Complementary 3-domination number in some Special graphs and Cubic graphs

[1]V.G. Bhagavathi Ammal, [2]M.K. Anushya

[1]Assistant Professor, Department of Mathematics, S.T.Hindu College, Nagercoil, Tamil Nadu, India, bhagavathianand@gmail.com,

[2]Research Scholar, Department of Mathematics, S.T.Hindu College, Nagercoil, Tamil Nadu, India, mkanushya1996@gmail.com

(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012), Tamil Nadu, India

Article History: Received: 14 June 2020; Accepted: 30 September 2020; Published Online: 19 November 2020

ABSTRACT: A subset S of a graph G is called a dominating set of G if every vertex in V−S is adjacent to atleast one vertex in S. The domination number $\gamma(G)$ is the minimal cardinality of a dominating set. A dominating set S in a graph G is said to be a ***complementary 3-dominating set*** of G if any vertex in S has atleast three neighbours in V−S. The complementary 3-domination number $\gamma_3'(G)$ of a graph G is the minimum cardinality of a complementary 3-dominating set. We determine complementary 3-domination number for some special graphs and proved some theorem in cubic graphs.

Keywords: Domination Number, Complementary 3-Domination Number, Chromatic Number

AMS: 05C69

1.Introduction

By a ***graph*** we mean a simple, connected, finite and undirected graph G = (V,E) where V is the vertex set whose elements are vertices or nodes and E is the edge set. Unless otherwise stated the graph G with $|V| = n \ and \ |E| = q$. Degree of a vertex v is denoted by $d(v)$. Let $\Delta(G)$ and $\delta(G)$ denotes the maximum and minimum degree of a graph respectively. A subset S of V is called a ***dominating set*** of G if every vertex In V-S is adjacent to atleast one vertex in S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set. The chromatic number χ(G) of a graph G is the smallest number of colors for V(G) so that adjacent vertices are colored differently. In this paper we introduce the concept of complementary 3-domination number and we present some results related to this parameter.

Definition:1.1 A dominating set of a graph G is called a complementary 3-dominating set of G if for every vertex in S has atleast three neighbors in $V - S$. The complementary 3-domination number $\gamma_3'(G)$ is the minimum cardinality taken over all complementary 3-dominating sets.

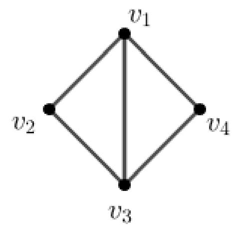
Theorem: 1.3 If G is any connected graph then $1 \leq \gamma_3'(G) \leq n$

Remark: 1.4 For any graph G with $\Delta(G) \leq 2$ then $\gamma_3'(G) = n$.

2. γ_3' value for some special graphs:

Definition: 2.1 A diamond graph G is a planar, undirected graph with 4 vertices and 5 edges as shown in figure 2.1. It consists of a complete graph K_4 minus one edge.

For any diamond graph G of order 4, $\gamma_3'(G) = 1$.



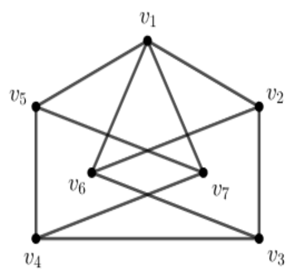
G

$Fig : 2.1$

In the above figure 2.1 the set $\{v_1\}$ forms a complementary 3-dominating set and hence $\gamma_3'(G) = 1$.

Definition: 2.2 The Hajos graph G is an undirected graph with seven vertices and eleven edges as shown in figure 2.2

For a Hajos graph G, $\gamma_3'(G) = 2$.



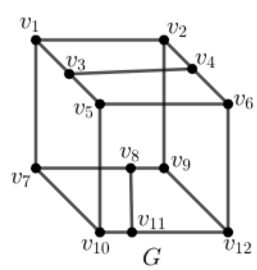
G

$Fig : 2.2$

In figure 2.2, the set $S = \{v_6, v_7\}$ forms a complementary 3-dominating set and hence $\gamma_3'(G) = 2$.

Definition: 2.3 The Bidiakis cube G is a 3-regular graph with 12 vertices and 18 edges as shown in figure 2.3

For a Bidiakis cube G, $\gamma_3'(G) = 3$.



