SYNCHRONIZATION OF TWO COMPLEX IDENTICAL SYSTEMS USING NON-LINEAR CONTROL

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Abstract: - Synchronization of two complex identical system using nonlinear control is presented by this paper. For emphasis on the chaos, initial conditions sensitivity and Lyapunov exponent is calculated. For the synchronization of the complex master and slave system nonlinear control method is proposed effectively

Keywords: Complex Identical System, Numerical Simulation, Nonlinear Control, Synchronization

1. Introduction

After the pioneering work on the chaos control [1-2], synchronization has attracts the wide attention. Generally two systems are used in synchronization, one master as an input system and other is slave as an output System. Synchronization becomes a very active area of interest in the nonlinear science and also in the area of applied mathematics and automation engineering [7-13]. Many more application as secure communication [3-6] the topic of synchronization has various application. Different effective methods are used for the different chaotic systems which are based on the different methods Recently backstepping and active control method gain the popularity in the area of synchronization as these method are more powerful and effective. Through the transmission of the signal the trajectory of slave system approaches asymptotically to the trajectory of master system which is input system so that the error dynamics converges to zero [14].When several single oscillators are coupled together then a complicated system is obtained. For the study of these types of oscillators complex variables are used which are more convenient. Based on Lyapunov function for determination of the controllers the nonlinear control technique is used and also for synchronize two identical complex chaotic system.The paper is planned as: In section II, Design of non linear control method. In section III, System describtion and nonlinear control method is used for the synchronization of two identical complex system. Simulation agreements are shown in section IV which presents the proposed system is efficient.

II. Design of Non-Linear Control Method

By using the relation consider the master system

$$\dot{x_1} = A_1 x_1 + g_1(x_1)$$

Where x_1 (t) is the state vector of n- dimension of the master system $A_1 \in R^{n*n}$ is the system parameters matrix, and the function $g_1(x_1): R^n \rightarrow R^n$ is a nonlinear function. The slave system is defined by adding the control input vector.

$$\dot{x_2} = A_2 x_2 + g_2(x_2) + u$$

Where x_2 (t) is the state vector of n- dimension of the slave system, the slave system parameters matrix is $A_2 \in R^{n*n}$, and the function $g_2(x_2): R^n \rightarrow R^n$ is a nonlinear function.when the two chaotic systems are identical the $A_1 = A_2$ and $g_1(x_1) = g_2(x_2)$ but when the system is not identical then $A_1 \neq A_2$ and also $g_1(x_1) \neq g_2(x_2)$

By using the master and slave equations consider the error dynamical system

$\dot{e} = A_2 x_2 + g_2(x_2) + u - (A_1 x_1 + g_1(x_1))$, where $e = x_2 - x_1$ is the error state vector. The considered problem is to design an appropriate controller u such that trajectory of the slave system asymptotically approaches to the master system. When error vector converges to zero as time goes to infinity the synchronization is obtained.

Consider the error function in terms of Lyapunov.

$$v(\dot{e}) = \frac{1}{2} e^t e$$

Here the positive definite function is $v(\dot{e})$. For obtaining the synchronization the controller u is selected in such a way that $v(\dot{e}) < 0$, then. Assume that the states of input and output system are measurable, and parameters are known.

III.System Description and Synchronization Of Two Identical Systems

The Rossler system [15] is given by Rossler for verification of effectiveness of chaos control technique.

The coupled non-linear differential equation is

$$\dot{x_1} = \alpha(x_2 - x_1)$$
$$\dot{x_2} = (\Upsilon - \alpha)x_1 - x_1x_3 + \Upsilon x_2$$
$$\dot{x_3} = -\beta x_3 - \delta x_4 + x_1x_2$$
$$\dot{x}_4 = -dx_4 + fx_3 + x_1x_2$$

Now replace the real variable of the system (x_1, x_2, x_3, x_4) by the complex variables

$$x_1 = X_1 + \mathbf{i}\, X_2$$
$$x_2 = X_3 + \mathbf{i}\, X_4$$
$$x_3 = X_5 + \mathbf{i}\, X_6$$
$$x_4 = X_7 + \mathbf{i}\, X_8 \qquad (1.1)$$

We will get the following complex system.

