Research Article

DIFFERENTIAL-ALGEBRAIC EQUATION FOR DECENTRALIZED SLIDING CONTROL METHOD

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ABSTRACT

A Differential-Algebraic Equations (DAE) system is a system of equations that either contains differential equations and algebraic equations, or is equivalent to such a system. In mathematics these are examples of differential algebraic varieties and correspond to ideals in differential polynomial rings. It is used to control the method to simulate the system, while satisfying the algebraic constraints which is formulated. Because of the large amount of computation and communication associated with large scale matrix inversion problems in the existing centralized approaches, this new distributed method is much more efficient. Therefore, by using this novel method for distributed simulation of Differential Algebraic Equation systems is developed based on purely decentralized sliding mode is controlled for applying simulation of a multiple- pendulum system. Hence, this method performs better results interms of performance and stability.

Keywords: Algebra, Differential-Algebraic Equations (DAE), Distribution

INTRODUCTION

Various applications involve mixed systems of Differential and Algebraic Equations (DAE's). For instance, Gear's basic article was stimulated by problems from network analysis and continuous system simulation. A different example occurs in the mathematical modeling of electrophoretic separation processes and further DAE's are found in connection with certain problems in nonlinear mechanics. nonlinear mechanics [1]. In many cases, DAE's can be solved efficiently by means of standard numerical methods for Ordinary Differential Equations (ODE's). But DAE's also have properties which may cause such ODE-solvers to run into difficulties or failures. Some interesting results are presented about the causes o f such difficulties in the case of a class of linear DAE's.

The techniques used in these studies are algebraic in nature and do not provide complete information about the existence and uniqueness of solutions [2].

Differential Algebraic Equation (DAE) systems provide a more general description of dynamical systems than Ordinary Differential Equations (ODEs). However, DAE systems present a number of difficulties in simulation and control [3]. The main problem is that most methods require an explicit state variable model. One method to address this problem is to reformulate the DAE system into an equivalent nonlinear control problem, in which the algebraic constraints are satisfied by sliding manifolds. This approach results in a state space approximation to the DAE [4]. A robust sliding mode controller can be designed to achieve a reasonable approximation error. However, when the order of the system is very large, it is computationally too expensive to control the system on a single computer. The sliding mode simulation method is inherently centralized due to the input decoupling Jacobian matrix which must be inverted at each time step. This can cause significant problems since the matrix inversion problem is difficult to divide onto multiple processors. In this paper, the problem mentioned above is investigated, and a method based on decentralized sliding mode is proposed to apply to the state space approximation of the DAE. The system is decoupled into many subsystems, and a separate sliding mode controller is designed for each subsystem on a separate computer. Since these subsystems have some common states, linear controllers are designed to make the errors between the common states in different subsystems zero. Because the internal dynamic of the system is stable, the decentralized sliding controller makes the whole system stable [5].

This method makes the distributed control possible, while eliminating the interconnection between the decentralized computers, which in turn substantially decreases the information flow rate. Application to simulation of a double pendulum system confirms the validity of the proposed method [6]. Sliding-Mode Control (SMC) is one of the robust and nonlinear control methods. Systematic design procedure of the method provides a straightforward solution for the control input. The method has several advantages such as robustness against matched external disturbances and unpredictable parameter variations.

LITERATURE SURVEY

Xuehui Chen, Liang Wei, Jizhe Sui, Xiaoliang Zhang and Liancun Zheng, et.al [7] generalized differential transform method is implemented for solving several linear fractional partial differential equations arising in fluid mechanics. This method is based on the two-dimensional Differential Transform Method (DTM) and generalized Taylor's formula. Numerical illustrations of the time-fractional diffusion equation and the time-fractional wave equation are investigated to demonstrate the effectiveness of this method. Results obtained by using the scheme presented here agree well with the analytical solutions and the numerical results presented elsewhere. The results reveal the method is feasible and convenient for handling approximate solutions of linear or nonlinear fractional partial differential equations.

N. Kumaresan, K. Ratnavelu and B. R. Wong, et.al [8] optimal control for Fuzzy linear Partial Differential Algebraic Equations (FPDAE) with quadratic performance is obtained using Simulink. By using the method of lines, the FPDAE is transformed into a Fuzzy Differential Algebraic Equations (FDAE). Hence, the optimal control of FPDAE can be found out by finding the optimal control of the corresponding FDAE. The goal is to provide optimal control with reduced calculus effort by the solutions of the Matrix Riccati Differential Equation (MRDE) obtained from Simulink. Accuracy of the solution of the Simulink approach to the problem is qualitatively better. The advantage of the proposed approach is that, once the Simulink model is constructed, it allows to evaluate the solution at any desired number of points spending negligible computing time and memory and the solution curves can be obtained from the model without writing any code. An illustrative numerical example is presented for the proposed method.

