# Intuitionistic Fuzzy $\widehat{g}^*$ Semi Continuous Mapping and Irresolute Function in Intuitionistic Fuzzy Topological Spaces

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***Abstract:*** The purpose of this paper is to introduce and study the concepts of ***Intuitionistic Fuzzy $\widehat{g}^*$ Semi Continuous Mapping and Intuitionistic Fuzzy $\widehat{g}^*$ Semi Irresolute Function in Intuitionistic Fuzzy Topological Spaces*** and some of their characterizations are discussed. Additionally, we examine its driving effects by contrasting them with known irresolute functions and continuous mappings. A thorough verification has been carried out using relevant cases for the irresolute function and continuity mentioned above.

***Key Words:*** Intuitionistic Fuzzy $\widehat{g}$-Open set ($\mathcal{IF}\widehat{g}\mathcal{OS}$), Intuitionistic Fuzzy $\widehat{g}^*$ Semi Closed set($\mathcal{IF}\widehat{g}^*s\mathcal{CS}$), Intuitionistic Fuzzy $\widehat{g}^*$ Semi Continuous Function($\mathcal{IF}\widehat{g}^*s$-continuous) and Intuitionistic Fuzzy $\widehat{g}^*$ Semi Irresolute Function ($\mathcal{IF}\widehat{g}^*s$-irresolute).

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## I. INTRODUCTION

In order to explain the uncertainty in mathematics, Zadeh (1965)[15] ignited the lamp with his theory of fuzzy sets. A brilliant thinker, Chang [2] carried out additional research in this area and introduced fuzzy topology in 1968. Many research works have focused on the generalizations of the concepts of fuzzy sets and fuzzy topological spaces. In order to generalize fuzzy sets, Atanassov [1] developed the novel idea of Intuitionistic Fuzzy Sets, which was made possible by developments in this field of ongoing study. Coker [3] constructed Intuitionistic Fuzzy Topological spaces in 1997. This is a minor attempt, based on this heritage, to explore a new class of **Intuitionistic** ***Fuzzy $\widehat{g}^*$ Semi Continuous Mappings and Intuitionistic Fuzzy $\widehat{g}^*$ Semi Irresolute Function in Intuitionistic Fuzzy Topological Spaces***. The characterization and properties have been highlighted with the support of explicable examples.

## II. PRELIMINARIES

**Definition 2.1:** [1] Let $\mathbb{X}$ be a universal set. Let $\mathcal{A}_{\text{if}}$ be an object having the following form, i.e., $\mathcal{A}_{\text{if}} = \{\langle x, \mu_{\mathcal{A}_{\text{if}}}(x), \upsilon_{\mathcal{A}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$ is called as an intuitionistic fuzzy subset ($\mathcal{IFS}$ in short) $\mathcal{A}_{\text{if}}$ in $\mathbb{X}$. Here the functions $\mu_{\mathcal{A}_{\text{if}}}: \mathbb{X} \to [0,1]$ and $\upsilon_{\mathcal{A}_{\text{if}}}: \mathbb{X} \to [0,1]$ denote the degree of membership (namely $\mu_{\mathcal{A}_{\text{if}}}(x)$) and the degree of non-membership (namely $\upsilon_{\mathcal{A}_{\text{if}}}(x)$) of each element $x \in \mathbb{X}$ to the set $\mathcal{A}_{\text{if}}$ respectively, and $0 \le \mu_{\mathcal{A}_{\text{if}}}(x) + \upsilon_{\mathcal{A}_{\text{if}}}(x) \le 1$ for each $x \in \mathbb{X}$. The set of all intuitionistic fuzzy sets in $\mathbb{X}$ is denoted by $\mathcal{IFS}(\mathbb{X})$. For any two $\mathcal{IFS}$s $\mathcal{A}_{\text{if}}$ and $\mathcal{B}_{\text{if}}$, $(\mathcal{A}_{\text{if}} \cup \mathcal{B}_{\text{if}})^C = \mathcal{A}_{\text{if}}^C \cap \mathcal{B}_{\text{if}}^C$ ; $(\mathcal{A}_{\text{if}} \cap \mathcal{B}_{\text{if}})^C = \mathcal{A}_{\text{if}}^C \cup \mathcal{B}_{\text{if}}^C$.

**Definition2.2:** [1] If $\mathcal{A}_{\text{if}} = \{\langle x, \mu_{\mathcal{A}_{\text{if}}}(x), \upsilon_{\mathcal{A}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$ and $\mathcal{B}_{\text{if}} = \{\langle x, \mu_{\mathcal{B}_{\text{if}}}(\text{x}), \upsilon_{\mathcal{B}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$ be two $\mathcal{IFS}(\mathbb{X})$, then

(a) $\mathcal{A}_{\text{if}} \subseteq \mathcal{B}_{\text{if}}$ if and only if $\mu_{\mathcal{A}_{\text{if}}} \le \mu_{\mathcal{B}_{\text{if}}}$ and $\upsilon_{\mathcal{A}_{\text{if}}}(x) \ge \upsilon_{\mathcal{B}_{\text{if}}}(x)$ for all $x \in \mathbb{X}$ ,
(b) $\mathcal{A}_{\text{if}} = \mathcal{B}_{\text{if}}$ if and only if $\mathcal{A}_{\text{if}} \subseteq \mathcal{B}_{\text{if}}$ and $\mathcal{A}_{\text{if}} \supseteq \mathcal{B}_{\text{if}}$,
(c) $\mathcal{A}_{\text{if}}^C = \{\langle x, \upsilon_{\mathcal{A}_{\text{if}}}(x), \mu_{\mathcal{A}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$ (complement of $\mathcal{A}_{\text{if}}$),
(d) $\mathcal{A}_{\text{if}} \cup \mathcal{B}_{\text{if}} = \{\langle x, \mu_{\mathcal{A}_{\text{if}}}(x) \vee \mu_{\mathcal{B}_{\text{if}}}(\text{x}), \upsilon_{\mathcal{A}_{\text{if}}}(x) \wedge \upsilon_{\mathcal{B}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$,
(e) $\mathcal{A}_{\text{if}} \cap \mathcal{B}_{\text{if}} = \{\langle x, \mu_{\mathcal{A}_{\text{if}}}(x) \wedge \mu_{\mathcal{B}_{\text{if}}}(\text{x}), \upsilon_{\mathcal{A}_{\text{if}}}(x) \vee \upsilon_{\mathcal{B}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$,
(f) $(\mathcal{A}_{\text{if}} \cup \mathcal{B}_{\text{if}})^C = \mathcal{A}_{\text{if}}^C \cap \mathcal{B}_{\text{if}}^C$ and $(\mathcal{A}_{\text{if}} \cap \mathcal{B}_{\text{if}})^C = \mathcal{A}_{\text{if}}^C \cup \mathcal{B}_{\text{if}}^C$.
(h) $\tilde{\mathbf{0}} = \langle x, 0, 1\rangle$(empty set) and $\tilde{\mathbf{1}} = \langle x, 1, 0\rangle$ (whole set).

