MTBF of a Repairable Units with CCS Failures and M L Estimation

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ABSTRACT

In this article, we discussed the method of maximum likelihood (M L) estimation to assess the estimates of mean time between failures (MTBF) of a redundant system. 2-unit identical system in case of series and parallel modes subject to individual, lethal and non-lethal common cause shock (LCCS & NCCS) failures is studied. System life times and repair times of the units are exponential. We developed the estimates of mean time between failures functions in case of series and parallel systems in the time off of analytical approach. We have presented the validity of the resulting estimates with the help of simulation.

Keywords: Reliability, Estimation, Mean Time between Failures, Common Cause Shock failures, Simulation

1. INTRODUCTION

Common Cause Shock (CCS) failures induce simultaneous failure in the system and drastically reduce reliability of the system. These events may be outside atmosphere such as fire, thunderstorm, lightning, flood, earthquake, error due to human interventions etc. Occur at random times producing simultaneous failure of several components. The reliability analysts and researchers were considered two types of CCS failures in literature. (i) Lethal common cause shocks results failure of all the components in the system and (ii) A non-lethal common cause shocks results failure of some components at random. Reliability analysts were discussed them in the evaluation of reliability measures and routine of the system very much. The CCS failure models can be found in the literature by Billinton and Allan [2]. Some other reliability models with CCS failures were analyzed by Chari et al [3], Verma [9], Sagar et al [6].

Mathematical modelling, estimation and life testing is vital interest in order to measure mean life of the system. In truth life testing experiments are intended to measure the average life of the units and also fascinated to counter such questions as 'what is the probability that the system will down in the interval (0, t). Awgichew et al [1] derived M L estimates of availability measures for two unit system with CCS failures and also established simulation study on reliability estimates of a repairable system. Sreedhar et al [8] proposed maximum likelihood estimation approach for estimating reliability indices of a two unit system in the presence of CCS failures. Levitin [4] proposed the universal generating function of multi-state system dependability analysis to include CCS failures. A procedure is proposed to evaluate reliability functions of no repairable series-parallel multistate systems under the influence of CCS failures. Reddy [5] developed reliability measures with the effect of lethal and non-lethal common cause shock failures of a two unit non-identical system.

The present investigation is to integrate M L estimation approach in Sagar et al [7]. We estimate the M L estimates of mean time between failure functions for series system as well as parallel system.

2. ASSUMPTIONS

We consider a two component identical system.

i) The units fail individually and also simultaneously due to two kinds of CCS (lethal or non-lethal) failures in Poisson fashion.
ii) Individual, LCCS and NCCS failures are independent to each other.
iii) There is a single repair facility to repair failed units whether they are failed individually or simultaneously due to common cause shock.
iv) Repair times of failed units depend on the failure mode and are assumed exponentially distributed.

3. NOTATIONS

λ : Failure rate (individual)

ω : LCCS failure rate

β : NCCS failure rate

μ_1 : repair rate when one unit is down and other one is working

μ_2 : Repair rate when second has failed whereas one unit was already down

μ_c : repair rate when both units fail simultaneously

$p(q)$: The probability of simultaneous failures of units due to NCCS (LCCS)

$R_{LNS}(t)$: Reliability of the series system in (0, t) with LCCS and NCCS failures

$R_{LNP}(t)$: Reliability of the parallel system in (0, t) with LCCS and NCCS failures

$E_{LNS}(T)$: MTBF of the system with LCCS and NCCS failures when units are in series

$E_{LNP}(T)$: MTBF of parallel system in case of LCCS and NCCS failures

$\hat{E}_{LNS}(T)$: M L Estimate of MTBF function for series system under the influence of LCCS & NCCS failure mode

$\hat{E}_{LNP}(T)$: Estimate of MTBF function for parallel system in the presence of LCCS and NCCS failures

$\bar{x}, \bar{y}, \bar{w}$: Sample means of individual, NCCS and LCCS failure occurrence

