Research Article

FUZZY ZAGREB INDEX AND ITS APPLICATION IN FINANCIAL INTERCONNECTION NETWORKS

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ABSTRACT

In contrast to conventional models, fuzzy graphs offer an enhanced level of precision and adaptability for systems. These graphs provide a numerical representation of a molecule's structural graph through topological indices. In the realm of graph theory, numerous topological indices can be applied to the domain of fuzzy graphs. This paper elucidates the utilization of fuzzy graph theory indices within interconnection networks. The scope of this analysis encompasses the derivation of analytical results for the fuzzy Zagreb index and its practical applications in interconnecting companies within the field of financial mathematics.

Additionally, we leverage fuzzy graph modeling to investigate a significant approach to designing efficient investment strategies, harnessing the power of the fuzzy Zagreb index as a key tool in this endeavor.

Key words: Fuzzy Graphs; Fuzzy Zagreb indices; Interconnection Networks; Financial Mathematics.

1. INTRODUCTION

In a general context, graphs serve as a fundamental framework for representing the connections among various entities and data-driven relationships. They provide a concise means to articulate the underlying structure, facilitating comprehension and analysis of the behaviors inherent to the studied concepts. In graph theory, two essential components are the vertex set, representing objects within the context, and the edge set, signifying relationships between vertices. When uncertainties surround the characterization of both vertices and edges, the concept of fuzzy graphs becomes indispensable. Within the realm of structural molecular graphs, numerical measures known as topological indices play a pivotal role. These indices find applications across diverse fields, including engineering, pharmacology, graph theory, and mathematics. Notably, I. Gutman introduced the first Zagreb index in 1972 [1], and the Randic index, pioneered by Randic [2], marked the inception of topological indices. Various experts, including I. Gutman and Eliasi et al., have explored the development of multiple Zagreb and Randic indices [3]. Fuzzy graphs emphasize the significance of vertices, edges, and their associated fuzzy membership values.

The foundational work by Zadeh in 1965 introduced the concept of fuzzy relations and sets [4]. Recent research delves into the Wiener index of fuzzy networks and explores the correlations between the connectivity index and the fuzzy graph of Wiener index [5]. Fuzzy Zagreb indices have been introduced, and bounds for fuzzy Zagreb energy have been investigated [6], along with discussions on bounds for the fuzzy Zagreb Estrada index [7]. Recent studies have also addressed topics such as Hamiltonian fuzzy graphs, transitive blocks, and applications in fuzzy interconnection networks [8, 9]. Hayat and Imran have contributed to the field by exploring the topological features of specific networks [10].

With advancements in large-scale integrated circuit technology, complex interconnectivity networks have become more feasible to construct. Graph theory serves as the primary tool for the construction and analysis of such networks. These interconnected subjects find clearer elucidation through the lens of graph theory and connectivity networks [10, 11]. The advantage of employing fuzzy graphs lies in their capacity to represent intricate relationships and uncertainties, especially when the strength of connections remains imprecise or ambiguous. The dissociation of vertex memberships and edge membership values provides a flexible framework for modeling and analyzing diverse real-world scenarios. For instance, a fuzzy graph can effectively depict the design of an interconnected network of industries, with nodes representing various companies and edges signifying correlated relationships among them.

The global financial landscape is an intricate network where businesses, institutions, and markets are intricately interwoven in the pursuit of economic growth and stability. This intricate interconnection has given rise to a dynamic ecosystem characterized by complex relationships and dependencies that extend well beyond individual entities. In this context, comprehending the structure and dynamics of these interconnections is of utmost significance for financial mathematics researchers and practitioners.

The study of network theory and its associated metrics has become a potent tool for dissecting and analyzing the intricacies of financial systems. Network analysis allows us to model and explore the relationships among financial entities, providing valuable insights into risk assessment, systemic stability, portfolio optimization, and other vital aspects of financial mathematics.

Among the various network metrics, the 'Zagreb Index' has gained attention for its capability to quantify the topological characteristics of networks. Originally rooted in chemical graph theory, the Zagreb Index calculates the degree-based connectivity of nodes within a network. However, interconnection networks in the financial sector often involve fuzzy and uncertain relationships. These uncertainties arise from factors like market volatility, regulatory changes, and shifting investor sentiment. To address these complexities, there is a growing demand for network metrics capable of accommodating fuzzy relationships and imprecise data.

In this research paper, we delve into the application of a novel adaptation of the Zagreb Index, known as the 'Fuzzy Zagreb Index,' within the realm of financial interconnection networks. The Fuzzy Zagreb Index extends the traditional Zagreb Index by incorporating the concept of fuzziness, enabling a more nuanced representation of network connectivity. By embracing fuzziness, we acknowledge the inherent uncertainty surrounding financial interconnections, providing a more realistic and adaptable framework for analysis.