$$\dot{x_1} = \alpha(x_3 - x_1)$$
$$\dot{x_2} = \alpha(x_4 - x_2)$$
$$\dot{x_3} = (\Upsilon - \alpha)x_1 - x_1x_5 + \Upsilon x_3$$
$$\dot{x}_4 = (\Upsilon - \alpha)x_2 - x_2x_6 + \Upsilon x_4$$
$$\dot{x_5} = -\beta x_5 - \delta x_7 + x_1x_3$$
$$\dot{x_6} = -\beta x_6 - \delta x_8 + x_2x_4$$
$$\dot{x_7} = -dx_7 + fx_5 + x_1x_3$$
$$\dot{x}_8 = -dx_8 + fx_6 + x_2x_4 \qquad (1.2)$$

System (1.2) is considered as a master system and the matrix for the system i

$$A = \begin{bmatrix} -a & 0 & a & 0 & 0 & 0 & 0 & 0 \\ 0 & -a & 0 & a & 0 & 0 & 0 & 0 \\ \gamma - a - x_5 & 0 & \gamma & 0 & -x_1 & 0 & 0 & 0 \\ 0 & \gamma - a - x_6 & 0 & \gamma & 0 & -x_2 & 0 & 0 \\ x_3 & 0 & x_1 & 0 & -\beta & 0 & -\delta & 0 \\ 0 & x_4 & 0 & x_2 & 0 & -\beta & 0 & -\delta \\ x_3 & 0 & x_1 & 0 & f & 0 & -d & 0 \\ 0 & x_4 & 0 & x_2 & 0 & f & 0 & -d \end{bmatrix}$$

The slave that is output system is.

$$\dot{y_1} = \alpha(y_3 - y_1) + u_1$$
$$\dot{y_2} = \alpha(y_4 - y_2) + u_2$$
$$\dot{y_3} = (\Upsilon - \alpha)y_1 - y_1y_5 + \Upsilon y_3 + u_3$$
$$\dot{y_4} = (\Upsilon - \alpha)y_2 - y_2y_6 + \Upsilon y_4 + u_4$$
$$\dot{y_5} = -\beta y_5 - \delta y_7 + y_1y_3 + u_5$$
$$\dot{y_6} = -\beta y_6 - \delta y_8 + y_2y_4 + u_6$$

$$\dot{y}_7 = -dy_7 + fy_5 + y_1y_3 + u_7$$
$$\dot{y}_8 = -dy_8 + fy_6 + y_2y_4 + u_8 \qquad (1.3)$$

Where $u = [u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8]^T$ is the control input vector to be determined and T is the transpose. Trajectory of slave system asymptotically approaches to that of master system, we wish to estimate appropriate nonlinear controller u_i where $i = 1,2,3, \dots .8$

Now consider the error dynamical system is.

$$\dot{e_i} = y_i - x_i \qquad i = 1, 2, \dots 8 \qquad (1.4)$$
$$\dot{e_i} = y_1 - x_1$$
$$\dot{e_i} = \alpha(y_3 - y_1) + u_1 - [\alpha(x_3 - x_1)]$$
$$\dot{e_i} = \alpha(e_3 - e_1) + u_1$$
$$\dot{e_2} = y_2 - x_2$$
$$\dot{e_2} = \alpha(y_4 - y_2) + u_2 - [\alpha(x_4 - x_2)]$$
$$\dot{e_2} = \alpha(e_4 - e_2) + u_2$$
$$\dot{e_3} = y_3 - x_3$$
$$\dot{e_3} = (\Upsilon - \alpha)y_1 - y_1y_5 + \Upsilon y_3 + u_3 - [(\Upsilon - \alpha)x_1 - x_1x_5 + \Upsilon x_3]$$
$$\dot{e_3} = (\Upsilon - \alpha)e_1 - y_1y_5 + \Upsilon e_3 + u_3 - x_1x_5$$
$$\dot{e_4} = y_4 - x_4$$
$$\dot{e_4} = (\Upsilon - \alpha)y_2 - y_2y_6 + \Upsilon y_4 + u_4 - [(\Upsilon - \alpha)x_2 - x_2x_6 + \Upsilon x_4]$$
$$\dot{e_4} = (\Upsilon - \alpha)e_2 - y_2y_6 + \Upsilon e_4 + u_4 - x_2x_6$$
$$\dot{e_5} = y_5 - x_5$$
$$\dot{e_5} = -\beta y_5 - \delta y_7 + y_1y_3 + u_5 - [-\beta x_5 - \delta x_7 + x_1x_3]$$
$$\dot{e_6} = y_6 - x_6$$
$$\dot{e_6} = -\beta y_6 - \delta y_8 + y_2y_4 + u_6 - [-\beta x_6 - \delta x_8 + x_2x_4]$$
$$\dot{e_6} = -\beta e_6 - \delta y_8 + y_2y_4 + u_6 - x_2x_4$$
$$\dot{e_7} = y_7 - x_7$$
$$\dot{e_7} = -dy_7 + fy_5 + y_1y_3 + u_7 - [-dx_7 + fx_5 + x_1x_3]$$
$$\dot{e_7} = de_7 + fe_5 + y_1y_3 + u_7 - x_1x_3$$
$$\dot{e_8} = y_8 - x_8\text{-}$$
$$\dot{e_8} = -dy_8 + fy_6 + y_2y_4 + u_8 - [-dx_8 + fx_6 + x_2x_4]$$
$$\dot{e_8} = -de_8 + fe_6 + y_2y_4 + u_8 - x_2x_4 \qquad (1.5)$$