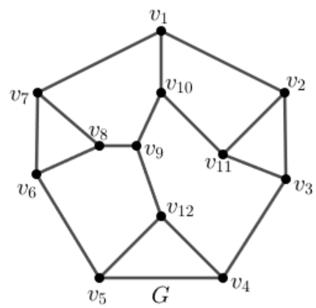
G

$Fig : 2.3$

In figure 2.3 the set $S = \{v_1, v_6, v_{11}\}$ forms a complementary 3-dominating set.

Definition: 2.4 A Frucht graph G is a cucic graph with 12 vertices, 18 edges and no non trivial symmetries as shown in figure 2.4

For a Frucht graph G, $\gamma_3'(G) = 4$.

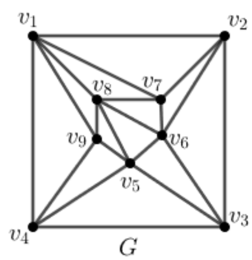


$Fig : 2.4$

In figure 2.4, th set $S = \{v_1, v_8, v_3, v_2\}$ forms a complementary 3-dominating set .

Definition: 2.5 A Soifer graph G is an undirected planar graph with 9 vertices and 20 edges as shown in figure 2.5

For a Soifer graph G $\gamma_3'(G) = 2$.

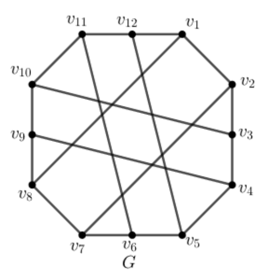


$Fig : 2.5$

In figure 2.5, the set $S = \{v_1, v_3\}$ forms a complementary 3-dominating set.

Definition: 2.6 The Franklin graph G is a 3-regular graph with 12 vertices and 18 edges.

For a Franklin graph G $\gamma_3'(G) = 4$.

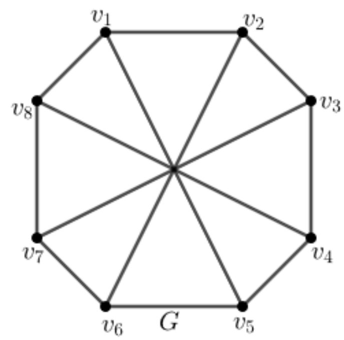


$Fig : 2.6$

In the above figure 2.6, the set S=$\{v_2, v_4, v_8, v_{11}\}$ forms a complementary 3-dominating set.

Definition: 2.7 The Wagner graph G is a 3-regular graph with 8 vertices and 12 edges as shown in figure 2.7

For a Wagner graph G, $\gamma_3'(G) = 3$.

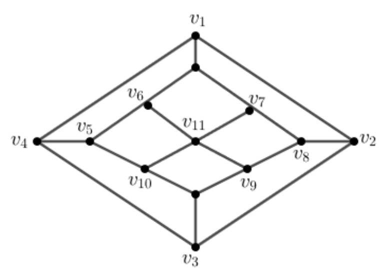


G
$Fig : 2.7$

In figure 2.7, the set $S = \{v_1, v_3, v_6\}$ forms a complementary 3-dominating set.

Definition: 2.8 The Herschel graph G is a bipartite undirected graph with eleven vertices and eighteen edges. It is a smaller polyhedral graph that does not have a Hamiltonian cycle, a cycle passing through all its vertices. It is named after a British Astronomer Alexander Stewart Herschel.

For a Herschel graph G, $\gamma_3'(G) = 3$.

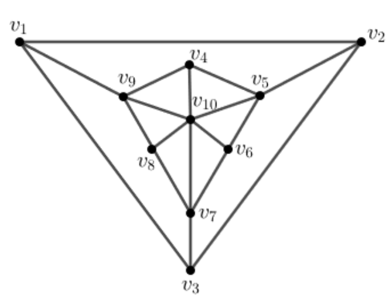


G
$Fig : 2.8$

In Fig 2.8 the set $S = \{v_1, v_3, v_{11}\}$ forms a complementary 3-dominating set.

Definition: 2.9 The Golomb graph G is a polyhedral graph with 10 vertices and eighteen edges. It is named after Solomon W.Golomb.

For a Golomb graph G, $\gamma_3'(G) = 2$.



G
$Fig : 2.9$

In figure 2.9 the set $S = \{v_3, v_{10}\}$ forms a complementary 3-dominating set.

