Variation Of Labeling Some Family Of Graph

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Article History: Received: 28/10/2020 **Accepted:** 19/11/2020; **Published online:** 14/12/2020.

Abstract
In this paper, we investigate mean square cordial labeling of Windmill graph and star graph and 1 - Near mean cordial labeling of Helm graph

AIMS CLASSIFICATION: 05C78, 65C38

Keywords: Mean Labeling of graphs, cubic root mean labeling of graphs

1. Introduction

Many graph Labeling [1] techniques have been discussed by different researchers and it is still getting enriched due to its broad range of applications in various fields like electical circuit, social psychology addressing communication network system, finding optimal circuit layouts, channel assignment process etc, for the basic notation follows from [2] cahit discussed the cardial labeling [3] and ponraj etal [4] initiated mean square cordial labeling was first introduced by A. Mellai Murugan and etal And they proved that path related graphs
$Sp(p_n,K_{1,n}),(P_2 \cup m_k) + N_2(m - odd),P_n \circ c_3,P_n \circ 2k_{1,m}P_n \circ S_m(neven)$
are mean square cordial graphs K palani J Rejila Jeya Surya introduced a new concept of 1-Near mean square cordial Labeling [5]. In [6], sri Ram Govindaraj showed that the graphs $D_2(P_n),P_n(+1)N_m$, where n is even, Jelly fish $J(m,n)$ are 1-Near mean cordial Labeling of graphs. In this paper we have proved that Helm graph H_n is 1-Near mean cordial Labeling and stargraph and Windmill graph $Nd(L,n),K \geq 2,\ n \geq 2$ is mean square cordial labeling.

2. Preliminaries

Definition 2.1 *The Windmill graph Nd(K,n) is an undirected graph formed for* $K \geq 2$ *and* $n \geq 2$ *by joining n copies of the complete graph* K_n *at a shaned universal vertex.*
Definition 2.2 *A star graph* S_n *is the complete bipartite graph* $K_{1,k}$
Definition 2.3 *A Helm graph* H_n*,* $n \geq 3$ *is a graph formed by attaching a single edge to all the rim vertices of wheel graph*
Definition 2.4 *Let* $G = (V,E)$ *be a simple graph. A surjective function* $f : V(G) \to \{0,1,2\}$ *is said to be 1-Near mean cordial labeling if for each edge uv The induced map*

$$f^*(uv) = \begin{cases} 0, & if\ \frac{f(u)+f(v)}{2} \quad is\ an\ integer \\ 1, & otherwise \end{cases}$$

Satisfies the condition $|ef(0) - ef(1)| \leq 1$*, where* $ef(0)$ *and* $ef(1)$ *is the number of edges label with zero and the number of edges label with one respectively.*

Definition 2.5 *A mean square cordial labeling of a graph* $G = (V,E)$ *with p vertices and q- edges is a surjection from V(G) into* $\{0,1\}$ *such that each edge uv is assigned . The label* $\left\lceil \frac{f(u)^2+f(v)^2}{2} \right\rceil$, $\lceil x \rceil$ *is least square integer greater than or equal to x and satisfies the condition* $|vf(0) - vf(1)| \leq 1$ *and* $|ef(0) - ef(1)| \leq 1$ *where* $vf(0)$ *and* $vf(1)$ *is the number of edges with total zero and the number of vertices label one respectively similar* $ef(0)$ *and* $ef(1)$ *is the number of edge label with zero and the number of edges label with one respectively*

3. Main Result

Theorem 3.1 *The Helm graph* H_n*,* $n \geq 3$ *admits 1-Near mean cordial labeling.*
Proof. Let G be a Helm graph H_n, $n \geq 3$
Let $V(G) = \{c,u_i,v_i / 1 \leq i \leq n\}$

Let
$E(G) = \{cu_i / 1 \leq i \leq n\} \cup \{u_i u_{u+1} / 1 \leq i \leq n-1\} \cup \{u_i u_n\} \cup \{\{u_i v_i\} / 1 \leq i \leq n\}$

$$|V(G)| = 2n + 1$$
$$|E(G)| = 3n$$

Define the vertex labeling $f : V(G) \rightarrow \{0,1,2\}$ by $f(c) = 1$

$$f(v_i) = \begin{cases} 0, & \text{if } i \equiv 1 \pmod 2 \\ 2, & \text{if } i \equiv 0 \pmod 2 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & \text{if } i \equiv 1 \pmod 2 \\ 2, & \text{if } i \equiv 2 \pmod 2 \end{cases}$$