$\bar{z}$: Average repair times

$\hat{\bar{x}}, \hat{\bar{y}}, \hat{\bar{w}}$: Estimates for individual, NCCS and LCCS failure rates of occurrence respectively

$\hat{\bar{z}}$: Estimate of repair times

n : Smple size

N : Simulated samples

MSE : Mean Square Error

4. STOCHASTIC MODEL

Under the stated assumptions Markov model can be formulated to derive the mean time between failure function E (T) under the influence of individual as well as CCS failures and the state transition diagram is given in Figure.1. The numerals in Figure.1 denote the system state.

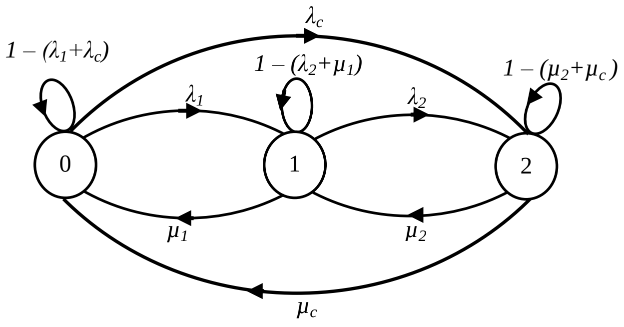

Figure.1 State transition diagram

The quantities in the above model are as follows:

$$\lambda_1 = 2(\lambda + \beta pq)$$

$$\lambda_2 = (\beta p^2 + \omega)$$

$$\lambda_c = (\lambda + \beta p)$$

Mathematical model

Based on the arguments of stochastic theory, we can develop the set of differential equations associated with the existing mathematical model for the above mentioned state transition diagram.

$$P_0^{'}(t) = -(\lambda_1 + \lambda_c)P_0(t) + \mu_1 P_1(t) + \mu_c P_2(t) \qquad (1)$$

$$P_1^{'}(t) = \lambda_1 P_0(t) - (\lambda_2 + \mu_1)P_1(t) + \mu_2 P_2(t) \qquad (2)$$

$$P_2^{'}(t) = \lambda_c P_0(t) + \lambda_2 P_1(t) - (\mu_2 + \mu_c)P_2(t) \qquad (3)$$

Initial conditions, $P_0(0) = 1$, and other state probabilities are zero at $t = 0$ (4)

Taking Laplace transformation of equations (1) to (3) and using equation (4), we obtain

$$P_0(t) = \frac{r_1^2 + r_1 l_1 + m_1}{r_1(r_1 - r_2)}\exp(r_1 t) - \frac{r_2^2 + r_2 l_1 + m_1}{r_2(r_1 - r_2)}\exp(r_2 t) + \frac{m_1}{r_1 r_2} \qquad (5)$$

$$P_1(t) = \frac{r_1 l_2 + m_2}{r_1(r_1 - r_2)}\exp(r_1 t) - \frac{r_2 l_2 + m_2}{r_2(r_1 - r_2)}\exp(r_2 t) + \frac{m_2}{r_1 r_2} \qquad (6)$$

$$P_2(t) = \frac{r_1 l_3 + m_3}{r_1(r_1 - r_2)}\exp(r_1 t) - \frac{r_2 l_3 + m_3}{r_2(r_1 - r_2)}\exp(r_2 t) + \frac{m_3}{r_1 r_2} \qquad (7)$$