This error system can be considered as control problem with the control input vector u_i

Consider the Lyapunov function.

$$\dot{v(e)} = \frac{1}{2}e^te \qquad (1.6)$$

In order to make (1.6) negative, the controllers u_i are.

$$u_1 = 2\alpha e_1 - \alpha e_3$$
$$u_2 = 2\alpha e_2 - ae_4$$
$$u_3 = -[\gamma - \alpha]e_1 + y_1y_5 + x_1x_5 + (1 - \gamma)e_3$$
$$u_4 = -[\gamma - \alpha]e_2 + y_2y_6 + x_2x_6 + (1 - \gamma)e_4$$

$$u_5 = \delta e_7 - y_1y_3 + x_1x_3 + (1+\beta)e_5$$
$$u_6 = \delta e_8 - y_2y_4 + x_2x_4 + (1+\beta)e_6$$
$$u_7 = -fe_5 - y_1y_3 + x_1x_3 + (1+d)e_7$$
$$u_8 = -fe_6 - y_2y_4 + x_2x_4 + (1+d)e_8 \qquad (1.7)$$

By using appropriate controllers u_i the equation (1.6) becomes

$$v(\dot{e}) = -\alpha e_1{}^2 - \alpha e_2{}^2 - e_3{}^2 - e_4{}^2 - e_5{}^2 - e_6{}^2 - e_7{}^2 - e_8{}^2 < 0 \qquad (1.8)$$

Since the function $v(\dot{e})$ is a negative definite function and the error state $\lim_{t\to\infty}\left||e(t)|\right| = 0$, approaching synchronization of master slave system.

The final slave system by using the equations (1.7) is

$$\dot{y_1} = \alpha(x_3 - 2x_1 + y_1)$$
$$\dot{y_2} = \alpha(x_4 - 2x_2 + y_2)$$
$$\dot{y_3} = \gamma(x_1 + x_3) + x_1x_5 + y_3 - x_3 + \alpha x_1$$
$$\dot{y_4} = \gamma(x_2 + x_4) + x_2x_6 + y_4 - x_4 + \alpha x_2$$
$$\dot{y_5} = -x_5 - \delta x_7 + x_1x_3 + y_5$$
$$\dot{y_6} = -\beta x_6 - \delta x_8 + x_2x_4 + y_6$$
$$\dot{y_7} = -dx_7 + fx_5 + x_1x_3 + y_{7\_}x_7$$
$$\dot{y_8} = -dx_8 + fx_6 + x_2x_4 + y_8 - x_8 \qquad (1.9`)$$

IV. Numerical Simulation

The initials conditions are $x(0) = (0.1,0,0.1,1,0.1,1,0.1,1)^T$ and $y(0) = (0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1)^T$ of two equations (1.2) and (1.9) and the values of $(\alpha,\beta,\gamma,\delta,d,f)$ as (10,10,20,10,10,10) are selected in such a way that the system is solved numerically By using the MATLAB . Fig 3(a) to 3(d) shows the error dynamics withtime t, shows that error system converges to zero and the two system are synchronized. Fig 4 (a) to 4(d) shows the time series of signals between x_i and y_i where $i = 1,2,\ldots.8$. Fig 2(a) to 2(e) shows the chaotic behaviour of the system (1.2) for the values of $\alpha,\beta,\gamma,\delta,d,f)$ as $(10,10,\frac{8}{3},10,10,10)$