3. γ'_3- number for cubic graph with 10 vertices

Theorem: 3.1 For a cubic graph G with 10 vertices, $G \cong G_i$ where $1 \leq i \leq 20$ then $\gamma'_3(G) = \chi(G) = 3$.

Proof: To prove this theorem we will discuss the following cases.

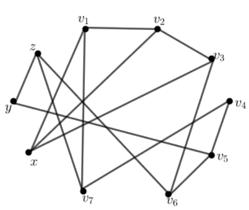
Let G be a cubic graph wuth 10 vertices. Let $D = \{x, y, z\}$ and $H_1 = \{v_1, v_2, v_3\} = N(x)$

Now, $< S > \neq P_3 \ or \ K_3$. Therefore <S>= $P_2 \cup P_1$ or $\overline{K}_3$.

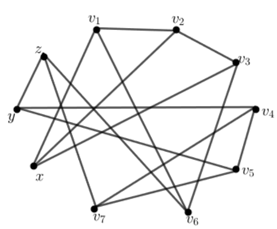
Now we consider the graphs with $< S > = P_2 \cup P_1$. Let v_4 and v_5 be the remaining two vertices which is adjacent to z, and v_6 and v_7 be the two remaining other vertices which is adjacent to y. Now, let us assume that $H_2 = \{v_4, v_5\}$ and $H_3 = \{v_6, v_7\}$.

Now let us consider the following cases.

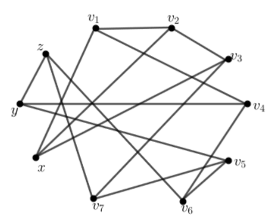
Case (1): $< S > = P_2 \cup P_1 \ and \ < H_1 > = P_3$