**Definition 2.3:** [3] An intuitionistic fuzzy topology ($\mathcal{IFT}$ in short) on $\mathbb{X}$ is a family of $\mathcal{IFS}s$ in $\mathbb{X}$, satisfying the following axioms.

1. $\tilde{0}$ , $\tilde{1} \in \tau_{\text{if}}$
2. $\mathcal{A}_{\text{if}} \cap \mathcal{B}_{\text{if}} \in \tau_{\text{if}}$ for any $\mathcal{A}_{\text{if}}, \mathcal{B}_{\text{if}} \in \tau_{\text{if}}$
3. $\cup \mathcal{A}_{\text{if}_i} \in \tau_{\text{if}}$ for any family $\{\mathcal{A}_{\text{if}_i} / i \in \mathcal{J}\} \subseteq \tau_{\text{if}}$.

The pair $(\mathbb{X}, \tau_{\text{if}})$ is called an intuitionistic fuzzy topological space ($\mathcal{IFTS}$in short) and any $\mathcal{IFS}$ in $\tau_{\text{if}}$ is known as an $\mathcal{IF}$ open set ($\mathcal{IFOS}$ in short) in $\mathbb{X}$. The complement ($\mathcal{A}_{\text{if}}^C$ ) of an $\mathcal{IFOS}$ $\mathcal{A}_{\text{if}}$ in an $\mathcal{IFTS}(\mathbb{X}, \tau_{\text{if}})$ is called an intuitionistic fuzzy closed set($\mathcal{IFCS}$ in short) in $\mathbb{X}$ . In this paper, $\mathcal{IF}$ interior is denoted by $int_{\text{if}}$ and $\mathcal{IF}$ closure is denoted by $cl_{\text{if}}$.

**Definition 2.4:** [3] Let $(\mathbb{X}, \tau_{\text{if}})$ be an $\mathcal{IFTS}$ and $\mathcal{A}_{\text{if}} = \{\langle x, \mu_{\mathcal{A}_{\text{if}}}(x), \upsilon_{\mathcal{A}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$ be an $\mathcal{IFS}$ in $\mathbb{X}$. Then the interior and closure of the above $\mathcal{IFS}$ are defined as follows,

(i) $int_{\text{if}}(\mathcal{A}_{\text{if}}) = \cup \{\mathcal{G}_{\text{if}} \mid \mathcal{G}_{\text{if}}$ is an $\mathcal{IFOS}$ in $\mathbb{X}$ and $\mathcal{G}_{\text{if}} \subseteq \mathcal{A}_{\text{if}}\}$,
(ii) $cl_{\text{if}}(\mathcal{A}_{\text{if}}) = \cap \{\mathcal{K}_{\text{if}} \mid \mathcal{K}_{\text{if}}$ is an $\mathcal{IFCS}$ in $\mathbb{X}$ and $\mathcal{A}_{\text{if}} \subseteq \mathcal{K}_{\text{if}}\}$

**Definition 2.5:** [9] An $\mathcal{IFS}$ $\mathcal{A}_{\text{if}}$ of an $\mathcal{IFTS}$ $(\mathbb{X}, \tau_{\text{if}})$ is called an intuitionistic fuzzy $\hat{g}^*$ semi closed set (in short $\mathcal{IF}\hat{g}^*s\mathcal{CS}$), if $scl_{\text{if}}(\mathcal{A}_{\text{if}}) \subseteq \mathcal{O}_{\text{if}}$ whenever $\mathcal{A}_{\text{if}} \subseteq \mathcal{O}_{\text{if}}$ and $\mathcal{O}_{\text{if}}$ is any intuitionistic fuzzy $\hat{g}$-open in $(\mathbb{X}, \tau_{\text{if}})$.

**Definition 2.6:** [4] Let $f: (\mathbb{X}, \tau_{\text{if}}) \to (\mathbb{Y}, \sigma_{\text{if}})$ be a mapping. Then $f$ is said to be intuitionistic fuzzy continuous ($\mathcal{IF}$ $-$continuous in short) if $f^{-1}(\mathcal{N}_{\text{if}})$ is $\mathcal{IFCS}$ in $(\mathbb{X}, \tau_{\text{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\text{if}}$ in $(\mathbb{Y}, \sigma_{\text{if}})$.

**Definition 2.7:** Let $f: (\mathbb{X}, \tau_{\mathrm{if}}) \to (\mathbb{Y}, \sigma_{\mathrm{if}})$ be a mapping. Then $f$ is said to be

(i) An $\mathcal{IF}\mathcal{s}$-continuous [5] if $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IFsC}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathrm{if}}$ in $(\mathbb{Y}, \sigma_{\mathrm{if}})$.

(ii) An $\mathcal{IF}\alpha$-continuous [5] if $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IF}\alpha\mathcal{C}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathrm{if}}$ in $(\mathbb{Y}, \sigma_{\mathrm{if}})$.

(iii) An $\mathcal{IF}p$-continuous [5] if $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IF}p\mathcal{C}$ in$(\mathbb{X}, \tau_{\mathrm{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathrm{if}}$ in$(\mathbb{Y}, \sigma_{\mathrm{if}})$.

(iv) An $\mathcal{IF}gs$-continuous[6] if $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IF}gs\mathcal{C}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathrm{if}}$ in$(\mathbb{Y}, \sigma_{\mathrm{if}})$.

(v) An $\mathcal{IF}g$-continuous [14] if $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IF}g\mathcal{C}$ in$(\mathbb{X}, \tau_{\mathrm{if}})$ for every $\mathcal{IFC}$ set $\mathcal{N}_{\mathrm{if}}$ in$(\mathbb{Y}, \sigma_{\mathrm{if}})$.

(vi) An $\mathcal{IF}g^*$-continuous[10] if $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IF}g^*\mathcal{C}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathrm{if}}$ in$(\mathbb{Y}, \sigma_{\mathrm{if}})$.

(vii) An $\mathcal{IF}w$-continuous [13] or $\mathcal{IF}\ \widehat{g}$-continuous if $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IF}\widehat{g}\mathcal{C}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathrm{if}}$ in$(\mathbb{Y}, \sigma_{\mathrm{if}})$.

(viii) An $\mathcal{IF}g^*s$-continuous [8] if $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IF}g^*s\mathcal{C}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathrm{if}}$ in$(\mathbb{Y}, \sigma_{\mathrm{if}})$.

(ix) An $\mathcal{IF}\Psi$-continuous [7] if $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IF}\Psi\mathcal{C}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathrm{if}}$ in $(\mathbb{Y}, \sigma_{\mathrm{if}})$.

