Constant of Pentagonal Fuzzy Number Matrices

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Abstract:

The fuzzy set theory has been applied in many fields such as management, engineering, theory of matrices and so on. In this paper, some elementary operations on proposed pentagonal fuzzy numbers are defined. We also have been defined some operations on pentagonal fuzzy matrices. The notion of constant of pentagonal fuzzy matrices is proposed and some properties of constant pentagonal fuzzy matrix are verified. Some of their relevant examples are also included to justify the proposed notions.

Keywords: Fuzzy number, Fuzzy arithmetic, Pentagonal fuzzy number, Pentagonal fuzzy matrix, Constant of pentagonal fuzzy matrix**.**

I. Introduction

Fuzzy sets have been introduced by Lofti. A. Zadeh [13]. Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer [2] in 1951, by means of Zadeh's extension principle [12,14], the usual arithmetic operations on real numbers can be extended to the ones defined on fuzzy numbers. Dubosis and Prade [1] has defined any of the fuzzy numbers. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single-valued numbers.

Fuzzy matrices were introduced for the first time by Thomason [11] who discussed the convergence of power of fuzzy matrix. Fuzzy matrices play an important role in scientific development. Two new operations and some applications of fuzzy matrices are given in [6,7,8]. A presented new ranking function and arithmetic operations on type-2 generalized type-2 trapezoidal fuzzy numbers by Stephen Dinagar and Anbalagan [9]. Stephan Dinagar et.al [10] presented some important properties on constant type-2 triangular fuzzy matrices. Jaisankar and Mani [3] proposed the Hessenberg of trapezoidal fuzzy number matrices.

The paper organized as follows, Firstly in section 2, we recall the definition of pentagonal fuzzy number and some operations on pentagonal fuzzy numbers, In section 3, we have reviewed the definition of pentagonal fuzzy matrix and some operations on pentagonal fuzzy matrices. In section 4, we defined the notion of constant of pentagonal fuzzy matrix. In section 5, we have presented some properties of constant of pentagonal fuzzy matrix. Finally in section 6, conclusion is included.

II. Preliminaries

In this section, we recapitulate some underlying definitions and basic results of fuzzy numbers.

Definition: 2.1 (Fuzzy set)

A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A=\{(x,\mu_A(x))\ ;\ x\in X\}$$

Here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x\in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

Definition: 2.2 (Normal fuzzy set)

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_A(X) = 1$.

Definition: 2.3 (Convex fuzzy set)

A fuzzy set $A = \{X, \mu_A(X)\} \subseteq X$ is called convex fuzzy set if all A_α are convex set (i.e.,) for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ for every $\alpha \in [0,1], \lambda x_1 + (1-\lambda)x_2 \in A_\alpha$ for all $\lambda \in [0,1]$ otherwise the fuzzy set is called non-convex fuzzy set.

Definition: 2.4 (Fuzzy number)

A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

i. $\tilde{A}$ is normal
ii. $\tilde{A}$ is convex
iii. The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

Definition: 2.5 (Pentagonal fuzzy number)

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ is said to be a pentagonal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for \quad x < a_1, a_5 \leq x \\ \frac{x-a_1}{a_2-a_1} & for \quad a_1 \leq x \leq a_2 \\ \frac{x-a_2}{a_3-a_2} & for \quad a_2 \leq x \leq a_3 \\ 1 & for \quad x = a_3 \\ \frac{a_4-x}{a_4-a_3} & for \quad a_3 \leq x \leq a_4 \\ \frac{a_5-x}{a_5-a_4} & for \quad a_5 \leq x \leq a_4 \end{cases}$$