Where

$$l_1 = \lambda_2 + \mu_1 + \mu_2 + \mu_c$$

$$l_2 = \lambda_1$$

$$l_3 = \lambda_c$$

$$m_1 = \lambda_2 \mu_c + \mu_1 \mu_c + \mu_1 \mu_2$$

$$m_2 = \lambda_1 \mu_c + \lambda_1 \mu_2 + \lambda_c \mu_2$$

$$m_3 = \lambda_2 \mu_c + \lambda_c \mu_1 + \lambda_1 \lambda_2$$

$$r_1, r_2 = -(\lambda_1 + \lambda_2 + \lambda_c + \mu_1 + \mu_2 + \mu_c) \pm sqrt[(\lambda_1 + \lambda_2 + \lambda_c + \mu_1 + \mu_2 + \mu_c)^2 - 4(\mu_2(\lambda_1 + \lambda_c + \mu_1) + \mu_c(\lambda_1 + \lambda_2 + \mu_1) + \lambda_2(\lambda_1 + \lambda_c) + \lambda_c + \mu_1)] \qquad (8)$$

5. ESTIMATION OF MEAN TIME BETWEEN FAILURES

In this section, we have used maximum likelihood estimation approach to estimate the reliability measures such as mean time between failures of two unit repairable systems in the presence of NCCS and LCCS failures for both series and parallel cases.

Let the samples $x_1, x_2, ..., x_n$; $y_1, y_2, ..., y_n$ and $w_1, w_2, ..., w_n$ with size '*n*' representing times between individual, NCCS and LCCS failures which will obey exponential population respectively.

Let $z_{11}, z_{12}, ..., z_{1n}$; $z_{21}, z_{22}, ..., z_{2n}$ & $z_{31}, z_{32}, ..., z_{3n}$ be '*n*' number of times between repairs of the units with exponential population law.

Where, $\hat{\bar{x}}, \hat{\bar{y}}, \hat{\bar{w}}, \hat{\bar{z}}_1, \hat{\bar{z}}_2$ & $\hat{\bar{z}}_3$ are the M L estimates of individual failure rate λ, NCCS failure rate (β), LCCS failure rate (ω) and repair rates of (μ_1, μ_2, μ_c) of the system respectively.

Where, $\hat{\hat{x}} = \frac{1}{\bar{x}}; \hat{\hat{y}} = \frac{1}{\bar{y}}; \hat{\hat{w}} = \frac{1}{\bar{w}}; \hat{\hat{z}}_1 = \frac{1}{\bar{z}_1};$

$$\hat{\hat{z}}_2 = \frac{1}{\bar{z}_2}; \hat{\hat{z}}_3 = \frac{1}{\bar{z}_3}; \bar{x} = \frac{\sum x_i}{n}; \bar{y} = \frac{\sum y_i}{n}$$

$$\bar{w} = \frac{\sum w_i}{n}; \bar{z}_1 = \frac{\sum z_{1i}}{n}; \bar{z}_2 = \frac{\sum z_{2i}}{n}; \bar{z}_3 = \frac{\sum z_{3i}}{n}$$

5.1 Series System

In this case, we have reliability function for two unit system with LCCS and NCCS failures as

$$R_{LNS}(t) = P_0(t)$$

$$= \frac{1}{(r_1 - r_2)}\left[(\lambda_2 + r_1)\exp(r_1 t) - (\lambda_2 + r_2)\exp(r_2 t)\right] \quad (9)$$

Where r_1, r_2 obtained in (8)

The mean time between failure function is

$$E_{LNS}(T) = \int_0^\infty R_{LNS}(t).dt$$

$$= \lambda_2 / [\lambda_2(\lambda_1 + \lambda_c) + \mu_2(\lambda_1 + \lambda_c + \mu_1) + \mu_c(\lambda_1 + \lambda_2 + \mu_1) + \lambda_c \mu_1] \quad (10)$$

Therefore, the M L estimate of MTBF function is given by

$$\hat{E}_{LNS}(T) = (\hat{\hat{y}}p^2 + \hat{\hat{w}}) / [(\hat{\hat{y}}p^2 + \hat{\hat{w}})(2(\hat{\hat{x}} + \hat{\hat{y}}pq) + \hat{\hat{x}} + \hat{\hat{y}}p) + \hat{\hat{z}}_2(2(\hat{\hat{x}} + \hat{\hat{y}}pq) + \hat{\hat{x}} + \hat{\hat{y}}p + \hat{\hat{z}}_1) + \hat{\hat{z}}_3 (2(\hat{\hat{x}} + \hat{\hat{y}}pq) + \hat{\hat{y}}p^2 + \hat{\hat{w}} + \hat{\hat{z}}_1) + (\hat{\hat{x}} + \hat{\hat{y}}p)\hat{\hat{z}}_1] \quad (11)$$