## III. INTUITIONISTIC FUZZY $\widehat{g}^*$ SEMI CONTINUOUS MAPPINGS

**Definition 3.1:** A mapping $f: (\mathbb{X}, \boldsymbol{\tau}_{\mathbf{if}}) \to (\mathbb{Y}, \boldsymbol{\sigma}_{\mathbf{if}})$ is called **an intuitionistic fuzzy $\widehat{g}^*$ Semi Continuous Mapping ($\mathcal{IF}\widehat{g}^*s$ -continuous in short) if $f^{-1}(\mathcal{B}_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$ in** $(\mathbb{X}, \boldsymbol{\tau}_{\mathbf{if}})$ **for every $\mathcal{IFCS}$ $\mathcal{B}_{\mathrm{if}}$ of** $(\mathbb{Y}, \boldsymbol{\sigma}_{\mathbf{if}})$**.**

**Example 3.2:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$, $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$, $\tau_{\mathrm{if}} = \{\tilde{0},\ \mathcal{A}_{\mathrm{if}},\ \tilde{1}\}$ and $\sigma_{\mathrm{if}} = \{\tilde{0},\ \mathcal{B}_{\mathrm{if}},\ \tilde{1}\}$ where $\mathcal{A}_{\mathrm{if}}$={< $\mathbb{e}$, 0.2, 0.8>, < $\mathbb{f}$, 0.3, 0.7>} and $\mathcal{B}_{\mathrm{if}}$={< $\mathbb{g}$, 0.5, 0.4>, < $\mathbb{h}$, 0.6, 0.4>}. Then $\tau_{\mathrm{if}}$ and $\sigma_{\mathrm{if}}$ are $\mathcal{IFT}$s on spaces $\mathbb{X}$ and $\mathbb{Y}$. We define a mapping $f: (\mathbb{X}, \tau_{\mathrm{if}}) \to (\mathbb{Y}, \sigma_{\mathrm{if}})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. Then for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathrm{if}}$ in $(\mathbb{Y}, \sigma_{\mathrm{if}})$, $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$. Hence $f$ is $\mathcal{IF}\widehat{g}^*s$-continuous mapping.

**Theorem 3.3:** Every $\mathcal{IF}$-continuous mapping is an $\mathcal{IF}\widehat{g}^*s$-continuous mapping but not conversely.

**Proof:** Let $f: (\mathbb{X}, \tau_{\mathrm{if}}) \to (\mathbb{Y}, \sigma_{\mathrm{if}})$ be an $\mathcal{IF}$-continuous mapping and let $\mathcal{N}_{\mathrm{if}}$ be an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{\mathrm{if}})$. Since $f$ is an $\mathcal{IF}$-continuous mapping, $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is an $\mathcal{IFCS}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$. Since every $\mathcal{IFCS}$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$, $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$. Hence $f$ is an $\mathcal{IF}\widehat{g}^*s$-continuous mapping.

**Example 3.4:** Let $\mathbb{X} = \{m, n\}$, $\mathbb{Y} = \{u, v,\}$, $\tau_{\mathrm{if}} = \{\tilde{0},\ \mathcal{A}_{\mathrm{if}},\ \tilde{1}\}$ and $\sigma_{\mathrm{if}} = \{\tilde{0},\ \mathcal{B}_{\mathrm{if}},\ \tilde{1}\}$ where $\mathcal{A}_{\mathrm{if}}$={< $m$, 0.6, 0.4>, < $n$, 0.5, 0.4>} and $\mathcal{B}_{\mathrm{if}}$={< $u$, 0.3, 0.6>, < $v$, 0.4, 0.6>}. Then $\tau_{\mathrm{if}}$ and $\sigma_{\mathrm{if}}$ are $\mathcal{IFT}$s on spaces $\mathbb{X}$ and $\mathbb{Y}$. We define a mapping $f: (\mathbb{X}, \tau_{\mathrm{if}}) \to (\mathbb{Y}, \sigma_{\mathrm{if}})$ by $f(m) = u$ and $f(n) = v$. Then $f$ is an $\mathcal{IF}\widehat{g}^*s$ -continuous mapping but not an $\mathcal{IF}$-continuous mapping, since $\mathcal{IFS}$ $\mathcal{N}_{\mathrm{if}} = \{< u,\ 0.6,\ 0.3 >, < v,\ 0.6,\ 0.4 >\}$ is an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{\mathrm{if}})$ but $f^{-1}(\mathcal{N}_{\mathrm{if}})$ is not an $\mathcal{IFCS}$ in $(\mathbb{X}, \tau_{\mathrm{if}})$.

**Theorem 3.5:** Every $\mathcal{IF}s$-continuous mapping is an $\mathcal{IF}\widehat{g}^*s$- continuous mapping but not conversely.

**Proof:** Let $f: (\mathbb{X}, \tau_{\text{if}}) \rightarrow (\mathbb{Y}, \sigma_{\text{if}})$ be an $\mathcal{IF}s$-continuous mapping and let $\mathcal{N}_{\text{if}}$ be an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. Since $f$ is an $\mathcal{IF}s$-continuous mapping, $f^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Since every $\mathcal{IF}s\mathcal{CS}$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$, $f^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Hence $f$ is an $\mathcal{IF}\widehat{g}^{*}s$ -continuous mapping.

**Example 3.6:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$, $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$, $\tau_{\text{if}} = \{\tilde{0}, \mathcal{A}_{\text{if}}, \tilde{1}\}$ and $\sigma_{\text{if}} = \{\tilde{0}, \mathcal{B}_{\text{if}}, \tilde{1}\}$ where $\mathcal{A}_{\text{if}}=\{<\mathbb{e}, 0.36, 0.62>, <\mathbb{f}, 0.28, 0.72>\}$ and $\mathcal{B}_{\text{if}}=\{<\mathbb{g}, 0.2, 0.8>, <\mathbb{h}, 0.21, 0.74>\}$. Then $\tau_{\text{if}}$ and $\sigma_{\text{if}}$ are $\mathcal{IFT}$s on spaces $\mathbb{X}$ and $\mathbb{Y}$. We define a mapping $f: (\mathbb{X}, \tau_{\text{if}}) \rightarrow (\mathbb{Y}, \sigma_{\text{if}})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. Then $f$ is an $\mathcal{IF}\widehat{g}^{*}s$-continuous mapping but not an $\mathcal{IF}s$-continuous mapping, since $\mathcal{IFS}$ $\mathcal{N}_{\text{if}} = \{<\mathbb{g}, 0.8, 0.2>, <\mathbb{h}, 0.74, 0.21>\}$ is an $\mathcal{IFCSS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$ but $f^{-1}(\mathcal{N}_{\text{if}})$ is not an $\mathcal{IF}s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$.