Our primary objective is to explore how the Fuzzy Zagreb Index can serve as a robust tool for quantifying the connectivity and risk profiles within the interconnection networks of financial companies. We aim to demonstrate its potential to capture the dynamic and often uncertain nature of relationships between financial entities, thereby offering practitioners and researchers a more comprehensive understanding of systemic risk and financial stability.

Throughout this paper, we will establish the theoretical foundations of the Fuzzy Zagreb Index, discuss its advantages in the context of financial networks, and provide practical examples and case studies to illustrate its applicability. Our research seeks to bridge the gap between network theory and financial mathematics, providing a unique perspective on the analysis and management of interconnection networks in the financial sector.

In subsequent sections, we will continue with an exploration of relevant literature, a formal definition of the Fuzzy Zagreb Index, and a presentation of our research methodology, results, and discussions. Our ultimate goal is to contribute to the growing body of knowledge in financial mathematics and network analysis, empowering stakeholders in the financial industry to make informed decisions and navigate the intricate web of interconnections with greater precision and confidence.

2. PRELIMINARIES

In this section, we will review key definitions related to fuzzy Zagreb indices and fundamental concepts in fuzzy graph theory, as they constitute a foundational framework for our subsequent discussions in this research paper. For a comprehensive understanding of the Zagreb indices, readers are encouraged to refer to [12] and [3], which provide an in-depth exploration of these metrics. Additionally, those seeking a solid grasp of graph theory and fuzzy graphs can find valuable resources in [13] and [14]. Furthermore, we will discuss the precise definitions and mathematical formulations of the fuzzy Zagreb first index, the associated fuzzy Zagreb matrix, and the concept of fuzzy Zagreb energy. These definitions are instrumental in our subsequent analyses and discussions and can be found in [6] and [7]. By establishing a clear foundation through these references, we aim to provide readers with the necessary background to comprehend the intricate nuances and applications of fuzzy Zagreb indices in our research.

DEFINITION 2.1. *A fuzzy graph $G(V,\sigma,\mu)$; which also can simply be denoted by $G(\sigma,\mu)$, is a graph with a vertex-membership function $\sigma: V \to [0,1]$ and edge-membership function $\mu: V \times V \to [0,1]$. The corresponding crisp graph is denoted by $G(\sigma^*,\mu^*)$.*

Also, we denote the strength of vertex u by $\mu(u)$, it represents the minimum strengths of edges incident to the vertex u

$$\mu(u) = \bigwedge_{uv_i \in \mu^*} \mu(u, v_i)$$

With the provided definition, it is important to emphasize that there exist no constraints or prerequisites regarding the interdependence of vertex membership values and edge membership values. These two sets of values remain entirely autonomous, permitting their definition based on distinct criteria or considerations. This inherent flexibility opens up a wide spectrum of applications and modeling scenarios in which relationships among elements can be effectively represented with varying degrees of uncertainty and fuzziness.

To illustrate this versatility, let us consider the application of fuzzy graphs to the representation of a social network. In this context, the membership values assigned to vertices could capture the extent to which individuals belong to various social groups or communities. Conversely, the membership values assigned to edges could signify the intensity of social connections between individuals, influenced by factors such as the frequency of interaction, shared interests, or geographic proximity.

Importantly, it is not mandatory for a direct correlation to exist between an individual's membership in a particular social group and the strength of their connections to other individuals within the same group. The inherent adaptability of fuzzy graphs allows for the modeling of intricate relationships and uncertainties. In such models, the membership values associated with vertices and edges can be determined independently, taking into account different facets and aspects of the complex system under consideration.

This flexibility empowers researchers and practitioners to embrace a comprehensive approach to modeling real-world systems, where multifaceted and nuanced relationships can be accurately represented. By permitting the independent definition of membership values for vertices and edges, fuzzy graphs provide a powerful framework for capturing the intricate dynamics of systems characterized by uncertainty, fuzziness, and diverse criteria for assessing relationships.

DEFINITION 2.2. *The fuzzy Zagreb first index of $G(\sigma,\mu)$ is defined as*

$$FM_1(G) = \sum_{uv\in\mu^*} [\sigma(u)\mu(u) + \sigma(v)\mu(v)]$$

Equivalently, the index can also be defined as

$$FM_1(G) = \sum_{u\in\sigma^*} [\sigma(u)\mu(u)d_u]$$

DEFINITION 2.3. *If $G(\sigma,\mu)$ is a fuzzy graph and $\sigma^* = \{u_1, u_2, \dots, u_n\}$ then first fuzzy Zagreb matrix is defined as $FZ^{(1)} = (fz^{(1)})_{i,j}$ where*

$$(fz^{(1)})_{i,j} = \begin{cases} \sigma(u_i)\mu(u_i) + \sigma(u_j)\mu(u_j) & ,\text{if } \ i \neq j \ \text{ and } \ u_i.u_j \in \mu^* \\ 0 & ,\text{if } \ u_i.u_j \notin \mu^* \\ 0 & ,\text{if } \ i = j \end{cases}$$