Whereas $\hat{\hat{x}}, \hat{\hat{y}}, \hat{\hat{w}}, \hat{\hat{z}}_1, \hat{\hat{z}}_2$ & $\hat{\hat{z}}_3$ are the sample estimates.

5.2 Parallel System

In this case, we have reliability function for two unit system with LCCS and NCCS failures as

$$R_{LNP}(t) = \frac{1}{(r_1 - r_2)}[(\lambda_1 + \lambda_2 + r_1)\exp(r_1 t) - (\lambda_1 + \lambda_2 + r_2)\exp(r_2 t)] \quad (12)$$

Where r_1 and r_2 are seen in (8)

The mean time between failure function is

$$E_{LNP}(T) = \int_0^\infty R_{LNP}(t).dt$$

$$= (\lambda_1 + \lambda_2) / [\mu_2(\lambda_1 + \lambda_c + \mu_1) + \mu_c (\lambda_1 + \lambda_2 + \mu_1) + \lambda_2(\lambda_1 + \lambda_c) + \lambda_c \mu_1] \quad (13)$$

In this case, the M L estimate of MTBF function is given by

$$\hat{E}_{LNP}(T) = [2(\hat{\hat{x}} + \hat{\hat{y}}pq) + (\hat{\hat{y}}p^2 + \hat{\hat{w}})] / [\hat{\hat{z}}_2(2(\hat{\hat{x}} + \hat{\hat{y}}pq) + \hat{\hat{x}} + \hat{\hat{y}}p + \hat{\hat{z}}_1) + \hat{\hat{z}}_3(2(\hat{\hat{x}} + \hat{\hat{y}}pq) + \hat{\hat{y}}p^2 + \hat{\hat{w}} + \hat{\hat{z}}_1) + (\hat{\hat{y}}p^2 + \hat{\hat{w}}) (2(\hat{\hat{x}} + \hat{\hat{y}}pq) + \hat{\hat{x}} + \hat{\hat{y}}p) + (\hat{\hat{x}} + \hat{\hat{y}}p)\hat{\hat{z}}_1] \quad (14)$$

6. SIMULATION

The suggested estimates of the MTBF functions by M L E method do not find analytic shape of density or distribution. Hence it is not likely to try or extend analytical verification properties of proposed M L estimates such as $E_{LNS}(T), E_{LNP}(T)$ of the present model. We have attempted empirical approach by Monte-Carlo simulation to generate large samples of different sizes and mean squared error of the estimates have computed. The various sample of sizes '*n*' from 5 to 30 at step 5 were developed empirically with specified parameter value of failure rate and used them to obtain MSE for various sizes of samples with simulations. The detailed analysis is tabulated and is seen in table 1 and table 2.

For illustration purpose by fixing a range of particular value of the rate of individual (λ), NCCS failures (β), LCCS failures (ω) and repairs (μ_1, μ_2, μ_c) for the sample size *n=5(5)30* were simulated using C-Language and the sample estimates are computed for *N=10,000(20,000)90,000* and mean squared error of the estimates are evolved for $E_{LNS}(T), E_{LNP}(T)$ which gives reasonably small even for the small samples of size *n=5*. Therefore the research results established that maximum likelihood estimation approach is satisfactory to estimate reliability indices.