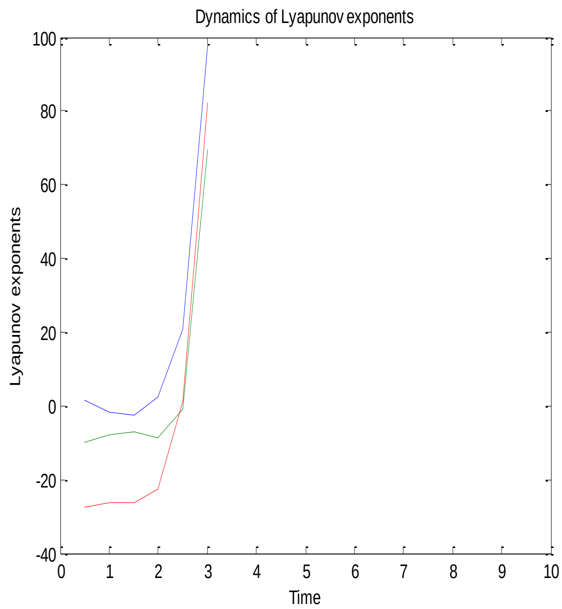

Fig 1. Lyapunov exponents of dynamical system

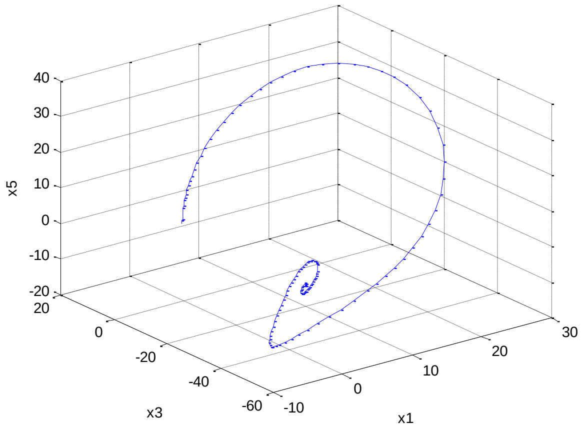

Fig 2(a). Chaotic behaviour of the system between $x_{1,}x_{3,}x_5$

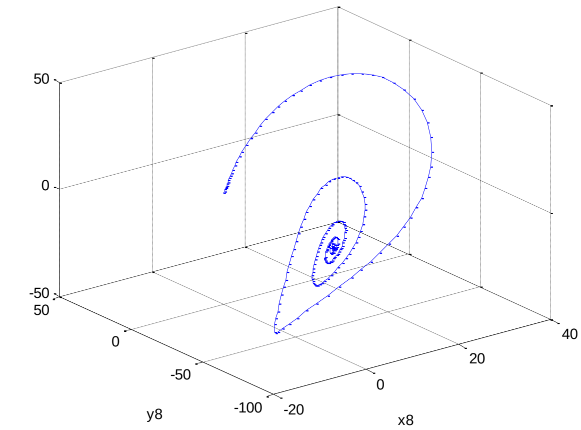

Fig 2(b). The Chaotic nature of the system between $x_{1,}x_{3,}x_8$

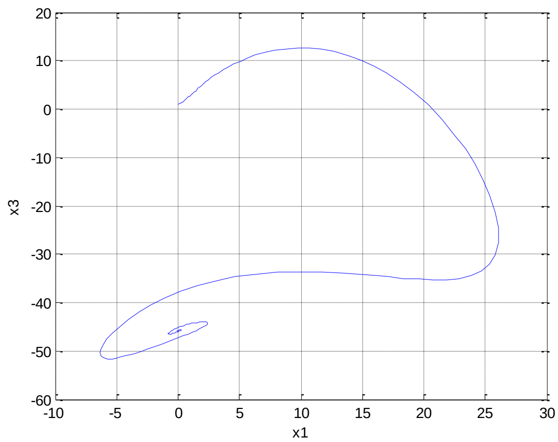

Fig. 2(c). The chaotic nature between $x_{1,}x_{3,}$

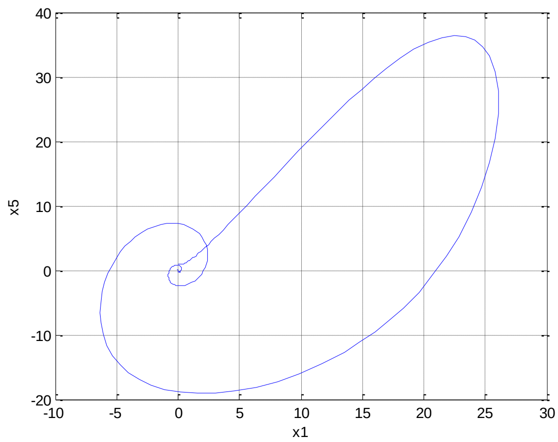

Fig. 2(d). The chaotic nature between the$x_{1,}x_5$

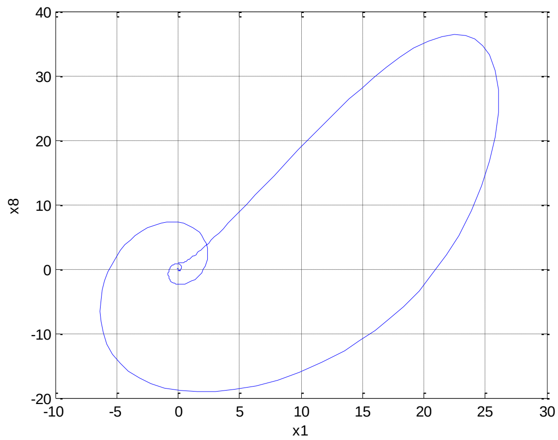

Fig. 2(e). The chaotic nature between the system $x_{1,}x_8$