DEFINITION 2.4. *If $G(\sigma,\mu)$ is a fuzzy graph and $\sigma^* = \{u_1, u_2, \dots, u_n\}$, if $FZ^{(1)}$ is the first fuzzy Zagreb matrix with its eigen values $\xi_1^{(1)}, \xi_2^{(1)}, \dots, \xi_n^{(1)}$ then the first fuzzy Zagreb energy is defined as*

$$FZE^{(1)} = \sum_{i=1}^{n} |\xi_i^{(1)}|$$

3. FINANCIAL PARAMETERS

In the intricate landscape of financial investments, the process of selecting shares or stocks necessitates a meticulous examination of various financial parameters. These parameters act as guiding principles, steering investors toward prudent decisions amidst the complex array of investment options. This section offers an extensive analysis of the multifaceted world of investment decision parameters and their pivotal role in the evaluation of risks and opportunities.

INVESTMENT DECISION PARAMETERS:

Company Fundamentals: A cornerstone of investment decision-making is the comprehensive evaluation of a company's financial health. Investors scrutinize a range of factors, including revenue growth, earnings per share (EPS), profit margins, debt levels, and cash flow, to assess the overall performance and stability of the company in question.

Valuation metrics: Valuation metrics provide insights into whether a stock is overvalued or undervalued. Common metrics include the Price-to-Earnings ratio (P/E), Price-to-Book ratio (P/B), Price-to-Sales ratio (P/S), and Dividend Yield, offering investors a framework to gauge a stock's pricing relative to its intrinsic worth.

Dividends and income: For investors seeking a steady income stream, dividend-paying stocks hold particular appeal. Considerations such as dividend yield and a company's history of dividend payments play a crucial role in the decision-making process for those aiming to derive income from their investments.

INTERCONNECTION BETWEEN INVESTMENT DECIDING PARAMETERS:

Company fundamentals and value metrics: The financial robustness of a company, as reflected in key metrics like revenue growth and profitability, directly influences valuation metrics, notably the Price-to-Earnings (P/E) ratio. Companies characterized by strong fundamentals and growth potential often command higher P/E ratios, signaling that they are relatively more expensive compared to their earnings. Conversely, companies with weaker fundamentals may exhibit lower P/E ratios, indicating potential undervaluation.

Market trends and industry analysis: Broader market trends, whether categorized as bullish or bearish, wield substantial influence over individual industries. In a bullish market, most sectors typically perform favorably, while during bearish phases, defensive sectors such as utilities and consumer staples may exhibit greater resilience. Understanding these overarching market trends empowers investors to identify sectors with growth potential or stability.

Company fundamentals and management quality: Strong company fundamentals often stem from competent and effective management. Companies guided by capable leadership are more likely to demonstrate superior financial performance, strategic growth initiatives, and prudent decision-making factors that instill investor confidence.

In the complex fabric of investment decisions, these financial parameters serve as guiding stars, illuminating the path to well-informed choices. Recognizing the intricate web of interconnections among these parameters empowers investors to navigate the complexities of the financial realm, facilitating the formulation of more discerning investment strategies and decisions. These insights, in turn, foster a deeper understanding of the financial landscape, assisting investors in harnessing opportunities and effectively managing risks.

4. SECTOR ANALYSIS

In this phase of our research, we employ multidimensional fuzzy graphs to conduct a comprehensive sector analysis. When constructing fuzzy graphs to investigate the intricate interconnections within this complex network, we meticulously consider nine critical parameters. These parameters include dividend paid (p_1), return on capital employed (ROCE) (p_2), earnings per share (EPS) (p_3), profit growth (p_4), debt-to-equity ratio (p_5), price-to-book ratio (P/B) (p_6), earning yield (p_7), operating profit margin (OPM) (p_8), and price-to-earnings ratio (P/E) (p_9). Our analysis focuses on the top five companies in various sectors, classified by market capitalization. These sectors serve as the nodes within our graph, and their collective market capitalization percentages serve as the edge membership values. The sectors under consideration encompass Banking, IT (software industries), Electronics, Metal, and FMCG.

It is worth noting that our analysis is grounded in real-time market data, which is readily accessible through open-source financial platforms.

4.a INVESTIGATING THE BANKING SECTOR:

We present the vertex and edge membership values for various banks along with the corresponding parameters $p_1, p_2, \ldots, p_9$ in the tables below:

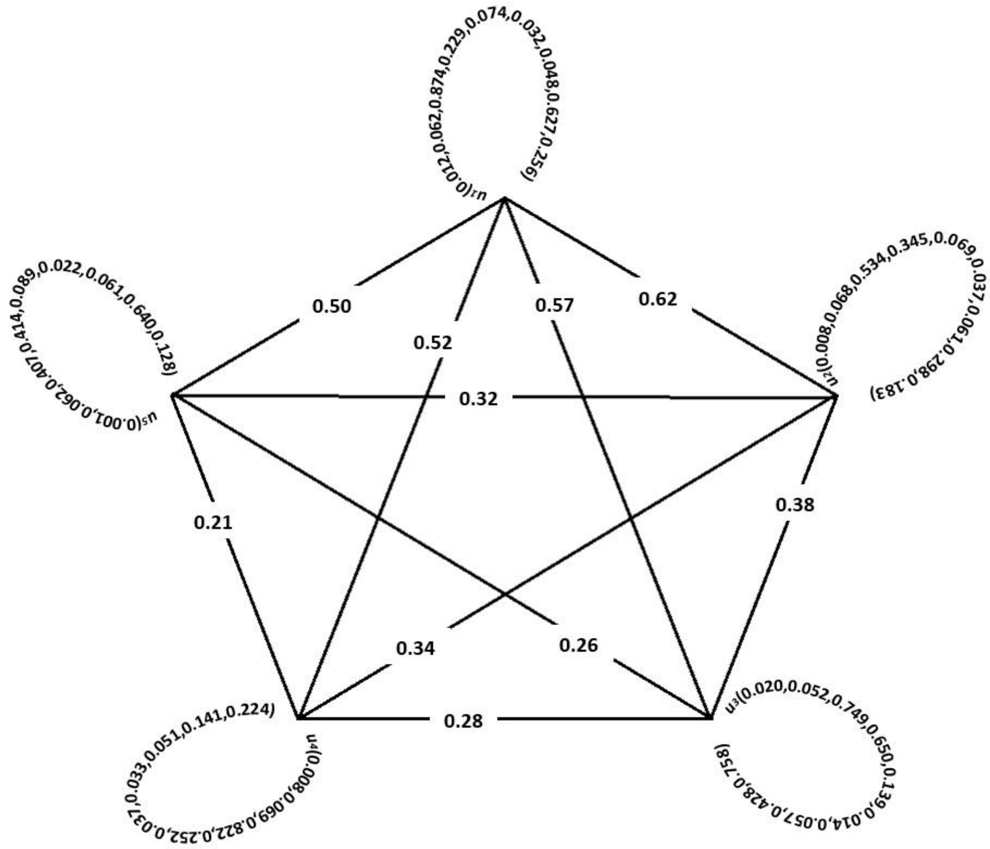
Table 4.a.1: Edge memberships

	Market Cap.(in Cr.)	u_1	u_2	u_3	u_4	u_5
HDFC Bank (u_1)	1248837		0.62	0.57	0.52	0.50
ICICI Bank (u_2)	685093	0.62		0.38	0.34	0.32
SBI (u_3)	507007	0.57	0.38		0.28	0.26
Kotak M. Bank (u_4)	364661	0.52	0.34	0.28		0.21
Axis Bank (u_5)	292850	0.50	0.32	0.26	0.21	

Table 4.a.2: Vertex memberships

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
u_1	0.012	0.062	0.874	0.229	0.074	0.032	0.048	0.627	0.256
u_2	0.008	0.068	0.534	0.345	0.069	0.037	0.061	0.298	0.183
u_3	0.020	0.052	0.749	0.650	0.139	0.014	0.057	0.428	0.758
u_4	0.008	0.069	0.822	0.252	0.037	0.033	0.051	0.141	0.224
u_5	0.001	0.062	0.407	0.414	0.089	0.022	0.061	0.640	0.128
FM_1	0.06	0.38	4.24	2.13	0.49	0.17	0.33	2.74	1.83

Fig. 4.a.3: *Fuzzy Graph of Banking Sector*



Hence, the fuzzy Zagreb index FM_1 (Banking Sector) = 12.37.

4.b EXPLORING THE IT SECTOR

In the subsequent tables, we provide the vertex and edge membership values for different companies with the IT sector, categorised under various parameters $p_1, p_2, ..., p_9$.

Table 4.b.1: Edge memberships

	Market Cap.(in Cr.)	v_1	v_2	v_3	v_4	v_5
TCS (v_1)	1270019		0.73	0.62	0.59	0.56
Infosys (v_2)	577063	0.73		0.35	0.32	0.29
HCL Tech (v_3)	309656	0.62	0.35		0.21	0.18
Wipro (v_4)	228612	0.59	0.32	0.21		0.15
LTI Mindtree (v_5)	150474	0.56	0.29	0.18	0.15	

Table 4.b.2: Vertex memberships

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
v_1	0.014	0.591	1.000	0.128	0.001	0.014	0.047	0.262	0.290
v_2	0.025	0.407	0.590	0.108	0.001	0.078	0.061	0.242	0.234
v_3	0.042	0.283	0.557	0.112	0.001	0.047	0.068	0.221	0.205
v_4	0.002	0.177	0.212	0.087	0.002	0.029	0.069	0.187	0.196
v_5	0.002	0.377	1.000	0.533	0.000	0.091	0.041	0.183	0.338
FM_1	0.09	2.33	4.05	0.86	0.01	0.23	0.29	1.25	1.39

Hence, the fuzzy Zagreb index FM_1 (I.T. Sector) = 10.50.