Table 1. *Simulation results for MTBF function in the case of series system with $\lambda = 0.1$, $\beta = 0.5$, $\omega = 0.1$, $\mu_1 = 1$, $\mu_2 = 1.5$, $\mu_c = 2$, $p = 0.7$*

N	*Sample Size*	$E_{LNS}(T)$	$\hat{E}_{LNS}(T)$	*M.S.E*
10,000	5	0.048959	0.041318	0.000448
	10	0.048959	0.043380	0.000236
	15	0.048959	0.044162	0.000161
	20	0.048959	0.044326	0.000127
	25	0.048959	0.044507	0.000104
	30	0.048959	0.044595	0.000090
30,000	5	0.048959	0.041478	0.000457
	10	0.048959	0.043287	0.000236
	15	0.048959	0.044089	0.000165
	20	0.048959	0.044368	0.000126
	25	0.048959	0.044602	0.000105
	30	0.048959	0.044753	0.000088
50,000	5	0.048959	0.041448	0.000458
	10	0.048959	0.043368	0.000239

	15	0.048959	0.044081	0.000164
	20	0.048959	0.044337	0.000127
	25	0.048959	0.044637	0.000103
	30	0.048959	0.044718	0.000089
70,000	5	0.048959	0.041517	0.000456
	10	0.048959	0.043406	0.000238
	15	0.048959	0.044050	0.000163
	20	0.048959	0.044427	0.000128
	25	0.048959	0.044616	0.000104
	30	0.048959	0.044787	0.000089
90,000	5	0.048959	0.041462	0.000459
	10	0.048959	0.043366	0.000239
	15	0.048959	0.044039	0.000165
	20	0.048959	0.044461	0.000126
	25	0.048959	0.044616	0.000104
	30	0.048959	0.044747	0.000089

Table 2. *Simulation results for MTBF function in case of parallel system with* λ = 0.1, β = 0.5, ω = 0.1, μ_1 = 1, μ_2= 1.5, μ_c = 2, p = 0.7

N	*Sample Size*	$E_{LNP}(T)$	$\hat{E}_{LNP}(T)$	*M.S.E*
10,000	5	0.107142	0.090520	0.001792
	10	0.107142	0.094971	0.000973
	15	0.107142	0.096789	0.000671
	20	0.107142	0.096983	0.000532
	25	0.107142	0.097449	0.000440
	30	0.107142	0.097609	0.000384
30,000	5	0.107142	0.090907	0.001830
	10	0.107142	0.094900	0.000985
	15	0.107142	0.096504	0.000690
	20	0.107142	0.097089	0.000528
	25	0.107142	0.097711	0.000442
	30	0.107142	0.097887	0.000374
50,000	5	0.107142	0.090646	0.001837
	10	0.107142	0.095044	0.000981
	15	0.107142	0.096547	0.000684
	20	0.107142	0.097099	0.000533
	25	0.107142	0.097681	0.000433
	30	0.107142	0.097881	0.000375
70,000	5	0.107142	0.090801	0.001822
	10	0.107142	0.094989	0.000982
	15	0.107142	0.096358	0.000681
	20	0.107142	0.097249	0.000536
	25	0.107142	0.097656	0.000440
	30	0.107142	0.097994	0.000376
90,000	5	0.107142	0.090650	0.001823
	10	0.107142	0.094937	0.000991
	15	0.107142	0.096398	0.000688
	20	0.107142	0.097282	0.000528
	25	0.107142	0.097635	0.000438
	30	0.107142	0.097925	0.000376

7. Discussion

This paper discusses the estimation method which could give appropriate estimation practice of the reliability indices with specific sign to lethal and non-lethal CCS failures. For this purpose, we considered a reliability model with repair of down system. We have derived maximum likelihood estimators of the reliability indices such as MTBF failures of the existing model. The estimates of MTBF function were derived for both series and parallel systems. The performance of the proposed estimates has been developed in provisions of *MSE* using simulation. The simulation result suggest that M L estimate is sensibly very good and give exact estimate even for *n=5*. When n tending to large *MSE* is zero in all cases almost.

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