Cubic Root Mean Difference Labeling Of Graphs

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Abstract
In a graph, $G = (V,E)$ with p vertices and q edges is said to be cubic root mean difference Labeling of graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, \cdots q + 1$ in which each edge $e = uv$ is labeled with $f(e = uv) = \left|\sqrt{\frac{f(u)^3 - f(v)^3}{2}}\right|$ then the edge labels are distinct. Here f is called a cubic root mean labeling of G. In this paper we prove the Jewel graph, prism grpah, ladder graph, comb graph and Triangular graph are cubic root mean labeling of graphs.
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1. Introduction

Graceful and Harmonious Labeling, variation of graceful labeling are studied by Gallian [1]. The concepts of labeling by Somasundaram and ponraj[5]. Root square mean labeling of graphs has been introduced by sandhya , somosundaram and Anusa[8]. By the motivation of above works we introduce a new type of Labeling called cubic root Mean Labeling of graphs

2. Preliminaries

Definition 2.1 *The Jewel graph J_r is a graph with the vertex set*

$$V(J_n) = \{u, X, V, Y, V_i; 1 \le i \le n\}$$

and the edge set

$$E(J_n) = \{u_x, V_x, u_y, V_y, uV_i, V_v; 1 \le i \le n\}$$

Definition 2.2 *A Prism graph is a graph that has one of the prism as this skeleton. it is denoted by cL_n*
Definition 2.3 *The product graph $P_2 \times P_n$ is called a ladder graph and it is denoted by L_n.*
Definition 2.4 *The graph obtained by joining a single Pendent edge to each vertex of a path is called a comb graph*
Definition 2.5 *A triangular snake graph T_n is obtained from a path $u_1, u_2, \cdots u_n$ by joining u_i and u_{i+1} to a new vertex V_i for $1 \le i \le n - 1$. That is every edge of a path is replaced by a triangle C_3*
Definition 2.6 *Let G be the simple graph with the vertex set $V(G)$ and the edge set $E(G)$ vertex set $V(G)$ are labeled by positive integer and Let $E(e)$ denoted the edge label that its difference of label of vertices incident with the edge e.*

3. Cubic Root Mean difference Labeling of graphs

Definition 3.1 *In a graph $G = (V,E)$ with P vertices and q edges is said to be a cubic root mean difference labeling of graph if the vertices $x \in V$ with distinct element $f(x)$ form $1, 2 \cdots q + 1$ and each edge $e = uv$, is labeled with* $f(e = uv) = \left|\sqrt{\frac{f(u)^3 - f(v)^3}{2}}\right|$
then the resulting labels are distinct. Here f is called cubic root mean difference labeling of G

Theorem 3.1 *The Jewel graph J_r admits cubic root mean difference labeling of graph* Proof. Let $G = J_r$ be the graph with the vertices $u_1, u_2, \cdots u_n$ and the edges $e_1, e_2, \cdots e_q$

We define vertex and edge labels by

$$\begin{aligned} f(u_i) &= i-2,\ 1 \le i \le r \\ f(v_i) &= i+4,\ 1 \le i \le r \end{aligned}$$

and we induce the edge labeling function $f^* : E \to N$ define by

$$f^*(uv) = \left|\sqrt{\frac{f(u)^3 - f(v)^3}{2}}\right|$$

for every $uv \in E(J_r)$ and all distinct

$$\begin{aligned} f^*(u_iu_{i+1}) &= \left|\sqrt{\frac{f(u_i)^3 - f(u_{i+1})^3}{2}}\right| \\ &= \left|\sqrt{\frac{(i-2)^3 - (i-2+1)^3}{2}}\right| \end{aligned}$$

$$\begin{aligned} &= \left|\sqrt{\frac{(i-2)^3 - (i-1)^3}{2}}\right| \\ &= \left|\sqrt{\frac{[(i)^3 - 3(i)^2(2) + 3i(4) - 8 - [(i)^3 - 3(i)^2 + 3i - 1]]}{2}}\right| \\ f^*(u_iu_{i+1}) &= \left|\sqrt{\frac{9i-4}{2}}\right| \\ f^*(v_iu_i) &= \left|\sqrt{\frac{f(V_i)^3 - f(u_i)^3}{2}}\right| \\ &= \left|\sqrt{\frac{(i+4)^3 - (i-2)^3}{2}}\right| \\ &= \boxed{\left|\sqrt{\frac{36i+54}{2}}\right|} \end{aligned}$$

hence the edge labeling and vertex labeling are distinct. Hence the Jewel graph J_r admits cubic root mean difference labeling of graph

Theorem 3.2 *A prism graph Y_n admits cubic root mean labeling of graph*
Proof. Let $G = Y_n$ and $v_1, v_2, \cdots v_n$ are the vertices of G.
We define the function $f(v_i)(2i-1)$ for the vertex labeling the cubic root mean difference labeling of G

$$\begin{aligned} f^*(V = v_iv_{i+1}) &= \left|\sqrt{\frac{f(u_i)^3 - f(u_{i+1})^3}{2}}\right| \\ &= \left|\sqrt{\frac{(2i-1)^3 - (2i-1+1)^3}{2}}\right| \\ &= \left|\sqrt{\frac{(2i-1)^3 - (2i)^3}{2}}\right| \\ &= \boxed{\left|\sqrt{\frac{6i+1}{2}}\right|} \end{aligned}$$

Hence the vertex labeling is distinct. Hence the prism graph admits cubic root mean difference labeling of G.

