \{1\}, \{1,2\}, X\}$ and $\tau_2 = \{\phi, \{3,4\}, X\}$. Then $\tau_{1,2} = \{\phi, \{1\}, \{1,2\}, \{3,4\}, \{1,3,4\}, \{1,2,3,4\}, X\}$. Here, $J = \{1,2,4\}$ is (1,2)*-D**-cld (resp. not (1,2)*-pre-cld, not (1,2)*-α-cld, not (1,2)*-semi-cld).

Proposition 3.5 Every (1,2)*-D*-cld is (1,2)*-D-cld

Proof

Since (1,2)*-scl*(A) ⊆ (1,2)*-scl(A).

Example 3.6

Proposition 3.7

Each (1,2)*-D**-cld is (1,2)*-gspr-cld

Proof

Let A be any (1,2)*-D**-cld set. Let $A \subseteq U$ and U is regular (1,2)*-open in X. Since each regular (1,2)*-open set is $\tau_{1,2}$ -open and each $\tau_{1,2}$ -open is (1,2)*-D*-open, we get (1,2)*-spcl(A) ⊆U. Hence, A is (1,2)*-gspr-cld.

Remark 3.8

The reverse of the above proposition need not be true.

Example 3.9

Let X={1, 2, 3, 4} with $\tau_1 = \{\phi, \{1\}, \{2\}, \{1,2\}, X\}$ and $\tau_2 = \{\phi, \{4\}, \{2,4\}, X\}$. Then $\tau_{1,2} = \{\phi, \{1\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,2,4\}, X\}$. Here, $J = \{1,2,4\}$ is (1,2)*-gspr-cld but not (1,2)*-D**-cld.

Theorem 3.10

Each (1,2)*-ω-cld is (1,2)*-D**-cld.

Proof

Let A be (1,2)*-ω-cld in X. Let $A \subseteq U$ and U is (1,2)*-D*-open. Then $\tau_{1,2}$-cl(A) ⊆U. Since each (1,2)*-ω-cld set is (1,2)*-pre-cld and each (1,2)*-pre-cld set is (1,2)*-semi-pre-cld, A is (1,2)*-semi-pre-cld.

Then A ⊆(1,2)*-pcl(A) ⊆(1,2)*-ωcl(A), Since each $\tau_{1,2}$-closed is (1,2)*-ω-cld, (1,2)*-ω-cl(A) ⊆ $\tau_{1,2}$-cl(A). Therefore, (1,2)*-spcl(A) ⊆ (1,2)*-pcl(A) ⊆ $\tau_{1,2}$-cl(A) ⊆ U. Hence, A is (1,2)*-D**-cld.

Remark 3.11

The reverse of the above proposition need not be true.

Example 3.12

Let X={1, 2, 3} with $\tau_1 = \{\phi, \{1\}, X\}$ and $\tau_2 = \{\phi, \{2\}, X\}$. Then $\tau_{1,2} = \{\phi, \{1\}, \{2\}, \{1,2\}, X\}$. Here, $J = \{1\}$ is (1,2)*-D**-cld but not (1,2)*-ω-cld.

Proposition 3.13

Each (1,2)*-D**-cld is (1,2)*-gsp-cld.

Proof

Let A be any (1,2)*-D**-cld in X. Let $A \subseteq U$ and U is $\tau_{1,2}$-open set in X. Since every $\tau_{1,2}$-open is (1,2)*-D*-open, we get (1,2)*-spcl(A) $\subseteq$U. Hence A is (1,2)*-gsp-cld.

Remark 3.14

The reverse of the above proposition need not be true.

Example 3.15

Let X={1, 2, 3} with $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{1\}, X\}$. Then $\tau_{1,2} = \{\phi, \{1\}, X\}$. Here, $J = \{1,2\}$ is (1,2)*-gsp-cld but not (1,2)*-D**-cld.

Proposition 3.16

Each (1,2)*-D**-cld is (1,2)*-pre-semi-cld

Proof

Let A be any (1,2)*-D**-cld in X. Let $A \subseteq U$ and U is (1,2)*-g-open in X. Since each (1,2)*-g-open is (1,2)*-D**-open, we get (1,2)*-spcl(A) $\subseteq$U. Hence, A is (1,2)*-pre-semi-cld.

Remark 3.17

The reverse of the above proposition need not be true.

Example 3.18

Let X={1, 2, 3, 4} with $\tau_1 = \{\phi, \{1\}, X\}$ and $\tau_2 = \{\phi, \{1,2,3\}, X\}$. Then $\tau_{1,2} = \{\phi, \{1\}, \{1,2,3\}, X\}$. Here, $J = \{\phi\}$ is (1,2)*-pre-semi-cld but not (1,2)*-D**-cld.

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