**Theorem 3.7:** Every $\mathcal{IF}\alpha$-continuous mapping is an $\mathcal{IF}\widehat{g}^{*}s$- continuous mapping but not conversely.

**Proof:** Let $f: (\mathbb{X}, \tau_{\text{if}}) \rightarrow (\mathbb{Y}, \sigma_{\text{if}})$ be an $\mathcal{IF}\alpha$-continuous mapping and let $\mathcal{N}_{\text{if}}$ be an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. Since $f$ is an $\mathcal{IF}\alpha$-continuous mapping, $f^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\alpha\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Since every $\mathcal{IF}\alpha\mathcal{CS}$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$, $f^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Hence $f$ is an $\mathcal{IF}\widehat{g}^{*}s$-continuous mapping.

**Example 3.8:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$, $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$, $\tau_{\text{if}} = \{\tilde{0}, \mathcal{A}_{\text{if}}, \tilde{1}\}$ and $\sigma_{\text{if}} = \{\tilde{0}, \mathcal{B}_{\text{if}}, \tilde{1}\}$ where $\mathcal{A}_{\text{if}}=\{<\mathbb{e}, 0.3, 0.6>, <\mathbb{f}, 0.2, 0.5>\}$ and $\mathcal{B}_{\text{if}}=\{<\mathbb{g}, 0.2, 0.8>, <\mathbb{h}, 0.3, 0.7>\}$. Then $\tau_{\text{if}}$ and $\sigma_{\text{if}}$ are $\mathcal{IFT}$s on spaces $\mathbb{X}$ and $\mathbb{Y}$. We define a mapping $f: (\mathbb{X}, \tau_{\text{if}}) \rightarrow (\mathbb{Y}, \sigma_{\text{if}})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. Then $f$ is an $\mathcal{IF}\widehat{g}^{*}s$-continuous mapping but not an $\mathcal{IF}\alpha$-continuous mapping, since $\mathcal{IFS}$ $\mathcal{N}_{\text{if}} = \{<\mathbb{g}, 0.8, 0.2>, <\mathbb{h}, 0.7, 0.3>\}$ is an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$ but $f^{-1}(\mathcal{N}_{\text{if}})$ is not an $\mathcal{IF}\alpha\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$.

**Theorem 3.9:** Every $\mathcal{IF}\Psi$-continuous mapping is an $\mathcal{IF}\widehat{g}^{*}s$- continuous mapping but not conversely.

**Proof:** Let $f: (\mathbb{X}, \tau_{\text{if}}) \rightarrow (\mathbb{Y}, \sigma_{\text{if}})$ be an $\mathcal{IF}\Psi$-continuous mapping and let $\mathcal{N}_{\text{if}}$ be an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. Since $f$ is an $\mathcal{IF}\Psi$-continuous mapping, $f^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\Psi\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Since every $\mathcal{IF}\Psi\mathcal{CS}$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$ , $f^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Hence $f$ is an $\mathcal{IF}\widehat{g}^{*}s$-continuous mapping.

**Example 3.10:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$, $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$, $\tau_{\text{if}} = \{\tilde{0}, \mathcal{A}_{\text{if}}, \tilde{1}\}$ and $\sigma_{\text{if}} = \{\tilde{0}, \mathcal{B}_{\text{if}}, \tilde{1}\}$ where $\mathcal{A}_{\text{if}}=\{<\mathbb{e}, 0.34, 0.66>, <\mathbb{f}, 0.28, 0.72>\}$ and $\mathcal{B}_{\text{if}}=\{<\mathbb{g}, 0.22, 0.78>, <\mathbb{h}, 0.2, 0.8>\}$. Then $\tau_{\text{if}}$ and $\sigma_{\text{if}}$ are $\mathcal{IFT}$s on spaces $\mathbb{X}$ and $\mathbb{Y}$. We define a mapping $f: (\mathbb{X}, \tau_{\text{if}}) \rightarrow (\mathbb{Y}, \sigma_{\text{if}})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. Then $f$ is an $\mathcal{IF}\widehat{g}^{*}s$ -continuous mapping but not an $\mathcal{IF}\Psi$-continuous mapping, since $\mathcal{IFS}$ $\mathcal{N}_{\text{if}} = \{<\mathbb{g}, 0.78, 0.22>, <\mathbb{h}, 0.8, 0.2>\}$ is an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$ but $f^{-1}(\mathcal{N}_{\text{if}})$ is not an $\mathcal{IF}\Psi\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$.

**Theorem 3.11:** Every $\mathcal{IF}g^{*}$-continuous mapping is an $\mathcal{IF}\widehat{g}^{*}s$- continuous mapping but not conversely.

**Proof:** Let $f: (\mathbb{X}, \tau_{if}) \to (\mathbb{Y}, \sigma_{if})$ be an $\mathcal{IF}g^*$-continuous mapping and let $\mathcal{N}_{if}$ be an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{if})$. Since $f$ is an $\mathcal{IF}g^*$-continuous mapping, $f^{-1}(\mathcal{N}_{if})$ is an $\mathcal{IF}g^*\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$. Since every $\mathcal{IF}g^*\mathcal{CS}$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$, $f^{-1}(\mathcal{N}_{if})$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$. Hence $f$ is an $\mathcal{IF}\widehat{g}^*s$-continuous mapping.

**Example 3.12:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$, $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$, $\tau_{if} = \{\tilde{0}, \mathcal{A}_{if}, \tilde{1}\}$ and $\sigma_{if} = \{\tilde{0}, \mathcal{B}_{if}, \tilde{1}\}$ where $\mathcal{A}_{if}$={<$\mathbb{e}$, 0.34, 0.62>, <$\mathbb{f}$, 0.25, 0.72>} and $\mathcal{B}_{if}$={< $\mathbb{g}$, 0.12, 0.88 >, < $\mathbb{h}$, 0.18, 0.82 >}. Then $\tau_{if}$ and $\sigma_{if}$ are $\mathcal{IFT}$s on spaces $\mathbb{X}$ and $\mathbb{Y}$. We define a mapping $f: (\mathbb{X}, \tau_{if}) \to (\mathbb{Y}, \sigma_{if})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. Then $f$ is an $\mathcal{IF}\widehat{g}^*s$-continuous mapping but not an $\mathcal{IF}g^*$-continuous mapping, since $\mathcal{IFS}$ $\mathcal{N}_{if} = \{< \mathbb{g}, 0.88, 0.12 >, < \mathbb{h}, 0.82, 0.18 >\}$ is an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{if})$ but $f^{-1}(\mathcal{N}_{if})$ is not an $\mathcal{IF}g^*\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$.

**Theorem 3.13:** Every $\mathcal{IF}g^*s$-continuous mapping is an $\mathcal{IF}\widehat{g}^*s$- continuous mapping but not conversely.