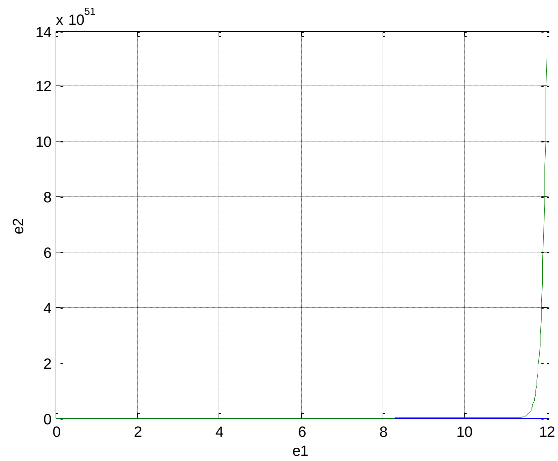

Fig. 3(a). The Error behavior between e_1 ande_2

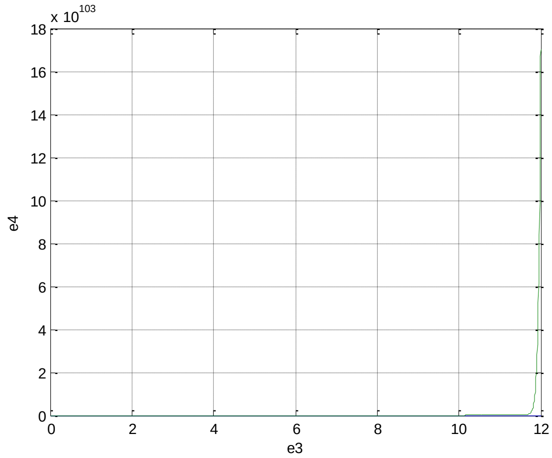

Fig. 3(b). The Error behavior between e_3 and e_4

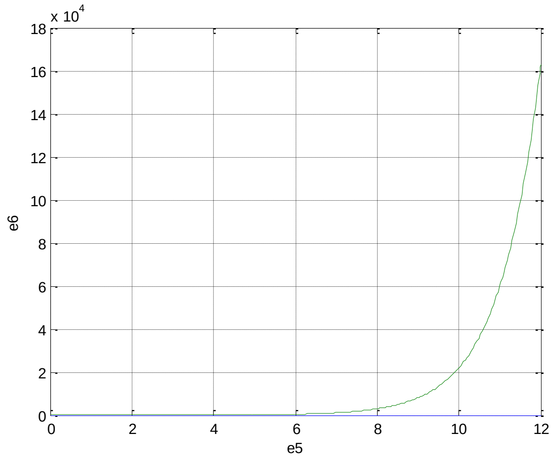

Fig. 3(d). The Error behaviour between $e_5$and e_6

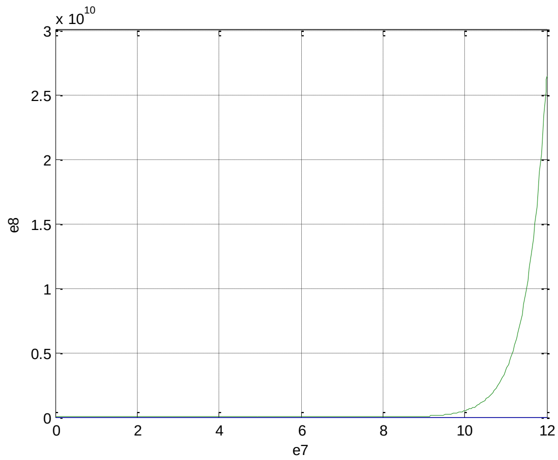

Fig. 3(c). The Error behaviour between $e_7$and e_8

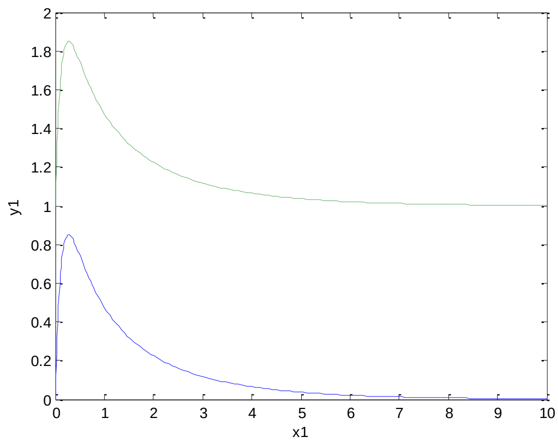

Fig. 4(a). Synchronization between x_1 and y_1 with time t

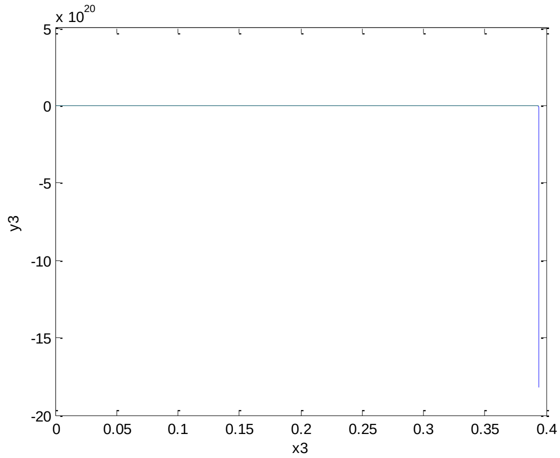

Fig. 4(b). Synchronization between x_3 and y_3 with time t

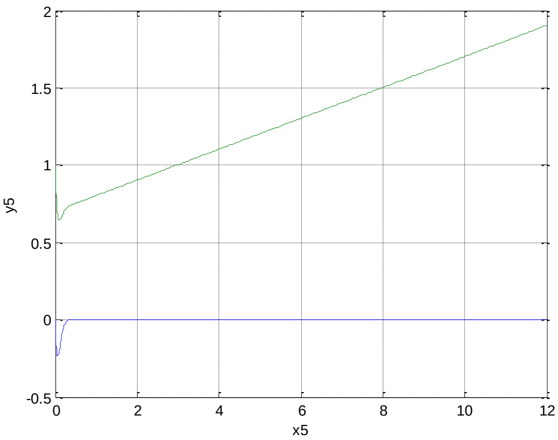