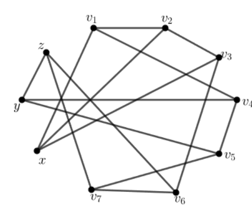
G_1



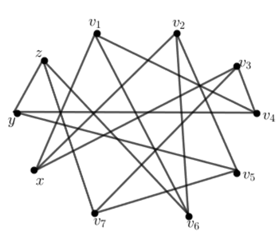
G_2

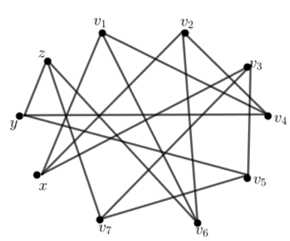


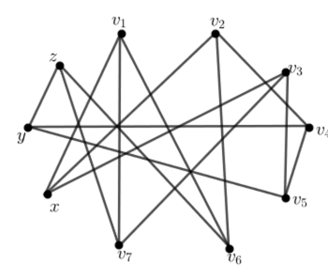
G_3



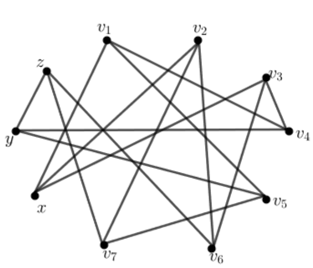
G_4

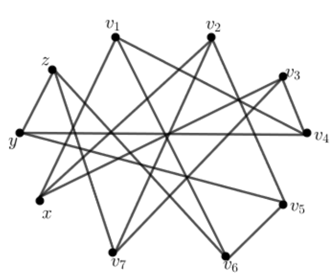


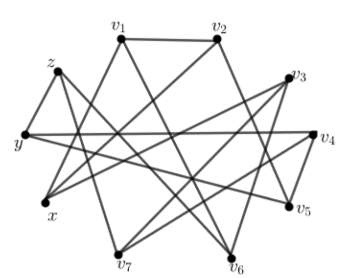




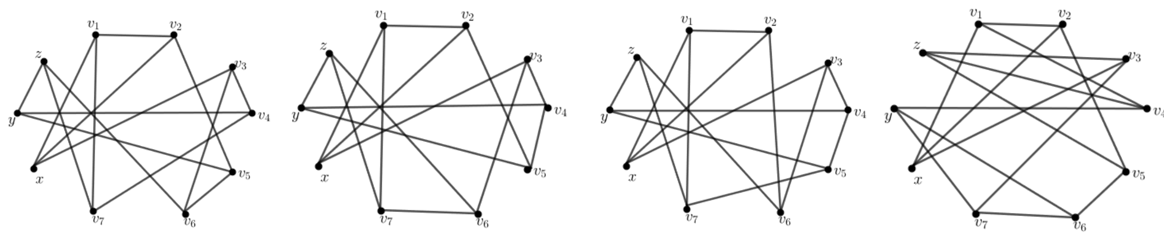
G_5 G_6 G_7



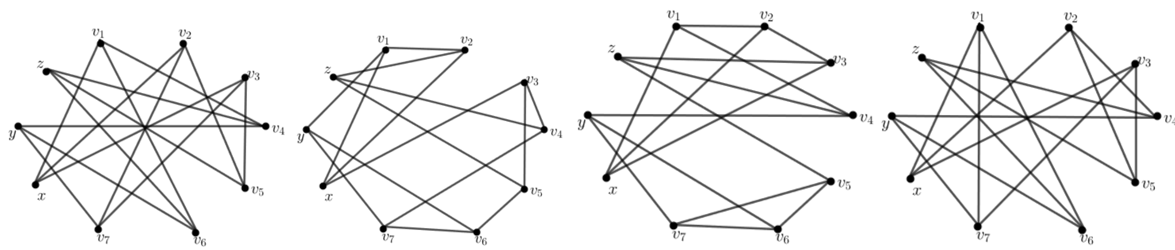




G_8 G_9 G_{10}



G_{11} G_{12} G_{13} G_{14}



G_{15} G_{16} G_{17} G_{18}

G_{19} G_{20}

Fig:3.1

Let $<H_1> = P_3 = v_1v_2v_3$

To prove this case we consider the following subcases.

Subcase(1a): $< H_1 > = < H_2 > = P_2$

Without loss of generality, let us assume that v_1 adjacent to v_6. Since G is a cubic graph v_3 non adjacent to v_5. Therefore v_3 adjacent to v_6(or equivalently v_7) and then v_5 adjacent to v_7 which implies G$\cong G_4$.

Subcase(1b): $<H_2 > = < H_3 > = \bar{P}_2$

Without loss of generality, let us assume that v_1 adjacent to v_4. Since G is cubic v_6 non adjacent to v_3. Therefore v_4 adjacent to v_6(or equivalently v_7) and so v_7 adjacent to v_3 and v_5 and since G is a cubic graph v_5 adjacent to v_6 which implies G$\cong G_3$.

Subcase(1c): $< H_2 > = P_2 \ and \ < H_3 > = \bar{P}_2$

Now v_5 adjacent to v_1 and v_3 or v_4 and v_5 or v_4 (or equivalently v_5) and v_1 (or equivalently v_3).

Suppose if v_7 adjacent to v_4 and v_5 then v_6 adjacent to v_1 and v_3 and so $G \cong G_1$. Suppose if v_7 adjacent to v_1 and v_3 then v_6 adjacent to v_4 and v_5 and so G≅ G_1. Suppose if v_7 adjacent to v_1 and v_4 then v_6 adjacent to v_3 and v_5 and so $G \cong G_2$.

Case: (2) $< D >= P_2 \cup P_1 \ and \ < H_1 >= \bar{K}_3$

To prove this case we consider the following subcases.

Subcase (2a): $< H_2 > = < H_3 > = P_2$

Since G is cubic in H_1 three vertices has to be incident with six edges. But in H_2 and H_3 four vertices can be incident with four edges which is a contradiction and hence in this subcase no graph exists.

Subcase (2b): $< H_2 > = P_2 \ and \ < H_3 > = \bar{P}_2$

Now, v_1 adjacent to v_6 and v_5 or v_4 and v_5 or v_4 (or equivalently v_5) and v_6 (or equivalently v_7). If v_1 adjacent to v_4 and v_5 then v_2 adjacent to v_6 and v_7 and then v_3 adjacent to v_6 and v_7.

Suppose if v_1 adjacent to v_4 and v_7, then v_2 adjacent to v_6 and v_7 or v_4 and v_5 or v_6 (or equivalently v_5). If v_2 adjacent to v_4 and v_5 then v_3 adjacent to v_6 and v_7. Suppose if v_2 adjacent to v_6 and v_7 then v_3 adjacent to v_4 and v_5. If v_2 adjacent to v_6 and v_4 then v_3 adjacent to v_7 and v_5 and so $G \cong G_7$.