**Proof:** Let $f: (\mathbb{X}, \tau_{if}) \to (\mathbb{Y}, \sigma_{if})$ be an $\mathcal{IF}g^*s$-continuous mapping and let $\mathcal{N}_{if}$ be an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{if})$. Since $f$ is an $\mathcal{IF}g^*s$-continuous mapping, $f^{-1}(\mathcal{N}_{if})$ is an $\mathcal{IF}g^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$. Since every $\mathcal{IF}g^*s\mathcal{CS}$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$, $f^{-1}(\mathcal{N}_{if})$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$. Hence $f$ is an $\mathcal{IF}\widehat{g}^*s$ -continuous mapping.

**Example 3.14:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$, $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$, $\tau_{if} = \{\tilde{0}, \mathcal{A}_{if}, \tilde{1}\}$ and $\sigma_{if} = \{\tilde{0}, \mathcal{B}_{if}, \tilde{1}\}$ where $\mathcal{A}_{if}$={< $\mathbb{e}$, 0.2, 0.8>, <$\mathbb{f}$, 0.3, 0.7>} and $\mathcal{B}_{if}$={< $\mathbb{g}$, 0.4, 0.6 >, < $\mathbb{h}$, 0.2, 0.8 >}. Then $\tau_{if}$ and $\sigma_{if}$ are $\mathcal{IFT}$s on spaces $\mathbb{X}$ and $\mathbb{Y}$. We define a mapping $f: (\mathbb{X}, \tau_{if}) \to (\mathbb{Y}, \sigma_{if})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. Then $f$ is an $\mathcal{IF}\widehat{g}^*s$-continuous mapping but not an $\mathcal{IF}g^*s$-continuous mapping, since $\mathcal{IFS}$ $\mathcal{N}_{if} = \{< \mathbb{g}, 0.6, 0.4 >, < \mathbb{h}, 0.8, 0.2 >\}$ is an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{if})$ but $f^{-1}(\mathcal{N}_{if})$ is not an $\mathcal{IF}g^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$.

**Remark. 3.15**: $\mathcal{IF}\widehat{g}^*s$-continuous mapping and $\mathcal{IF}\widehat{g}$-continuous mapping are independent.

**Example 3.16:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$ and $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$. Let $\tau_{if} = \{\tilde{0}, \mathcal{A}_{if}, \tilde{1}\}$ and $\sigma_{if} = \{\tilde{0}, \mathcal{B}_{if}, \tilde{1}\}$ where $\mathcal{A}_{if}$ = {< $\mathbb{e}$, 0.2, 0.8>, <$\mathbb{f}$, 0.1, 0.9>} and $\mathcal{B}_{if}$ = {< $\mathbb{g}$, 0.3, 0.7 >, < $\mathbb{h}$, 0.2, 0.8>}. Then $(\mathbb{X}, \tau_{if})$ and $(\mathbb{Y}, \sigma_{if})$ are $\mathcal{IFTS}s$. Define a mapping $f: (\mathbb{X}, \tau_{if}) \to (\mathbb{Y}, \sigma_{if})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. Then $f^{-1}(\mathcal{N}_{if})$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$ and not an $\mathcal{IF}\widehat{g}\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$ where $\mathcal{N}_{if} = \{< \mathbb{g}, 0.7, 0.3 >, < \mathbb{h}, 0.8, 0.2 >\}$ is an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{if})$. Therefore Then $f$ is an $\mathcal{IF}\widehat{g}^*s$-continuous mapping but not an $\mathcal{IF}\widehat{g}$-continuous mapping

**Example 3.17:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$ and $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$. Let $\tau_{if} = \{\tilde{0}, \mathcal{A}_{if}, \tilde{1}\}$ and $\sigma_{if} = \{\tilde{0}, \mathcal{B}_{if}, \tilde{1}\}$ where $\mathcal{A}_{if}$ = {< $\mathbb{e}$, 0.4, 0.6>, <$\mathbb{f}$, 0.2, 0.7>} and $\mathcal{B}_{if}$ = {< $\mathbb{g}$, 0.62, 0.35 >, < $\mathbb{h}$, 0.72, 0.12> }. Then $(\mathbb{X}, \tau_{if})$ and $(\mathbb{Y}, \sigma_{if})$ are $\mathcal{IFTS}s$. Define a mapping $f: (\mathbb{X}, \tau_{if}) \to (\mathbb{Y}, \sigma_{if})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. Then $f^{-1}(\mathcal{N}_{if})$ is an $\mathcal{IF}\widehat{g}\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$ and not an $\mathcal{IF}\widehat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$ where $\mathcal{N}_{if} = \{< \mathbb{g}, 0.35, 0.62 >, < \mathbb{h}, 0.72, 0.12 >\}$ is an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{if})$. Therefore Then $f$ is an $\mathcal{IF}\widehat{g}$-continuous mapping but not an $\mathcal{IF}\widehat{g}^*s$ -continuous mapping.

The diagram below depicts the interrelationship of $\mathcal{IF}\widehat{g}^*s$-continuous function with other continuous functions.

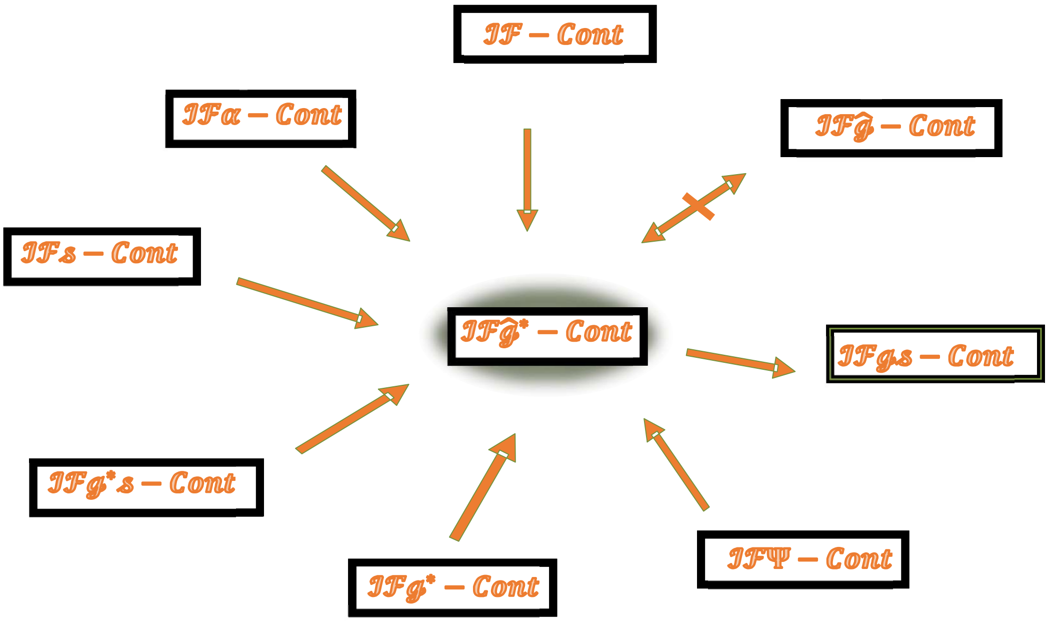


**Figure 3.1**

## IV. PROPERTIES OF INTUITIONISTIC FUZZY $\hat{g}^*$ SEMI CONTINUOUS MAPPINGS

**Theorem 4.1:** A mapping $f: (\mathbb{X}, \tau_{if}) \to (\mathbb{Y}, \sigma_{if})$ is an $\mathcal{IF}\hat{g}^*s$-continuous iff the inverse image of every $\mathcal{IFOS}$ in $(\mathbb{Y}, \sigma_{if})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{X}, \tau_{if})$.