Fig. 4(c). Synchronization between x_5 and y_5 with time t

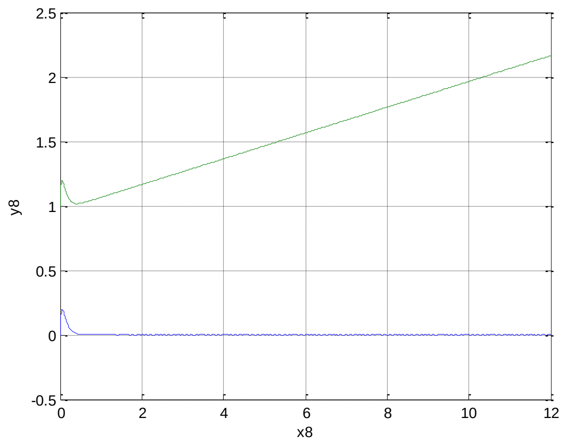

Fig. 4(d). Synchronization between$x_8$and y_8 with time t

V. CONCLUSION

Two identical complex system are synchronized using nonlinear control method. The initial conditions sensitivity and maximal Lyapunov exponent are calculated to show the chaotic behavior of that system. Many system as Lorenz system, Rössler system uses the technique which is effective and convenient for synchronization. Numerical calculation shows good agreement to the analytical results.

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