Theorem 3.3 *The Ladder graph L_n is a cubic root mean difference labeling of G.*
Proof. Let $G = L_n$ be the Ladder graph with the vertices $u_1, u_2, \cdots u_n$ and the edges $e_1, e_2, \cdots e_q$
Define the function $f : V(L_n) \to (1, 2, \cdots, q+1)$ and the induced edge labeling $f^* : E(G) \to N$ defined by

$$f^*(e = uv) = \left|\sqrt{\frac{f(u)^3 - f(v)^3}{2}}\right|$$

$$\begin{aligned} f^*(u_i\, u_{i+1}) &= \left|\sqrt{\frac{(2i+1)^3 - (2i+2)^3}{2}}\right| \\ &= \left|\sqrt{\frac{-4i + 3 + 6i - 2 + 8i}{2}}\right| \\ &= \left|\sqrt{\frac{10i + 1}{2}}\right| \end{aligned}$$

Now,

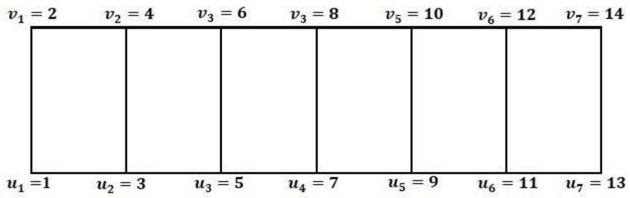
$$\begin{aligned} f^*(u_{n-1}\, u_n) &= \left|\sqrt{\frac{(2n-3)^3 - (2n+1)^3}{2}}\right| \\ &= \left|\sqrt{\frac{(2n-1) - (2n+1)(2n+1)^3 + (2n-1)(2n+1) + (2n+1)^2}{2}}\right| \\ &= \left|\sqrt{\frac{-12n^2 - 1}{2}}\right| \\ f^*(v_i v_{i+1}) &= \left|\sqrt{\frac{(3i)^3 - (3i+3)^2}{2}}\right| \\ &= \left|\sqrt{\frac{-3(27i^2 + 27i + 9)}{2}}\right| \\ &= \left|\sqrt{\frac{-81i^2 - 81i - 27}{2}}\right| \end{aligned}$$

Clearly,

$$\begin{aligned} f^*(v_{n-1}\, v_n) &= \left|\sqrt{\frac{(3n-3)^3 - (3n)^3}{2}}\right| \\ &= \left|\sqrt{\frac{-3(27n^2 - 27n + 9)}{2}}\right| \\ &= \left|\sqrt{\frac{81n^2 + 81n - 27}{2}}\right| \\ f^*(u_i\, v_i) &= \left|\sqrt{\frac{(2i+1)^3 - (3i)^3}{2}}\right| \end{aligned}$$

Hence the edge labels are distinct. Hence the ladder graph admits a cubic root mean difference labeling of graphs.

Example 3.1 *Cubic root mean difference labeling*



Hence $f(u_1) = 1, f(u_2) = 3, f(u_3) = 5, \cdots f(u_8) = 15$ $f(v_1) = 2, f(v_2) = 4, f(v_3) = 6, \cdots f(v_8) = 16$

Theorem 3.4 *Comb graph is a cubic root mean difference labeling of graphs.*
Proof. Let G be the comb graph with the vertices $u_1, u_2, \cdots u_n$ and the edges $e_1, e_2, \cdots e_q$. Let P_n be the path $u_1, u_2, \cdots, u_n$ in G and the vertices v_i to u_i for
$1 \le i \le n$ we define the function $f(u_i) = 2i + 1$ and $f(v_i) = 3i$ for $1 \le i \le n$ and we induce edge labeling $f^*(e = uv)$: $E(G) \to N$ defined by $f^*(e = uv) = \left|\sqrt{\frac{f(u)^3 - f(v)^3}{2}}\right|$
Now

$$f(u_i u_{i+1}) = \left|\sqrt{\frac{10i + 1}{2}}\right|$$

Clearly,

$$f^*(u_{n-1}u_n) = \left|\sqrt{-12n^2 - 1}\right|$$
$$f^*(u_i v_i) = \left|\sqrt{\frac{-19i^3 + 21i^2 + 6i + 1}{2}}\right|$$
$$f^*(u_n v_n) = \left|\sqrt{\frac{-19n^3 + 21n^2 + 6n + 1}{2}}\right|$$

Thus the edge labels are distinct. Hence the comb graph admits cubic root mean difference labeling of graphs

Example 3.2 .