**Proof: Necessary Part:** Let $\mathcal{A}_{if}$ be an $\mathcal{IFO}$ set in $(\mathbb{Y}, \sigma_{if})$. This implies $\mathcal{A}_{if}{}^{C}$ is an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{if})$. Since $f$ is an $\mathcal{IF}\hat{g}^*s$- continuous. $f^{-1}(\mathcal{A}_{if}{}^{C})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$. Since $f^{-1}(\mathcal{A}_{if}{}^{C}) = (f^{-1}(\mathcal{A}_{if}))^{C}$, $f^{-1}(\mathcal{A}_{if})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{X}, \tau_{if})$.

**Sufficient Part:** Let $\mathcal{A}_{if}$ be an $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{if})$. This implies $\mathcal{A}_{if}{}^{C}$ is an $\mathcal{IFOS}$ in $(\mathbb{Y}, \sigma_{if})$. By hypothesis, $f^{-1}(\mathcal{A}_{if}{}^{C})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{X}, \tau_{if})$. Since $f^{-1}(\mathcal{A}_{if}{}^{C}) = (f^{-1}(\mathcal{A}_{if}))^{C}$, $f^{-1}(\mathcal{A}_{if})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$. Hence $f$ is an $\mathcal{IF}\hat{g}^*s$ -continuous mapping.

**Theorem 4.2: Let** $f: (\mathbb{X}, \tau_{if}) \to (\mathbb{Y}, \sigma_{if})$ be an $\mathcal{IF}\hat{g}^*s$- continuous mapping. Then the following statements hold.

(i) $f(\mathcal{IF}\hat{g}^*scl_{if}(\mathcal{A}_{if})) \subseteq cl_{if}(f(\mathcal{A}_{if}))$ for every $\mathcal{IFS}$ $\mathcal{A}_{if}$ in $(\mathbb{X}, \tau_{if})$.

(ii) $\mathcal{IF}\hat{g}^*scl_{if}(f^{-1}(\mathcal{B}_{if})) \subseteq f^{-1}(cl_{if}(\mathcal{B}_{if}))$ for every $\mathcal{IFS}$ $\mathcal{B}_{if}$ in $(\mathbb{Y}, \tau_{if})$.

**Proof: (i)** Let $\mathcal{A}_{if} \subseteq (\mathbb{X}, \tau_{if})$. Then $cl_{if}(f(\mathcal{A}_{if}))$ is an $\mathcal{IFCS}$ in $(\mathbb{Y}, \tau_{if})$. Since $f$ is $\mathcal{IF}\hat{g}^*s$-continuous, $f^{-1}(cl_{if}\,(f(\mathcal{A}_{if})))$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{if})$. Since $\mathcal{A}_{if} \subseteq f^{-1}\left(f(\mathcal{A}_{if})\right) \subseteq$

$f^{-1}(cl_{if}\ (f(\mathcal{A}_{if})))$ and $f^{-1}(cl_{if}\ (f(\mathcal{A}_{if})))$ is an $\mathcal{IF}\widehat{g}^*\mathcal{sCS}$ in $(\mathbb{X},\tau_{if})$, we get $\mathcal{IF}\widehat{g}^*scl_{if}(\mathcal{A}_{if}) \subseteq f^{-1}(cl_{if}\ (f(\mathcal{A}_{if})))$. Hence $f(\mathcal{IF}\widehat{g}^*scl_{if}(\mathcal{A}_{if})) \subseteq cl_{if}\ (f(\mathcal{A}_{if}))$.

(ii) Substituting $f^{-1}(\mathcal{B}_{if})$ for $\mathcal{A}_{if}$ in (i) we get

$$f(\mathcal{IF}\widehat{g}^*scl_{if}\left(f^{-1}(\mathcal{B}_{if})\right)) \subseteq cl_{if}\ (f\left(f^{-1}(\mathcal{B}_{if})\right)) = cl_{if}\ (\mathcal{B}_{if})$$

$$f(\mathcal{IF}\widehat{g}^*scl_{if}\left(f^{-1}(\mathcal{B}_{if})\right)) \subseteq cl_{if}\ (\mathcal{B}_{if})$$

Therefore, $\mathcal{IF}\widehat{g}^*scl_{if}\left(f^{-1}(\mathcal{B}_{if})\right) \subseteq f^{-1}(cl_{if}\ (\mathcal{B}_{if}))$.

**Theorem 4.3:** Let $f:(\mathbb{X},\tau_{if}) \to (\mathbb{Y},\sigma_{if})$ be $\mathcal{IF}\widehat{g}^*\mathcal{s}$- continuous and $g:(\mathbb{Y},\sigma_{if}) \to (\mathbb{Z},\rho_{if})$ be an $\mathcal{IF}$- continuous. Then $g \circ f:(\mathbb{X},\tau_{if}) \to (\mathbb{Z},\rho_{if})$ is $\mathcal{IF}\widehat{g}^*\mathcal{s}$- continuous.

**Proof:** Let $\mathcal{B}_{if}$ be any $\mathcal{IFCS}$ in $(\mathbb{Z},\rho_{if})$. Since $g$ is $\mathcal{IF}$- continuous, $g^{-1}(\mathcal{B}_{if})$ is an $\mathcal{IFCS}$ in $(\mathbb{Y},\sigma_{if})$. Since $f$ is $\mathcal{IF}\widehat{g}^*\mathcal{s}$- continuous mapping $f^{-1}(g^{-1}(\mathcal{B}_{if}))$ is an $\mathcal{IF}\widehat{g}^*\mathcal{sCS}$ in $(\mathbb{X},\tau_{if})$. w.k.t $(g\circ f)^{-1} = f^{-1}(g^{-1}(\mathcal{B}_{if}))$. Therefore $(g\circ f)^{-1}(\mathcal{B}_{if})$ is an $\mathcal{IF}\widehat{g}^*\mathcal{sCS}$ in $(\mathbb{X},\tau_{if})$ for every $\mathcal{IFCS}$ $\mathcal{B}_{if}$ in $(\mathbb{Z},\rho_{if})$. Hence $g\circ f$ is $\mathcal{IF}\widehat{g}^*\mathcal{s}$- continuous