The individual edge labeling $f^* : E(G) \rightarrow \{0,1\}$ by

$$\begin{aligned} f(u_i) &= 1, \quad 1 \leq i \leq n \\ f(u_i u_{i+1}) &= 0, \quad 1 \leq i \leq n-1 \\ f(u_i u_n) &= 0 \end{aligned}$$

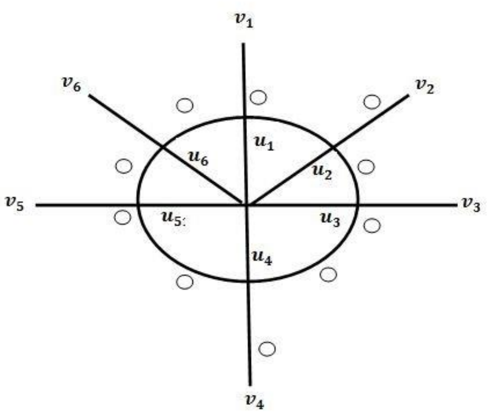
$$f(v_i) = \begin{cases} 1, & \text{if } i \equiv 1 \pmod 2 \\ 0, & \text{if } i \equiv 2 \pmod 2 \end{cases}$$

$$ef(0) = \begin{cases} \frac{3n-1}{2}, & \text{if n is odd} \\ \frac{3n}{2}, & \text{if n is even} \end{cases}$$

$$ef(1) = \begin{cases} \frac{3n+1}{2}, & \text{if n is odd} \\ \frac{3n}{2}, & \text{if n is even} \end{cases}$$

clearly $|ef(0) - ef(1)| \leq 1$
H_n, $n \geq 3$ admits 1-Near mean cordial labeling.

Illustration 3.1. *Helm Graph H_6*



Theorem 3.2 *The Windmill graph Wd(3,n),* $3 \geq 2$ *admits mean square cordial labeling.*
Proof. Let G be a Windmill graph Wd(3,n(, $n \geq 2$ n copies of K_3
$V(G) = \{Cu_i\ 1 \leq i \leq 2n\}$

$$|V(G)| = 2n + 1$$
$$|E(G)| = 3n$$

The vertex Labeling $f : V(G) \to \{0,1\}$ by

$$f(v_i) = 1, n+1 \leq 1 \leq 2n$$

The induced edge labeling

$$f^* : E(G) \to \{0,1\}$$

by

$$f^*(u_i u_{i+1}) = 0, 1 \leq i \leq n \qquad i \text{ is odd}$$
$$f^*(u_i u_{i+1} = 1, \qquad n+1 \leq 2n, \text{ i is odd}$$
$$f^*(u_i) = 0, \qquad 1 \leq i \leq n$$
$$f^*(u_i) = 1, \qquad n+1 \leq i \leq 2n$$
$$\begin{aligned} f(u_i u_{i+1}) &= 0, \quad 1 \leq i \leq n \quad \text{i is odd} \\ f(u_i u_{i+1}) &= n+1, \quad 1 \leq 2n \leq n \quad \text{i is odd} \\ f(u_i) &= 0, \quad n+1 \leq i \leq 2n \end{aligned}$$

$$vf(0) = \begin{cases} \frac{2n+2}{2}, & \text{if n is odd} \\ \frac{2n+2}{2}, & \text{if n is even} \end{cases}$$

$$vf(1) = \begin{cases} \frac{2n}{2}, & \text{if n is odd} \\ \frac{2n}{2}, & \text{if n is even} \end{cases}$$

$$ef(0) = \begin{cases} \frac{3n-1}{2}, & \text{if n is odd} \\ \frac{3n}{2}, & \text{if n is even} \end{cases}$$

$$ef(1) = \begin{cases} \frac{3n+1}{2}, & \text{if n is odd} \\ \frac{3n}{2}, & \text{if n is even} \end{cases}$$

$$\begin{aligned} |vf(0) - vf(1)| &\leq 1 \\ |ef(0) - ef(1)| &\leq 1 \end{aligned}$$

Hence the Windmill graph W(3,n) , $n \geq 2$ admits mean square cordial labeling.

Illustration 3.2.