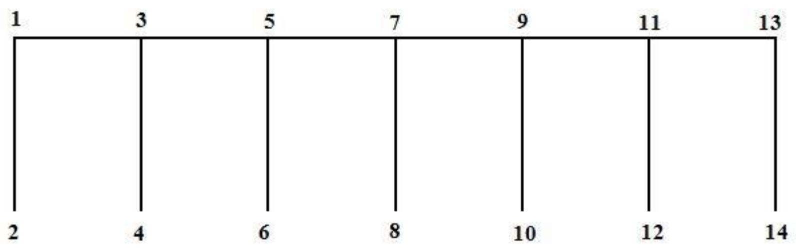


Figure 1: vertex Labels

Here the vertex Labels
$f(u_1) = 1, f(u_2) = 3, \cdots f(u_7) = 13$
and $f(v_1) = 2, f(v_2) = 4, \cdots f(v_7) = 14$
and the edge are
$f^*(u_1u_2) = 2, f^*(u_2u_3) = 3, \cdots f^*(u_6u_7) = 5$ $f^*(u_1v_1) = 1, f^*(u_2v_2) = 3, \cdots f^*(u_7v_7) = 127$
Hence the edge labels are distinct hence the comb graph obtain cubic root mean difference labeling of graphs.

Theorem 3.5 *A Triangular snake graph T_n admits cubic root mean difference labeling of graphs*

Proof. Let T_n be a triangular snake Define the function $f : V(T_n) \rightarrow (1,2,\cdots q+1)$ as follows $f(u_i) = (2i+1)$, $1 \le i \le n$, $f(v_i) = 3i$ and we induced edge labeling function $f^* : E(G) \rightarrow N$ is defined by

$$f^*(e = uv) = \left|\sqrt{\frac{f(u)^3 - f(v)^3}{2}}\right|$$

$$f^*(u_i\, u_{i+1}) = \left|\sqrt{\frac{f(u)^3 - f(v)^3}{2}}\right|$$

$$f^*(u_i\, u_{i+1}) = \left|\sqrt{\frac{15i+1}{2}}\right| \quad \text{for } 1 \le i \le n-1$$

Clearly

$$f^*(u_{n-1}u_n) = \left|\sqrt{-12n^2 - 1}\right|$$

$$f^*(u_i v_i) = \left|\sqrt{\frac{-19i^3 + 21i^2 + 6i + 1}{2}}\right|$$

$$f^*(u_n v_n) = \left|\sqrt{\frac{-19n^3 + 21n^2 + 6n + 1}{2}}\right|$$

$$f^*(u_{i+1} v_i) = \left|\sqrt{\frac{-11i + 32}{2}}\right|$$

$$f^*(u_n v_{n-1}) = \left|\sqrt{\frac{-19n^3 + 93n^2 - 75n + 28}{2}}\right|$$

Hence the edge labels are distinct mean difference Labeling of graph

4. conclusion

The study of cubic root mean difference labeling of graph is intersection and it is challenging to investigate some more graph. In this paper we prove Jewel graph, prism graph, Ladder graph comb grpah, Triangular graph.

References

[1]. GALLIAN , A Dynamic Survey of Graph Labeling,, *The Electronic Journal of Combinatiories*, 17, (2010),
[2]. HARARY. F, Graph theory, *Naroso Publishing house Reading, New Delhi* ,(1998). [3] HARARY. F, Graph theorey , *Addison-Wesley Reading ,Mass*,(1972).
[3]. MATHEW.T.K , Varkey some graph Theoritic Generations associated with graph labeling Ph.D Thesis, University of kerela (2000)
[4]. PONRAJ .R and SOMASUNDRANI. S, Mean labeling of graph , *National Academy of Science latters*,26,(2013) 2010-2013
[5]. ROSA . A On certain valuations of Graph, theory of graph (Rome , July 1966) Golden and Breach .N.Y and Paris (1967), 349-355
[6]. SANDHYA, S , SOMASUNDARAM .S and ANUSA .S, Root square Mean Labeling of Graphs, *International Journal contemporary Mathematics Sciences*, 9, (2014), 667-676.
[7]. SANDHYA, S , SOMASUNDARAM .S and ANUSA .S, Some more results on root square mean graph, *Journal Mathematics Research*, 7,No,1 (2015), 72-81.
[8]. SANDHYA, S , SOMASUNDARAM .S and ANUSA .S, Root square Mean Labeling of some New Disconnected Graphs, *International Journal Mathematics Trade and Technology*, 15, (2014), 85-92
[9]. SHIAMA . J, cube difference labeling of some graphs, *IJESIT*, 2(6) , (2013).