## V. INTUITIONISTIC FUZZY $\widehat{g}^*$ SEMI IRRESOLUTE FUNCTION

**Definition 5.1:** A mapping $f:(\mathbb{X},\boldsymbol{\tau_{if}}) \to (\mathbb{Y},\boldsymbol{\sigma_{if}})$ is called **an intuitionistic fuzzy $\widehat{g}^*$ Semi irresolute ($\mathcal{IF}\widehat{g}^*\mathcal{s}$-irresolute in short) if $f^{-1}(\mathcal{B}_{if})$ is an $\mathcal{IF}\widehat{g}^*\mathcal{sCS}$ in $(X,\tau)$ for every $\mathcal{IF}\widehat{g}^*\mathcal{sCS}$ $\mathcal{B}_{if}$ of $(\mathbb{Y},\sigma_{if})$.**

**Example 5.2:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$, $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$, $\tau_{if} = \{\tilde{0}, \mathcal{A}_{if}, \tilde{1}\}$ and $\sigma_{if} = \{\tilde{0}, \mathcal{B}_{if}, \tilde{1}\}$ where $\mathcal{A}_{if}$={< $\mathbb{e}$, 0.7, 0.3>, <$\mathbb{f}$, 0.6, 0.4>} and $\mathcal{B}_{if}$={< $\mathbb{g}$, 0.8, 0.2 >, < $\mathbb{h}$, 0.7, 0.3> }. Then $\tau_{if}$ and $\sigma_{if}$ are $\mathcal{IFT}$s on spaces $\mathbb{X}$ and $\mathbb{Y}$. We define a mapping $f:(\mathbb{X},\tau_{if}) \to (\mathbb{Y},\sigma_{if})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. $\mathcal{IF}\widehat{g}^*\mathcal{sC}(\mathbb{X},\tau_{if}) = \{\tilde{0}, \mathcal{P}, \tilde{1} / 0 < \mathcal{P} < \mathcal{A}_{if}{}^C\}$ and $\mathcal{IF}\widehat{g}^*\mathcal{sC}(\mathbb{Y},\tau_{if}) = \{\tilde{0}, \mathcal{Q}, \tilde{1} / 0 < \mathcal{Q} < \mathcal{B}_{if}{}^C\}$. Then for every $\mathcal{IF}\widehat{g}^*\mathcal{sCS}$ $\mathcal{N}_{if}$ in $(\mathbb{Y},\sigma_{if})$, $f^{-1}(\mathcal{N}_{if})$ is $\mathcal{IF}\widehat{g}^*\mathcal{sCS}$ in $(\mathbb{X},\tau_{if})$. Hence $f$ is $\mathcal{IF}\widehat{g}^*\mathcal{s}$- irresolute function.

**Theorem 5.3:** Let $f:(\mathbb{X},\tau_{if}) \to (\mathbb{Y},\sigma_{if})$ be an $\mathcal{IF}\widehat{g}^*\mathcal{s}$- irresolute function. Then $f$ is $\mathcal{IF}\widehat{g}^*\mathcal{s}$- continuous function but not conversely.

**Proof:** Let $f:(\mathbb{X},\tau_{if}) \to (\mathbb{Y},\sigma_{if})$ be an $\mathcal{IF}\widehat{g}^*\mathcal{s}$- irresolute function. Let $\mathcal{A}_{if}$ be an $\mathcal{IFCS}$ in $(\mathbb{Y},\sigma_{if})$. w.k.t., every $\mathcal{IFCS}$ is an $\mathcal{IF}\widehat{g}^*\mathcal{sCS}$. Therefore $\mathcal{A}_{if}$ is an $\mathcal{IF}\widehat{g}^*\mathcal{sCS}$ in $(\mathbb{Y},\sigma_{if})$. By hypothesis $f^{-1}(\mathcal{A}_{if})$ is an $\mathcal{IF}\widehat{g}^*\mathcal{sC}$ set in $(\mathbb{X},\sigma_{if})$. Hence $f$ is $\mathcal{IF}\widehat{g}^*\mathcal{s}$- continuous function.

**Example 5.4:** Let $\mathbb{X} = \{\mathbb{e}, \mathbb{f}\}$, $\mathbb{Y} = \{\mathbb{g}, \mathbb{h}\}$, $\tau_{if} = \{\tilde{0}, \mathcal{A}_{if}, \tilde{1}\}$ and $\sigma_{if} = \{\tilde{0}, \mathcal{B}_{if}, \tilde{1}\}$ where $\mathcal{A}_{if}$={< $\mathbb{e}$, 0.2, 0.8>, <$\mathbb{f}$, 0.3, 0.7>} and $\mathcal{B}_{if}$={< $\mathbb{g}$, 0.5, 0.4 >, < $\mathbb{h}$, 0.6, 0.4> }. Then $\tau_{if}$ and $\sigma_{if}$ are $\mathcal{IFT}$s on spaces $\mathbb{X}$ and $\mathbb{Y}$. We define a mapping $f:(\mathbb{X},\tau_{if}) \to (\mathbb{Y},\sigma_{if})$ by $f(\mathbb{e}) = \mathbb{g}$ and $f(\mathbb{f}) = \mathbb{h}$. Then $f$ is an $\mathcal{IF}\widehat{g}^*\mathcal{s}$-continuous mapping. Since $\mathcal{N}_{if} = \{<\mathbb{g}, 0.4, 0.5>, <\mathbb{h}, 0.4, 0.6>\}$ is an $\mathcal{IFCS}$ in $(\mathbb{Y},\sigma_{if})$ and $f^{-1}(\mathcal{N}_{if})$ is an $\mathcal{IF}g^*\mathcal{sCS}$ in $(\mathbb{X},\tau_{if})$. But $f$ is not an $\mathcal{IF}\widehat{g}^*\mathcal{s}$-irresolute function. Because $\mathcal{M}_{if} = \{<\mathbb{g}/0.2, 0.7>, <\mathbb{h}/0.2, 0.8>\}$ is an $\mathcal{IF}g^*\mathcal{sCS}$ in $(\mathbb{Y},\tau_{if})$ and $f^{-1}(\mathcal{M}_{if})$ is not an $\mathcal{IF}g^*\mathcal{sCS}$ in $(\mathbb{X},\tau_{if})$.

**Proposition 5.5:** If $f: (\mathbb{X}, \tau_{\text{if}}) \to (\mathbb{Y}, \sigma_{\text{if}})$ is $\mathcal{IF}\hat{g}^*s$- irresolute iff $f^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}g^*s\mathcal{OS}$ in $(\mathbb{X}, \tau_{\text{if}})$ for every $\mathcal{IF}g^*s\mathcal{OS}$ $\mathcal{N}_{\text{if}}$ of $(\mathbb{Y}, \tau_{\text{if}})$.