4.c EXPLORATION OF THE ELECTRONICS SECTOR

The tables below present the vertex and edge membership values for various companies within the Electronics sector, categorized according to different parameters $p_1, p_2, \ldots, p_9$.

Table 4.c.1: Edge memberships

	Market Cap.(in Cr.)	w_1	w_2	w_3	w_4	w_5
Bharat Electronics (w_1)	94405		0.85	0.68	0.66	0.63
Honeywell Auto. (w_2)	37264	0.85		0.31	0.30	0.26
Data Pattern (w_3)	11372	0.68	0.31		0.13	0.09
Syrma SGS Tech (w_4)	8410	0.66	0.30	0.13		0.08
Shivalik Bimetal (w_5)	3318	0.63	0.26	0.09	0.08	

Table 4.c.2: Vertex memberships

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
w_1	0.014	0.300	0.043	0.152	0.000	0.068	0.048	0.234	0.299
w_2	0.002	0.195	1.000	0.292	0.000	0.117	0.017	0.151	0.851
w_3	0.002	0.196	0.250	0.387	0.000	0.097	0.017	0.376	0.839
w_4	0.000	0.148	0.075	1.000	0.002	0.055	0.025	0.084	0.637
w_5	0.001	0.377	0.132	0.315	0.002	0.130	0.032	0.251	0.436
FM_1	0.04	1.20	1.31	1.25	0.00	0.39	0.16	0.99	2.28

Hence, the fuzzy Zagreb index FM_1 (Electronics Sector) = 7.61.

4.d EXPLORING THE METAL SECTOR

In the subsequent tables, we provide the vertex and edge membership values for diverse companies within the Metal sector, classified under various parameters $p_1, p_2, \ldots, p_9$.

Table 4.d.1: Edge memberships

	Market Cap.(in Cr.)	x_1	x_2	x_3	x_4	x_5
Adani ENT (x_1)	282098		0.59	0.53	0.47	0.46
JSW steel (x_2)	193691	0.59		0.42	0.36	0.35
TATA steel (x_3)	144397	0.53	0.42		0.30	0.29
Hindalco (x_4)	102192	0.47	0.36	0.30		0.24
Vedal (x_5)	89231	0.46	0.35	0.29	0.24	

Table 4.d.2: Vertex memberships

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
x_1	0.001	0.101	0.235	1.000	0.016	0.085	0.025	0.079	1.000
x_2	0.004	0.084	0.234	0.000	0.001	0.030	0.058	0.124	0.370
x_3	0.031	0.128	0.013	0.000	0.008	0.014	0.069	0.093	0.705
x_4	0.066	0.113	0.375	0.000	0.006	0.018	0.097	0.091	0.121
x_5	0.420	0.238	0.237	0.000	0.017	0.023	0.155	0.215	0.120
FM_1	0.51	0.79	1.36	1.84	0.06	0.25	0.45	0.72	3.41

Hence, the fuzzy Zagreb index FM_1 (Metal Sector) = 9.39.

4.e EXPLORING THE FMCG SECTOR

Below, we present the vertex and edge membership values for a range of companies within the Fast-Moving Consumer Goods (FMCG) sector, categorized under various parameters $p_1, p_2, \ldots, p_9$.

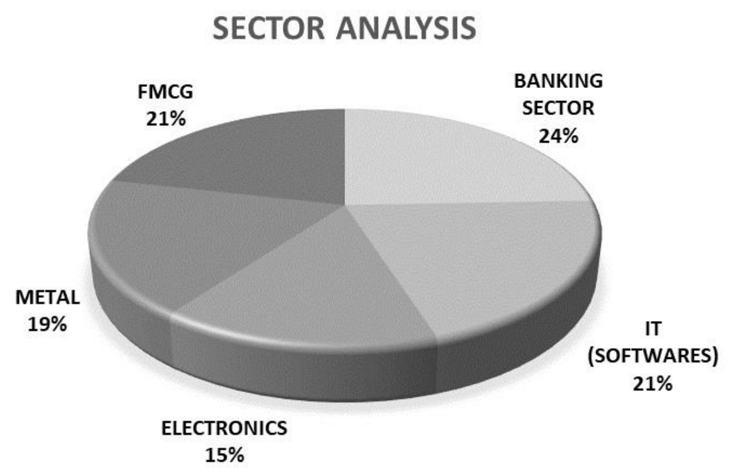
Table 4.e.1: Edge memberships

	Market Cap.(in Cr.)	y_1	y_2	y_3	y_4	y_5
Hindustan Unilever (y_1)	602869		0.74	0.52	0.45	0.43
ITC (y_2)	563847	0.74		0.50	0.43	0.41
Nestle India (y_3)	214324	0.52	0.50		0.21	0.19
Britannia (y_4)	110928	0.45	0.43	0.21		0.12
TATA Consumers (y_5)	78729	0.43	0.41	0.19	0.12	

Table 4.e.2: Vertex memberships

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
y_1	0.015	0.266	0.438	0.129	0.000	0.120	0.023	0.235	0.582
y_2	0.028	0.392	0.154	0.259	0.000	0.081	0.046	0.362	0.295
y_3	0.010	1.000	1.000	0.196	0.001	0.750	0.018	0.225	0.787
y_4	0.016	0.488	1.000	0.455	0.085	0.314	0.026	0.182	0.520
y_5	0.010	0.099	0.136	0.086	0.001	0.048	0.024	0.137	0.687
FM_1	0.09	2.14	2.31	1.06	0.04	1.08	0.15	1.32	2.66

Hence, the fuzzy Zagreb index FM_1 (FMCG Sector) = 10.86.