K. Kittipeerachon, N. Hori and Y. Tomita, et.al [9] exact method is presented for discretizing a constant-coefficient, non-square, matrix differential Riccati equation, whose solution is assumed to exist. The resulting discrete-time equation gives the values that have no error at discrete-time instants for any discrete-time interval. The method is based on a matrix fractional transformation, which is more general than existing ones, for linearizing the differential Riccati equation. A numerical example is presented to compare the proposed method with that based on gage invariance and bilinearization, which has better performances than the conventional forward-difference method.

X. Mao, X. Zhang and H. Zhou, et.al [10] well-known R0 implication is developed to pseudo-De Morgan algebras, which is called generalized pseudo-R0 implication. The notion of strong pseudo-De Morgan algebras is introduced, and its elementary properties are discussed. Secondly, two necessary and sufficient conditions are proved as follows: (1) A pseudo-De Morgan algebra A with generalized pseudo-R0 implication becomes a pseudo-involutive pseudo-BCK algebra if and only if A is a strong pseudo-De Morgan algebra. (2) A pseudo-De Morgan algebra A with generalized pseudo-R0 implication and corresponding operator becomes a pseudo-regular residuated lattice if and only if A is a strong pseudo-De Morgan algebra. Finally, all pseudo-De Morgan algebras, strong pseudo-De Morgan algebras and proper pseudo-involutive pseudo-BCK algebras are obtained by MATLAB software when the order number is smaller than or equal to 8. Furthermore, starting with bounded distributive lattices, we discussed the classification problem of lower-order pseudo-involutive pseudo-BCK algebras.

F. -a. Deng, T. Chen and S. Ren, et.al [11] aim of this study is Wd -fuzzy implication algebras which are subalgebra of fuzzy implication algebras. We showed that Wd -fuzzy implication algebras are regular fuzzy implication algebras, but the inverse is not true. The relations between Wd -fuzzy implication algebras and other fuzzy algebras are discussed. Properties and axiomatic systems for Wd -fuzzy implication algebras are investigated. Furthermore, a few new results on Wd -fuzzy implication algebras has been added.

J. Liu and W. Chen,et.al [12] introduce and investigate a non-commutative generalization of quasi-MV algebras, called pseudo-quasi-MV algebras (pseudo-qMV algebras for short). And then we characterize the bijective relation between ideal congruences and normal ideals of a pseudo-qMV algebra. Finally, we prove that pseudo-qMV algebras are categorically equivalent to pseudo-quasi-Wajsberg algebras which are the non-commutative generalization of quasi-Wajsberg algebras.
A. Pedram., et.al [13] show the design of specialized compute fabrics that maintain the efficiency of full custom hardware while providing enough flexibility to execute a whole class of coarse-grain linear algebra operations. The broad vision of this project is to develop integrated and specialized hardware/software solutions that are co-optimized and co-designed across all layers ranging from the basic hardware foundations all the way to the application through standard linear algebra packages. We have designed a specialized Linear Algebra Processor (LAP) that can perform level-3 BLAS and more complex LAPACK level operations like Cholesky, LU (with partial pivoting), and QR factorizations. We present a power performance model that compares state of the art CPUs and GPUs with our design. Our power model reveals sources of inefficiencies in CPUs and GPUs, and our

LAP design demonstrates how to overcome them. When compared to other conventional architectures for linear algebra applications, LAP is over orders of magnitude more power efficient. Based on our estimations up to 55 and 25 GFLOPS/W single- and double-precision efficiencies are achievable on a single chip in standard 45nm technology.

X. Wu, X. Liu, Li Zhou and J. Zhang, et.al [14] define a lower approximate operation and an upper approximate operation based on a Boolean partition on R 0 -algebras and discuss their properties. We then introduce a pair of belief measure and plausibility measure on R 0 -algebras and investigate the relationship between rough operations and belief measure and plausibility measure.