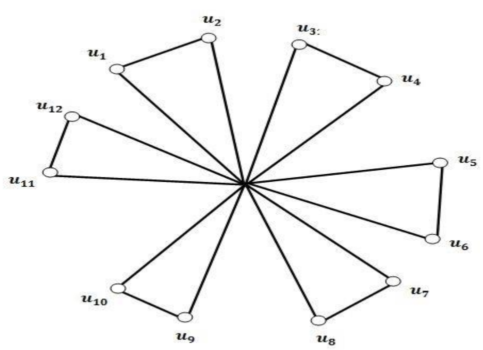


Figure 1: Windmill graph W(3,6)

Theorem 3.3 *The star graph S_n, $n \geq 2$ n is even admits mean square cordial labeling.*

Proof. Let G be the star graph $S_n, n \geq 2$

$$\begin{aligned} V(G) &= \{C, u_i / \ 1 \leq i \leq n-1\} \\ E(G) &= \{Cu_i / \ 1 \leq i \leq n-1\} \\ |V(G)| &= n \\ |E(G)| &= n-1 \end{aligned}$$

The vertex labeling $f : V(G) \to \{0,1\}$ by

$$\begin{aligned} f(c) &= 0, \\ f(u_i) &= 0, \quad 1 \leq i \leq n/2-1 \\ f(u_i) &= 1, \quad n/2 \leq i \leq n-1 \end{aligned}$$

The induced edge labeling $f^* : E(G) \to \{0,1\}$ by

$$\begin{aligned} f(u_i) &= 0, \quad 1 \leq i \leq n/2-1 \\ f(u_i) &= 1, \quad n/2 \leq i \leq n-1 \\ vf(0) &= n/2, \quad vf(1) = n/2 \\ ef(0) &= n-2/2, \quad ef(1) = n/2 \\ |vf(0) - vf(1) &\leq 1 \\ ef(0) - ef(1) &\leq 1 \end{aligned}$$

Therefore, S_n, $n \geq 2$ admits mean square labeling

Illustration 3.3. *Star graph S_9*

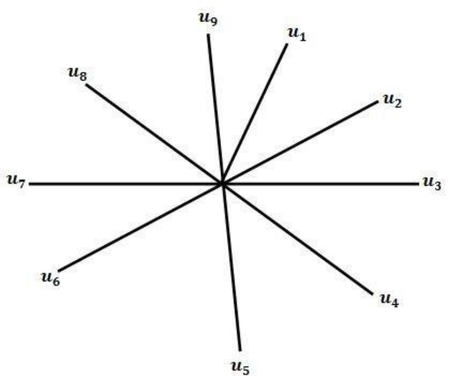


Figure 2: Star graph S_9

Theorem 3.4 *The star graph S_n, $n > 2$ n is odd admits mean square cordial labeling.*
Proof Let G be the star graph $S_n, n > 2$

$$\begin{aligned} V(G) &= \{C, u_i / \ 1 \leq i \leq n-1\} \\ E(G) &= \{Cu_i / \ 1 \leq i \leq n-1\} \\ |V(G)| &= n \\ |E(G) &= n-1 \end{aligned}$$

The vertex Labeling $f^* V(G) \to \{0,1\}$ by

$$\begin{aligned} f(c) &= 0, \\ f(u_i) &= 0, \quad 1 \leq i \leq \lceil n/2 \rceil \\ f(u_i) &= 1, \quad \lceil n/2 \rceil + 1 \leq i \leq n \end{aligned}$$

The individual edge labeling $f^* : E(G) \to \{0,1\}$ by

$$\begin{aligned} f^*(u_i) &= 0, \quad 1 \leq i \leq \lceil n/2 \rceil \\ f^*(u_i) &= 1, \quad \lceil n/2 \rceil + 1 \leq i \leq n \\ vf(0) &= n + 1/2, \quad vf(1) = n - 1/2 \\ ef(0) &= n - 1/2, \quad ef(1) = n - 1/2 \\ |vf(0) - vf(1) &\leq 1 \\ ef(0) - ef(1) &\leq 1 \end{aligned}$$

The star graph S_n n is odd admits mean square cordial labelling

4. conclusion

Here we investigated mean square labeling of windmill and star graphs and 1-Near cordial labeling of helm graph. Further it is open to all the researchers in this domain to discuss the same labeling techniques for the various types of graphs.

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