Based on the aforementioned analysis, it can be inferred that, given the current financial indicators, the **banking sector** exhibits favorable prospects for investment.

In the subsequent section, we investigate the banking sector further to identify specific companies with promising investment potential.

5. COMPANY ANALYSIS

Having identified the banking sector as the focal point for investment in Section 4, our current focus shifts towards the analysis of fuzzy graphs within different banks. Our objective is to calculate the Fuzzy Zagreb indices, ultimately aiding in the selection of the ideal company for investment.

In this phase, we construct fuzzy graphs for various companies within the banking sector, thereby completing the company analysis. When forming these fuzzy graphs to visualize the interconnectivity network of the banking sector, we consider nine distinct parameters $p_1, p_2, ..., p_9$ as nodes within the graph. The values of these parameters for each bank serve as the vertex memberships, while we also incorporate potential edge membership values based on hypothetical relationships. These relationships are derived by calculating the rank correlation coefficient between the parameters, and we base our considerations on the following factors

Dividend yield and EPS: Companies displaying both a high dividend yield, indicating a substantial payout of earnings as dividends, and a high EPS, receive a moderate to high edge membership value between the 'Dividend Yield' and 'EPS' vertices.

EPS and ROCE: Firms with high EPS and a robust Return on Capital Employed (ROCE), signifying efficient capital utilization, are assigned a moderate to high edge membership value connecting the 'EPS' and 'ROCE' vertices.

Debt-to-Equity Ratio and ROCE: Companies burdened with a high Debt-to-Equity ratio and a low ROCE, suggestive of potential difficulties in covering debt obligations, are linked with a moderate to high edge membership value between the 'Debt-to-Equity Ratio' and 'ROCE' vertices.

P/B ratio and ROCE: Companies with a high P/B ratio and strong ROCE are linked with a moderate to high edge membership value, illustrating the correlation between elevated market valuation and efficient capital management.

Earning Yield and P/E Ratio: A high earning yield, indicative of a more substantial return on investment based on current earnings, is associated with a relatively lower P/E ratio. Thus, a moderate to high edge membership value connects the 'Earning Yield' and 'P/E Ratio' vertices.

EPS and OPM: A moderate to high edge membership value is assigned to represent the correlation between EPS and Operating Profit Margin (OPM). This signifies that a higher profit margin can translate into improved earnings per share.

Debt-to-equity ratio and P/E ratio: A lower edge membership value suggests that the company's debt-to-equity ratio has a weak correlation with its price-to-earnings (P/E) ratio, implying that factors beyond debt structure influence the P/E ratio.

P/E and P/B ratio: A moderate to high correlation between the P/E and P/B ratios is reflected in a moderate to high edge membership value connecting the 'P/E Ratio' and 'P/B Ratio' vertices.

P/B ratio and dividend yield: In cases where no significant correlation exists between the P/B ratio and dividend yield, a low edge membership value is assigned to signify a weak relationship between these two parameters.

It is imperative to note that the assignment of edge membership values involves subjectivity and relies on domain expertise, data analysis, and expert judgment. The aim is to encapsulate the relationships between diverse parameters while accounting for the inherent uncertainty and vagueness within these relationships. The Fuzzy Zagreb index serves as a valuable tool for comprehending and interpreting the intricate interactions that define various aspects of a company's performance.