**Proof:** Let $f: (\mathbb{X}, \tau_{\text{if}}) \to (\mathbb{Y}, \sigma_{\text{if}})$ be an $\mathcal{IF}\hat{g}^*s$- irresolute function. Let $\mathcal{N}_{\text{if}}$ be an $\mathcal{IF}g^*s\mathcal{OS}$ in $(\mathbb{Y}, \tau_{\text{if}})$. Then ${\mathcal{N}_{\text{if}}}^{C}$ is an $\mathcal{IF}g^*s\mathcal{CS}$ in $(\mathbb{Y}, \tau_{\text{if}})$. By hypothesis we can say $f^{-1}({\mathcal{N}_{\text{if}}}^{C})$ is an $\mathcal{IF}g^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. But $f^{-1}({\mathcal{N}_{\text{if}}}^{C}) = (f^{-1}(\mathcal{N}_{\text{if}}))^{C}$. Therefore $(f^{-1}(\mathcal{N}_{\text{if}}))^{C}$ $\mathcal{IF}g^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Hence $f^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}g^*s\mathcal{OS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Converse is similar.

**Theorem 5.6:** Let $f: (\mathbb{X}, \tau_{\text{if}}) \to (\mathbb{Y}, \sigma_{\text{if}})$ and $g: (\mathbb{Y}, \sigma_{\text{if}}) \to (\mathbb{Z}, \rho_{\text{if}})$ be any two mappings. Then

(i) $g \circ f$ is $\mathcal{IF}\hat{g}^*s$- irresolute if both $f$ and $g$ are $\mathcal{IF}\hat{g}^*s$- irresolute.
(ii) $g \circ f$ is $\mathcal{IF}\hat{g}^*s$- continuous if $g$ is $\mathcal{IF}\hat{g}^*s$- continuous and $f$ is $\mathcal{IF}\hat{g}^*s$- irresolute.
(iii) $g \circ f$ is $\mathcal{IF}\hat{g}^*s$- continuous if $g$ is $\mathcal{IF}s$- continuous and $f$ is $\mathcal{IF}\hat{g}^*s$- irresolute.
(iv) $g \circ f$ is $\mathcal{IF}\hat{g}^*s$- continuous if $g$ is $\mathcal{IF}$- continuous and $f$ is $\mathcal{IF}\hat{g}^*s$- irresolute.
(v) $g \circ f$ is $\mathcal{IF}\hat{g}^*s$- continuous if $g$ is $\mathcal{IF}g^*s$ - continuous and $f$ is $\mathcal{IF}\hat{g}^*s$- irresolute.

**Proof: (i)** Let $\mathcal{N}_{\text{if}}$ be an $\mathcal{IF}g^*s\mathcal{CS}$ in $(\mathbb{Z}, \rho_{\text{if}})$. Since $g$ is $\mathcal{IF}\hat{g}^*s$- irresolute, $g^{-1}(\mathcal{N}_{\text{if}})$ is $\mathcal{IF}g^*s\mathcal{CS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. Since $f$ is $\mathcal{IF}\hat{g}^*s$- irresolute, $f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. w.k.t $(g \circ f)^{-1} = f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$. Therefore $(g \circ f)^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Hence $g \circ f$ is $\mathcal{IF}\hat{g}^*s$- irresolute.

**(ii)** Let $\mathcal{N}_{\text{if}}$ be an $\mathcal{IFCS}$ in $(\mathbb{Z}, \rho_{\text{if}})$. Since $g$ is $\mathcal{IF}\hat{g}^*s$-continuous, $g^{-1}(\mathcal{N}_{\text{if}})$ is $\mathcal{IF}g^*s\mathcal{CS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. Since $f$ is $\mathcal{IF}\hat{g}^*s$- irresolute, $f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. w.k.t $(g \circ f)^{-1} = f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$. Therefore $(g \circ f)^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Hence $g \circ f$ is $\mathcal{IF}\hat{g}^*s$- irresolute.

**(iii)** Let $\mathcal{N}_{\text{if}}$ be an $\mathcal{IFCS}$ in $(\mathbb{Z}, \rho_{\text{if}})$. Since $g$ is $\mathcal{IF}s$- continous, $g^{-1}(\mathcal{N}_{\text{if}})$ is $\mathcal{IF}s\mathcal{CS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. But every $\mathcal{IF}s\mathcal{CS}$ is $\mathcal{IF}\hat{g}^*s\mathcal{CS}$. Therefore $g^{-1}(\mathcal{N}_{\text{if}})$ $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. Since $f$ is $\mathcal{IF}\hat{g}^*s$- irresolute, $f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. w.k.t $(g \circ f)^{-1} = f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$. Therefore $(g \circ f)^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Hence $g \circ f$ is $\mathcal{IF}\hat{g}^*s$-continouous.

**(iv)** Let $\mathcal{N}_{\text{if}}$ be an $\mathcal{IFC}$ set in $(\mathbb{Z}, \rho_{\text{if}})$. Since $g$ is $\mathcal{IF}$- continous, $g^{-1}(\mathcal{N}_{\text{if}})$ is $\mathcal{IFCS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. But every $\mathcal{IFCS}$ is $\mathcal{IF}\hat{g}^*s\mathcal{CS}$. Therefore $g^{-1}(\mathcal{N}_{\text{if}})$ $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. Since $f$ is $\mathcal{IF}\hat{g}^*s$- irresolute, $f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. w.k.t $(g \circ f)^{-1} = f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$. Therefore $(g \circ f)^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Hence $g \circ f$ is $\mathcal{IF}\hat{g}^*s$-continouous.

**(v)** Let $\mathcal{N}_{\text{if}}$ be an $\mathcal{IFCS}$ in $(\mathbb{Z}, \rho_{\text{if}})$. Since $g$ is $\mathcal{IF}g^*s$-continuous, $g^{-1}(\mathcal{N}_{\text{if}})$ is $\mathcal{IF}g^*s\mathcal{CS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. But every $\mathcal{IF}g^*s\mathcal{CS}$ is $\mathcal{IF}\hat{g}^*s\mathcal{CS}$. Therefore $g^{-1}(\mathcal{N}_{\text{if}})$ $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{Y}, \sigma_{\text{if}})$. Since $f$ is $\mathcal{IF}\hat{g}^*s$-irresolute, $f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. w.k.t $(g \circ f)^{-1} = f^{-1}(g^{-1}(\mathcal{N}_{\text{if}}))$. Therefore $(g \circ f)^{-1}(\mathcal{N}_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{X}, \tau_{\text{if}})$. Hence $g \circ f$ is $\mathcal{IF}\hat{g}^*s$-continouous.

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