P. S. Budyakov, N. I. Chernov, V. Y. Yugai and N. N. Prokopenko,et.al [15] mathematical basics of the non-classical approach to the logical synthesis of k-valued digital structures based on the replacement of the classic mathematical apparatus of logic synthesis (Boolean algebra) to the proposed mathematical apparatus - linear algebra are considered. The logic synthesis process of two valued and multi-valued digital structures in linear algebra including the formation of bases of a linear space and original representation of the implemented logical function are discussed. Mathematical advantages of the proposed approach, which could be the basis for designing of high-speed digital logic structures for various applications are considered.

3. METHODOLGY

In this section, a design approach is proposed. Distributed Formulation of the System is Consider the DAE system. Assuming that the index of the system (with respect to each input) is three, the overall **Js** matrix of this system is obtained by.

$$J_s = \mu^2 . \frac{\partial}{\partial x}\left(\frac{\partial g}{\partial x} . f\right)\frac{\partial f}{\partial z} \qquad \text{---- (1)}$$

When the dimension of the system increases considerably, it is a significant problem to calculate the inverse of matrix Js on a single computer according to the timing restrictions of the real-time simulation. In this case, it is beneficial to decompose the system into some subsystems, and the Js matrix of each subsystem, which has a lower dimension compared with the overall system, is calculated on a separate computer. In order to decompose the overall system, the set of boundary algebraic equations is introduced.

The set of boundary algebraic equations is the set of algebraic equations that by eliminating them, the DAE system {(1),(2)} would be decomposed into two DAE subsystems which are independent from each other.

The schematic of a set of boundary algebraic equations is shown in Fig.1.

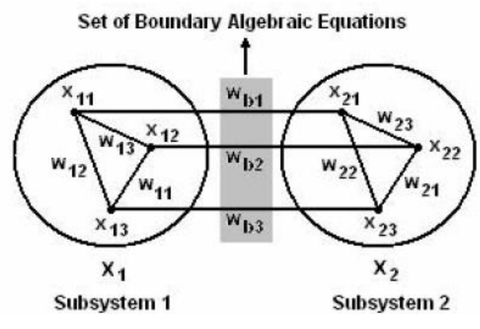


Figure 1: The Schematic of A Set Of Boundary Algebraic Equations.

In this figure, x_{ij} and w_{ij} 's are the states and algebraic equations, respectively. The set of boundary algebraic equations Wb = $\{w_{b1}, w_{b2}, w_{b3}\}$ decomposes the whole system into the two subsystems $X1 = [\, x_{11}\ x_{12}\ x_{13}]^T$ and $X2 = [x_{21}\ x_{22}\ x_{23}]T$, with the corresponding algebraic equations $W1 = [w11\ w12\ w13]^T$ and $W2 = [\, w_{21}\ w_{22}\ w_{23}]^T$, respectively.

Proposition.1: The DAE system can be reformulated as:

$X_1 = f_1 (X_1, Z_1, Z_b)$

$\dot{X}_2 = f_1 (X_1, Z_1, Z_b)$

$\hat{Z}_1 = V_1$

$\hat{Z}_2 = V_2$

$\hat{Z}_b = V_b$

$W_1 = g_1 (X_1)$

$W_1 = g_1 (X_1)$

$W_2 = g_2 (X_2)$

$W_b = g_b (X_1, X_2)$ ---- (2)

where $\mathbf{x}_i$, **v**i and $\mathbf{w}_i$'s are the state, input and output vectors, respectively. $\mathbf{f}_i$'s and $\mathbf{g}_i$'s are the vector functions with appropriate dimensions. Also, the dimensions of all the vectors and matrices are well defined. In equations (2), the set of equations $\mathbf{w}_b$ are the boundary algebraic equations. Eliminating these equations leads to the two subsets of DAE systems:

$\dot{X}_2 = f_1 (X_1, Z_1, Z_3)$

$\hat{Z}_1 = V_1$

W1 = g1 (X1) --- (3)

and

$\dot{X}_2 = f_2 (X_1, Z_1, Z_3)$

$\hat{Z}_2 = V_2$

W2 = g2 (X2)---- (4)

For the boundary algebraic equations, instead of one integrator, two integrators are applied to the corresponding input $\mathbf{v}_3$ to make the parameter **z** 3:
$\hat{\mathbf{z}}_b = \mathbf{v}_b$

$\mathbf{w}b = \mathbf{g}b\,(\mathbf{x}1, \mathbf{x}2)$ --- (5)