5.a ANALYSIS OF HDFC BANK

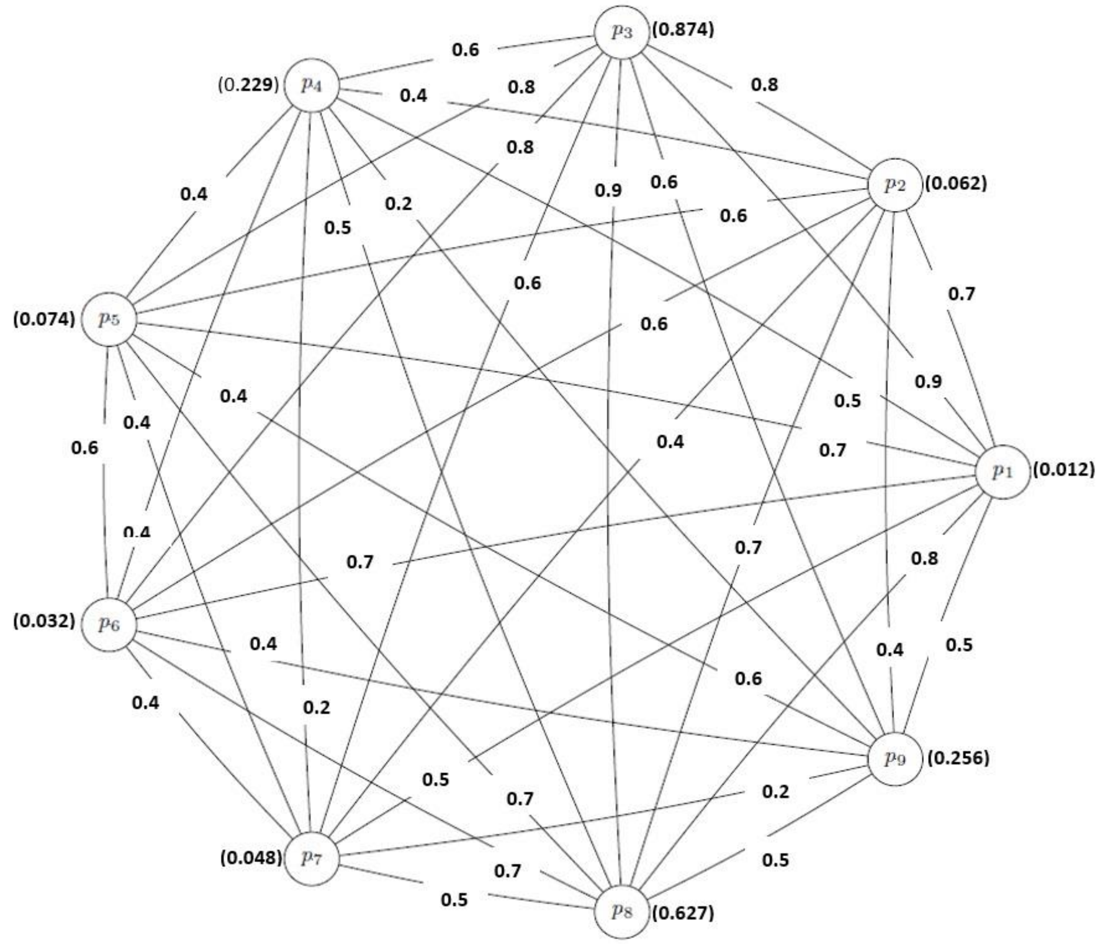
Vertex and edge membership values for the HDFC bank with vertices $p_1, p_2, ..., p_9$ are given in the following table.

Table 5.a.1:

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
u_1	0.012	0.062	0.874	0.229	0.074	0.032	0.048	0.627	0.256
p_1		0.7	0.9	0.5	0.7	0.7	0.5	0.8	0.5
p_2	0.7		0.8	0.4	0.6	0.6	0.4	0.7	0.4
p_3	0.9	0.8		0.6	0.8	0.8	0.6	0.9	0.6
p_4	0.5	0.4	0.6		0.4	0.4	0.2	0.5	0.2
p_5	0.7	0.6	0.8	0.4		0.6	0.4	0.7	0.4
p_6	0.7	0.6	0.8	0.4	0.6		0.4	0.7	0.4
p_7	0.5	0.4	0.6	0.2	0.4	0.4		0.5	0.2
p_8	0.8	0.7	0.9	0.5	0.7	0.7	0.5		0.5
p_9	0.5	0.4	0.6	0.2	0.4	0.4	0.2	0.5	

$FM_1(HDFC\ Bank) = 8.14$

Fig. 5.a.2: *Fuzzy Graph of HDFC Bank*



5.b ANALYSIS OF ICICI BANK

Vertex and edge membership values for the ICICI bank with vertices $p_1, p_2, \ldots, p_9$ are given in the following table.

Table 5.b.1:

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
u_2	0.008	0.068	0.534	0.345	0.069	0.037	0.061	0.298	0.183
p_1		0.7	0.5	0.6	0.7	0.8	0.8	0.5	0.6
p_2	0.7		0.6	0.7	0.8	0.9	0.9	0.6	0.7
p_3	0.5	0.6		0.5	0.6	0.7	0.7	0.4	0.5
p_4	0.6	0.7	0.5		0.7	0.8	0.8	0.5	0.6
p_5	0.7	0.8	0.6	0.7		0.9	0.9	0.6	0.7
p_6	0.8	0.9	0.7	0.8	0.9		1	0.7	0.8
p_7	0.8	0.9	0.7	0.8	0.9	1		0.7	0.8
p_8	0.5	0.6	0.4	0.5	0.6	0.7	0.7		0.5
p_9	0.6	0.7	0.5	0.6	0.7	0.8	0.8	0.5	

$FM_1(\text{ICICI Bank}) = 6.01$

5.c ANALYSIS OF SBI BANK

Vertex and edge membership values for the SBI bank with vertices $p_1, p_2, ..., p_9$ are given in the following table.