Suppose if v_1 adjacent to v_4 and v_6, G is cubic then v_2 non adjacent to v_6 and v_5 and so v_2 must be adjacent to v_7 and v_5 or v_6 and v_7. Suppose if v_2 adjacent to v_7 and v_5 then v_3 adjacent to v_6 and v_7 and so G≅ G_7. Suppose if v_2 adjacent to v_6 and v_7 then v_3 adjacent to v_7 and v_5 and hence G≅ G_7.

Subcase (2c): $<H_2> = <H_3> = \bar{P}_2$

Now suppose if v_1 adjacent to v_4 and v_5 (or equivalently v_6 and v_7) or v_4 (or equivalently v_5) and v_6 (or equivalently v_5) and v_6 (or equivalently v_7).

Suppose v_1 adjacent to v_4 and v_5. G is cubic then v_2 non adjacent to v_4 and v_5 and so v_2 adjacent to v_6 and v_7 or v_4 (or equivalently v_5) and v_6 (or equivalently v_7). If v_2 adjacent to v_6 and v_7 then v_3 non adjacent to v_4 and v_5 and so v_3 is non adjacent to v_6 and v_7. Therefore v_3 is adjacent to v_4 (or equivalently v_5) and v_6(or equivalently v_7) and so v_7 is adjacent to v_4 and hence $G \cong G_8$. If v_2 adjacent to v_4 and v_6 then v_7 adjacent to v_5 and v_3 and so v_3 adjacent to v_6 hence $G \cong G_8$. If v_1 adjacent to v_6 and v_4 then v_2 adjacent to v_6 and v_4 or v_7 and v_5 or v_6 and v_7 (or equivalently v_4 and v_5) or v_6 and v_5 (or equivalently v_7 and v_4).

Suppose v_2 adjacent to v_6 and v_4 then v_3 adjacent to v_7 and v_5 and so v_5 adjacent to v_7 hence G≅ G_6. If v_2 adjacent to v_7 and v_5 and so v_3 is non adjacent to v_4 and v_5(or equivalently v_6 or v_7). Therefore v_3 adjacent to v_4 and v_6 or v_5 and v_7 or v_4 and v_7 or equivalently v_6 and v_7. If v_3 adjacent to v_4 and v_6 and so v_7 adjacent to v_5 and hence G≅ G_6. Suppose if v_3 adjacent to v_5 and v_7 then v_6 adjacent to v_4 and hence G≅ G_6, if v_3 adjacent to v_3 and v_5 then v_6 adjacent to v_5 and hence G≅ G_9.

Suppose v_2 adjacent to v_6 and v_5 then v_3 is non adjacent to v_4 and v_5. Therefore v_3 adjacent to v_4 and v_7 or v_7 and v_5. If v_3 adjacent to v_7 and v_4 then v_7 adjacent to v_5 and hence G≅ G_5. Suppose v_3 adjacent to v_5 and v_7 then v_7 adjacent to v_4 and hence G≅ G_5. Suppose v_2 adjacent to v_6 and v_7 then v_3 is non adjacent to v_7 and v_4. Also v_3 non adjacent to v_7 and v_5. Therefore v_3 adjacent to v_4 and v_5, then v_5 adjacent to v_7 and hence G≅ G_8.

Case (3): $< D > = P_2 \cup P_1 \ and \ < H_1 > = P_2 \cup P_1$

To prove this case we discuss the following cases.

Subcase (3a): $< H_2 > = P_2$ and $< H_3 > = \bar{P}_2$

Let v_1v_2 be an edge in $< H_1 >$. Now, v_3 will not be adjacent to v_4 and v_5 because G is cubic. Therefore v_3 adjacent to v_6 and v_7 or v_4 (or equivalently v_5) and v_6 (or equivalently v_7).

Suppose v_3 adjacent to v_6 and v_7, then v_6 adjacent to v_1(or equivalently v_2) or v_4(or equivalently v_5). If v_6 adjacent to v_1 then v_7 non adjacent to v_2 and so v_7 adjacent to v_4(or equivalently v_5) and hence v_5 adjacent to v_2 and so G$\cong G_1$. Suppose v_6 adjacent to v_4 then v_7 non adjacent to v_5. Therefore v_7 adjacent to v_1(or equivalently v_2) and so v_2 adjacent to v_5. Hence G$\cong G_{10}$.