The set of equations {(3),(4),(5)} is the **distributed realization** of the DAE system. Assuming that the index of each subsystem (3) and (4) (with respect to each input - excluding the ones corresponding to the set of boundary algebraic equations) is three, the corresponding matrix **J**s for each of the subsystems (3) and (4) is calculated.

$$\boldsymbol{J}_s\mathbf{1} = \mu^2 . \frac{\partial}{\partial \boldsymbol{x}\mathbf{2}} \left(\frac{\partial \boldsymbol{g}^1}{\partial \boldsymbol{x}_1} . \boldsymbol{f}_1 \right) \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{z}_1}$$

$$\boldsymbol{J}_s\mathbf{2} = \mu^2 . \frac{\partial}{\partial \boldsymbol{x}\mathbf{2}} \left(\frac{\partial \boldsymbol{g}_2}{\partial \boldsymbol{x}_2} . \boldsymbol{f}_2 \right) \frac{\partial \boldsymbol{f}_2}{\partial \boldsymbol{z}_2}$$

-------- (6)

And assuming that the index of the set of boundary algebraic equations (5) (with respect to each corresponding input) is four (because of the extra integrator in the corresponding input), the **J**s for the boundary algebraic equations is calculated individually.

$J_s b= \mu^{3} \cdot \frac{\partial}{\partial x2}\left(\frac{\partial g^{1}}{\partial x_{1}} \cdot \boldsymbol{f}_{1}\right)\frac{\partial f_{1}}{\partial z6}+\frac{\partial}{\partial x2}\left(\frac{\partial g_{2}}{\partial x_{2}} \cdot \boldsymbol{f}_{2}\right)\frac{\partial f_{2}}{\partial z6}$

--- (7)

Decentralized sliding mode approach is useful in a variety of control fields. Because of the simplicity of the controller, it has been applied to many control applications. In order to control the distributed formulation of the system, decentralized sliding control is applicable.

4. RESULT ANALYSIS

In this section, the decentralized (distributed realization) and centralized sliding control approaches are applied in order to compare the performances.

Table.1: Performance Analysis

Parameters	Centralized	Decentralized
Performance	85	97
Stability	91	99

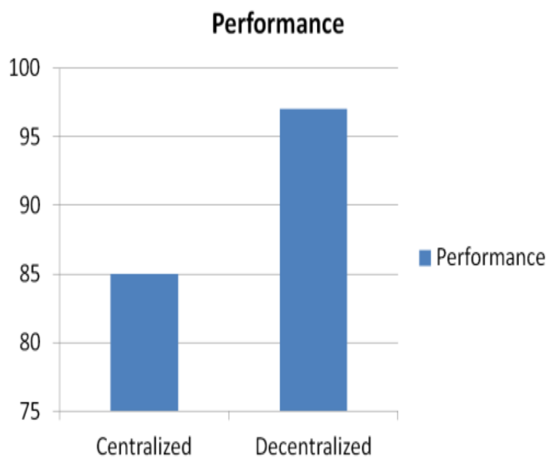
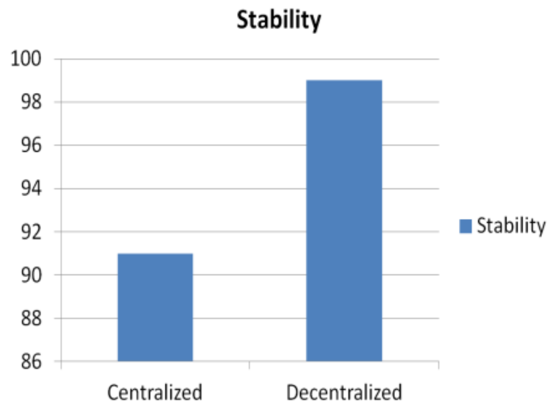


Figure 2: Performance Comparison Graph

In Fig.2 performance comparison graph is seen between centralized and decentralized.



In Fig.3 stability comparison graph is seen between centralized and decentralized.

5. CONCLUSIONS

A Differential-Algebraic Equations (DAE) system is a system of equations that either contains differential equations and algebraic equations, or is equivalent to such a system. In mathematics these are examples of differential algebraic varieties and correspond to ideals in differential polynomial rings. It is used to control the method to simulate the system, while satisfying the algebraic constraints which is formulated. This method divided the main system into numerous subsystems, and a sliding mode controller was designed for each subsystem individually. The necessary conditions for stability were developed. The new method was applied to a mutliple pendulum problem which demonstrated good performance and stability.

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