Table 5.c.1:

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
u_3	0.020	0.052	0.749	0.650	0.139	0.014	0.057	0.428	0.758
p_1		0.6	0.8	1	0.6	0.6	0.8	0.8	1
p_2	0.6		0.4	0.6	0.2	0.2	0.4	0.4	0.6
p_3	0.8	0.4		0.8	0.4	0.4	0.6	0.6	0.8
p_4	1	0.6	0.8		0.6	0.6	0.8	0.8	1
p_5	0.6	0.2	0.4	0.6		0.2	0.4	0.4	0.6
p_6	0.6	0.2	0.4	0.6	0.2		0.4	0.4	0.6
p_7	0.8	0.4	0.6	0.8	0.4	0.4		0.6	0.8
p_8	0.8	0.4	0.6	0.8	0.4	0.4	0.6		0.8
p_9	1	0.6	0.8	1	0.6	0.6	0.8	0.8	

FM_1(SBI Bank) = 11.13

5.d ANALYSIS OF KOTAK BANK

Vertex and edge membership values for the Kotak bank with vertices $p_1, p_2, ..., p_9$ are given in the following table.

Table 5.d.1:

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
u_4	0.008	0.069	0.822	0.252	0.037	0.033	0.051	0.141	0.224
p_1		0.6	0.5	0.3	0.6	0.5	0.3	0.2	0.3
p_2	0.6		0.9	0.7	1	0.9	0.7	0.6	0.7
p_3	0.5	0.9		0.6	0.9	0.8	0.6	0.5	0.6
p_4	0.3	0.7	0.6		0.7	0.6	0.4	0.3	0.4
p_5	0.6	1	0.9	0.7		0.9	0.7	0.6	0.7
p_6	0.5	0.9	0.8	0.6	0.9		0.6	0.5	0.6
p_7	0.3	0.7	0.6	0.4	0.7	0.6		0.3	0.4
p_8	0.2	0.6	0.5	0.3	0.6	0.5	0.3		0.3
p_9	0.3	0.7	0.6	0.4	0.7	0.6	0.4	0.3	

FM_1(Kotak Bank) = 5.43

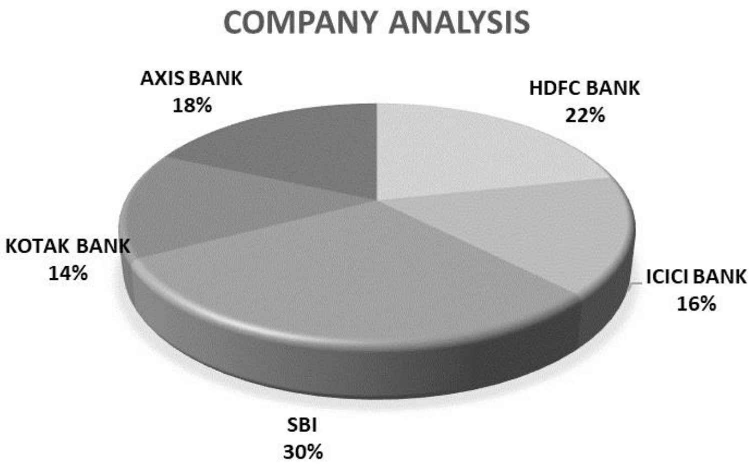
5.e ANALYSIS OF AXIS BANK

Vertex and edge membership values for the Axis bank with vertices $p_1, p_2, ..., p_9$ are given in the following table.

Table 5.e.1:

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
u_5	0.001	0.062	0.407	0.414	0.089	0.022	0.061	0.640	0.128
p_1		0.4	0.3	0.6	0.4	0.4	0.6	0.7	0.6
p_2	0.4		0.3	0.6	0.4	0.4	0.6	0.7	0.6
p_3	0.3	0.3		0.5	0.3	0.3	0.5	0.6	0.5
p_4	0.6	0.6	0.5		0.6	0.6	0.8	0.9	0.9
p_5	0.4	0.4	0.3	0.6		0.4	0.6	0.7	0.6
p_6	0.4	0.4	0.3	0.6	0.4		0.6	0.7	0.6
p_7	0.6	0.6	0.5	0.8	0.6	0.6		0.9	0.8
p_8	0.7	0.7	0.6	0.9	0.7	0.7	0.9		0.9
p_9	0.6	0.6	0.5	0.9	0.6	0.6	0.8	0.9	

FM_1(Axis Bank) = 6.88



Based on the preceding analysis, we draw the conclusion that, taking into account the current financial factors, 'State Bank of India' (SBI) presents a favorable investment opportunity within the Indian financial markets.
Given the expansive scope of applied fuzzy graphs, it is noteworthy that fuzzy topological indices hold potential applicability to various other interconnected graph networks, which we intend to explore in our forthcoming research endeavors.

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** The information utilized for the analysis in the paper across different sectors has been sourced from http://www.nseindia.com. This data is open-source and widely accessible on numerous financial websites.*