Suppose v_3 adjacent to v_4 and v_5, then v_7 adjacent to v_1 and v_2 or v_1(or equivalently v_2) and v_5. If v_7 adjacent to v_1 and v_2 then v_6 adjacent to v_5. Suppose v_7 adjacent to v_1 and v_5 then v_2 adjacent to v_6 and hence $G \cong G_{13}$.

Subcase (3b): $< H_2 > = < H_3 > = \bar{P}_2$

Let v_1v_2 be an edge in $< H_1 >$. Here v_3 adjacent to v_6 and v_7 (or equivalently v_4 and v_5) or v_6 and v_4 (or equivalently v_7 and v_5).

Now v_3 adjacent to v_6 and v_7. If v_3 adjacent to v_6 and v_4 then v_7 non adjacent to v_1 and v_2. Hence v_7 adjacent to v_4 and v_5 or v_1 and v_4 or v_1 and v_5.

If v_7 adjacent to v_4 and v_5 then v_5 non adjacent to v_6. Therefore v_5 adjacent to v_1 (or equivalently v_2) and so v_2 adjacent to v_6 . If v_7 adjacent to v_1 and v_4 then v_5 adjacent to v_2 and v_6 and hence $\{v_7, x, z\}$ is a γ_3'- set and $G \cong G_{12}$. Suppose v_5 adjacent to v_1 and v_7 then v_2 non adjacent to v_4. Therefore v_2 adjacent to v_6 or v_7. If v_2 adjacent to v_4 then v_5 adjacent to v_6. Suppose v_2 adjacent to v_5 then v_4 adjacent to v_6 and so $\{v_7, z, x\}$ is a γ_3'-set. Therefore $G \cong G_{12}$.

Subcase (3c): $< H_2 > = < H_3 > = P_2$

Here v_3 adjacent to v_4 and v_5 (or equivalently v_6 and v_7) or v_6 and v_4 (or equivalently v_7 and v_5). Suppose v_3 adjacent to v_4 and v_5 then v_2 adjacent to v_6 (or equivalently v_7) and so v_1 adjacent to v_7. If v_3 adjacent to v_6 and v_4 then v_1 adjacent to v_7 (or equivalently v_5) and so v_2 adjacent to v_5 and hence $\{v_7, x, z\}$ is a γ_3'- set. Therefore $G \cong G_{12}$.

Now let us consider the graphs with $< D > = \bar{K}_3$

Now, y will be adjacent to two points which are not in $N[x]$. Suppose x adjacent to v_6 and v_7 . Let $D_2 = \{v_6, v_7\}$. Then z adjacent to other two points v_4 and v_5. Let $D_3 = \{v_4, v_5\}$. Now we consider the following cases.

Case (4): $< D > = \bar{K}_3$ *and* $< H_1 > = P_3$

Let $< D_1 > = \{v_1, v_2, v_3\}$. To prove this we discuss the following subcases.

Subcase (4a): $< H_2 > = < H_3 > = P_2$

Now let us assume that $U = H_2 \cup \{y\}$ and $V = H_3 \cup \{z\}$ so that $< U > = < V > = C_3$. Since G is a cubic graph for some $u \in U$ and $v \in V$ then $D = \{x, u, v\}$ such that $< D > = P_2 \cup P_1$ which comes under the case $< D > = P_2 \cup P_1$.

Subcase (4b): $< H_2 > = < H_3 > = \bar{P}_2$

Now y adjacent to v_1 (or equivalently v_3) or v_4 (or equivalently v_5). In this cases no

graph exists.

Subcase (4c): $< H_2 > = P_2$ and $< H_3 > = \bar{P}_2$

Assume that v_6 and v_7 be an edge in H_2. y is non adjacent to v_1(or equivalently v_3) because G is cubic. Therefore y adjacent to v_4(or equivalently v_5). Suppose y adjacent to v_4(or equivalently v_5) then v_5 adjacent to v_1 and v_3 or v_7 and v_6 or v_6 (or equivalently v_7) and v_1 (or equivalently v_3).

Suppose v_5 adjacent to v_1 and v_3, then v_4 adjacent to v_6 (or equivalently v_7) and so z adjacent to v_7 and hence {v_5,z,x} is a $\gamma_3' - set$. Suppose v_5 adjacent to v_1 and v_6 then v_7 not adjacent to v_3. Therefore v_7 adjacent to v_4 or z. If v_7 adjacent to v_4 then z adjacent to v_3 and so $G \cong G_{17}$. Suppose v_7 adjacent to z then v_3 adjacent to v_4 and so {v_5,z,x} is a $\gamma_3' - set$. Suppose v_5 adjacent to v_7 and v_6 so that v_4 adjacent to v_1 (or equivalently v_3 and hence z adjacent to v_3. Therefore $G \cong G_{17}$.

Case (5): $< D > = \bar{K}_3, < H_1 > = P_2 \cup P_1$

To prove this case we discuss the following subcases.

Subcase (5a): $< H_2 > = P_2 \ and \ < H_3 > = \bar{P}_2$

Let us assume that $v_1 v_2$ be an edge in $< H_1 >$. Now v adjacent to any one of $\{v_6, v_7, y\}$ or z or v_4 (or equivalently v_5). Suppose v_1 adjacent to y then z adjacent to v_3 or v_6 (or equivalently v_7) or v_2.

Suppose z adjacent to v_3. If z adjacent to v_6 and v_2 then v_7 adjacent to v_3 or v_4 (or equivalently v_5). If v_7 adjacent to v_4 then v_5 adjacent to v_3 and v_6 and so v_3 adjacent to v_4. Hence {u,w, v_5} is a $\gamma_3' - set$ and therefore $G \cong G_{16}$.

Suppose v_1 adjacent to z then no graph exists. If v_1 adjacent to v_4 then v_4 adjacent to any one vertices of $\{v_6, v_7, y\}$ or v_3 or v_2. Suppose v_4 adjacent to y then z adjacent to v_3 or v_6 or v_2 (or equivalently v_7).

Suppose z adjacent to v_3, then v_2 adjacent to v_6(or equivalently v_7) or v_5. If v_2 adjacent to v_6 then v_5 adjacent to v_3 and v_7. If v_2 adjacent to v_5 and G is cubic, v_5 will not be adjacent to v_3. Therefore v_5 adjacent to v_6 (or equivalently v_7) and so v_3 adjacent to v_7 and hence {x,y,z} is a γ_3'-set. Therefore $G \cong G_{14}$. If z adjacent to v_2 or v_6 and v_4 adjacent to v_2 or v_3 no other new graph exists.

Subcase (5b): $< H_2 > = < H_3 > = \bar{P}_2$

Let $v_1 v_2$ be an edge in $< D_1 >$. Now let v_1 adjacent to y (or equivalently z) or v_6 (or equivalently v_7) or (equivalently v_4) or equivalently v_5. In these cases no other new graph exists.

Subcase (5c): $< H_2 > = < H_3 > = P_2$

Let $v_1 v_2$ be an edge in $< D_1 >$ and v_1 adjacent to one of the vertices {y, $v_6 v_7$} (or equivalently any one of {z, v_4, v_5}. Let v_1 adjacent to v_6 and hence no other new graphs exists.

Case (6): $< D > = \bar{K}_3$ and $< H_1 > = \bar{K}_3$

To prove this case we consider the following subcases.

Subcase (6a): $< H_2 > = < H_3 > = \bar{P}_2$

Suppose v_1 adjacent to v_6 (or equivalently v_7) or v_1 adjacent to y (or equivalently z).

Now v_1 adjacent to v_6 then y adjacent to v_1 or v_2 (or equivalently v_3) or v_4 (or equivalently v_5).

Suppose if y adjacent to v_1 then {x,y,z} is a γ_3'-set. If y adjacent to $v_2$then y adjacent to v_1 or v_2 (or equivalently v_6) or v_3 (or equivalently v_7). If z adjacent to v_2 then v_4 adjacent to v_3 and v_7 or v_1 and v_6 or v_3 (or equivalently v_7) and v_1 (or equivalently v_6). If v_4 adjacent to v_3 and v_7 then v_6 adjacent to v_3 or v_5. If v_6 adjacent to v_3 then v_5 adjacent to v_1 and v_7 and so $G \cong G_{19}$. Hence $\{x, y, z\}$ is a γ_3'-set.

Suppose v_6 adjacent to v_5 then v_1 adjacent to v_7 or v_5. If v_1 adjacent to v_7 then v_3 adjacent to v_5. If v_1 adjacent to v_7 then v_3 adjacent to v_5. If v_1 adjacent to v_5 then v_3 adjacent to v_7 . If v_4 adjacent to v_1 and v_6 then v_7 adjacent to v_3 and v_5 and so v_3 adjacent to v_5. If v_4 adjacent to v_1 and v_3 then v_7 adjacent to v_3 and v_5 and so v_5 adjacent to v_6 and hence G$\cong G_{20}$. Therefore $\{x, y, z\}$ is a γ_3'-set.

Suppose z adjacent to v_1 or v_3 and hence no other new graph exists. If y adjacent to v_4 then z adjacent to v_1 or v_6 or v_7 or v_2(or equivalently v_3). If z adjacent to v_1 then no other new graph exists. If z adjacent to v_6 then v_4 adjacent to v_1 or v_2 (or equivalently v_3). Suppose v_4 adjacent to v_1 then v_2 adjacent to v_7 and v_5 and so v_3 adjacent to v_7 and v_5. Hence G$\cong G_{15}$. Therefore $\{x, y, z\}$ is a γ_3'-set.

If v_4 adjacent to v_2 then v_1 adjacent to v_7 or v_5. Suppose v_1 adjacent to v_7 then v_3 adjacent to v_7 and v_5 and so v_2 adjacent to v_5. If v_1 adjacent to v_5 then v_3 adjacent to v_7 and v_5 and so v_2 adjacent to v_7. Hence G is isomorphic to G_{18}. Therefore $\{x, y, z\}$ is a γ_3'-set. If v_1 adjacent to y then no other graph exists.

Subcase (6b): $< H_2 > = P_2 \ and \ < H_3 > = \bar{P}_2$

Suppose z adjacent to one of the three vertices $\{v_1, v_2, v_3\}$. Now let z adjacent to v_1 and so y adjacent to v_1 or non adjacent to v_1. In these cases no other new graph exists.

Subcase (6c): $< H_2 > = < H_3 > = P_2$

Assume that v_6v_7 and v_4v_5 be the edge in $< H_2 >$ and assume that $< D_3 >$ respectively. If suppose v_1 adjacent to two vertices of {y, v_6, v_7} (or equivalently any two vertices of {z, v_4, v_5} or one vertex of $\{v_6, v_7, z\}$ and any one of $\{v_4, v_5, z\}$. In these cases no such new graph exists.

Conclusion: In this paper we successfully described the complementary 3-domination number of some special graphs, bounds and proved a theorem in cubic graphs.

References:

[1] Mahadevan G and Mydeen bibi A (2009), "Characterization of Two domination number and Chromatic number of a graph",-preprint.

[2] Mahadevan G, Selvam Avadayappan A and Mydeenbibi G, "Cubic graphs with equal domination number and Chromatic number", International Journal of Inormation Technology and Knowlwdge Management, July-December 2011, Volume 4, No.2,pp.379-383.

[3] Harary F, "Graph Theory", Addison-Wesley, Reading MA (1969).

[4] Kulli V R, "Theory of Domination in Graphs", Vishwa International Publications, Gulburga, India (2010).

[5] Mahadevan G, Selvam Avadayappan, Bhagavathi Ammal V.G and Subramanian T "Restrained Triple Connected Domination Number of a Graph", International Journal of Engineering Research and Applications(IJERA) ISSN: 2248-9622 www.ijera.com Vol.2, Issue 6,November-December 2012,pp[225-229].

[6] Mahadevan G, Selvam Avadayappan A and Mydeenbibi G, "Cubic graphs with equal Two Domination number and Chromatic Number", International Journal of Information Technology and Knowledge Management, July-December 2011,volume 4,No.2,pp.[379-383].

[7] Mahadevan G, Selvan Avadayappan A and Mydeenbibi A,"Characterization of Complementary connected Domination number of a Graph",International Journal of Mathematics and Soft Computing,Vol.2,No.1(2012),[119-129].

[8] Mehry S and Safakish R,"Three Domination number and Connectivity in Graphs:,arXiv:1502.04458v1[math.CO]16 Feb2015.

[9] Teresa W. Haynes, Stephen T. Hedetniemi, Peter J. Slater, "Fundamentals of Dominaion in Graphs", Marcel Dekker, Newyork (1998).