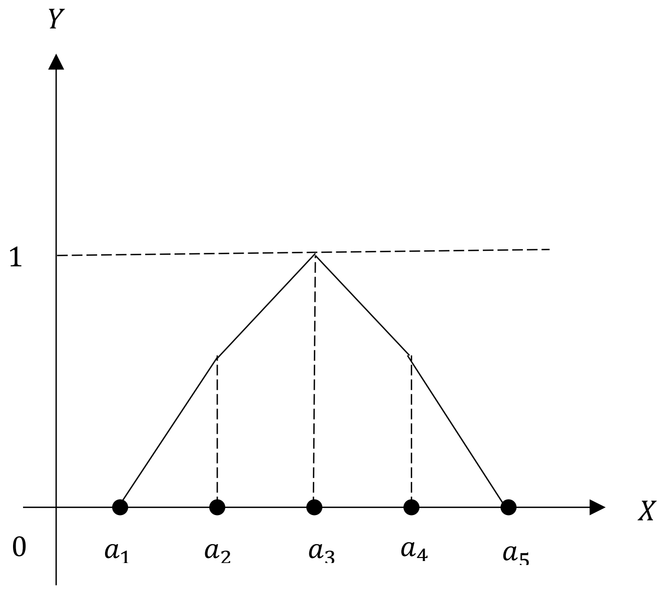

Fig. 1: Pentagonal fuzzy number

Definition: 2.6 (Ranking function)

We define a ranking function $\Re: F(R) \to R$ which maps each fuzzy numbers to real line $F(R)$ represent the set of all pentagonal fuzzy number. If R be any linear ranking function

$$\Re(\tilde{A}) = \left(\frac{a_1+a_2+a_3+a_4+a_5}{5}\right)$$

Also we defined orders on $F(R)$ by

$\Re(\tilde{A}) \geq \Re(\tilde{B})$ if and only if $\tilde{A} \geq_{\Re} \tilde{B}$

$\Re(\tilde{A}) \leq \Re(\tilde{B})$ if and only if $\tilde{A} \leq_{\Re} \tilde{B}$

$\Re(\tilde{A}) = \Re(\tilde{B})$ if and only if $\tilde{A} =_{\Re} \tilde{B}$

Definition: 2.7 (Arithmetic operations on pentagonal fuzzy numbers)

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ be pentagonal fuzzy numbers then we defined,

Addition

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$

Subtraction

$\tilde{A} - \tilde{B} = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1)$

Multiplication

$\tilde{A} \times \tilde{B} = \left(a_1\Re(\tilde{B}), a_2\Re(\tilde{B}), a_3\Re(\tilde{B}), a_4\Re(\tilde{B}), a_5\Re(\tilde{B})\right)$

where $\Re(\tilde{B}) = \left(\frac{b_1+b_2+b_3+b_4+b_5}{5}\right)$ or $\Re(\tilde{b}) = \left(\frac{b_1+b_2+b_3+b_4+b_5}{5}\right)$

Division

$\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{\Re(\tilde{B})}, \frac{a_2}{\Re(\tilde{B})}, \frac{a_3}{\Re(\tilde{B})}, \frac{a_4}{\Re(\tilde{B})}, \frac{a_5}{\Re(\tilde{B})}\right)$

where $\Re(\tilde{B}) = \left(\frac{b_1+b_2+b_3+b_4+b_5}{5}\right)$ or $\Re(\tilde{b}) = \left(\frac{b_1+b_2+b_3+b_4+b_5}{5}\right)$

Scalar multiplication

$$k\tilde{A} = \begin{cases}(ka_1, ka_2, ka_3, ka_4, ka_5) & if \quad k \geq 0 \\ (ka_5, ka_4, ka_3, ka_2, ka_1) & if \quad k < 0\end{cases}$$

Definition: 2.8 (Zero pentagonal fuzzy number)

If $\tilde{A} = (0,0,0,0,0)$ then $\tilde{A}$ is said to be zero pentagonal fuzzy number. It is defined by $\tilde{0}$.

Definition: 2.9 (Zero equivalent pentagonal fuzzy number)

A pentagonal fuzzy number $\tilde{A}$ is said to be a zero equivalent pentagonal fuzzy number if $\Re(\tilde{A}) = 0$. It is defined by $\tilde{0}$.

Definition: 2.10 (Unit pentagonal fuzzy number)

If $\tilde{A} = (1,1,1,1,1)$ then $\tilde{A}$ is said to be unit pentagonal fuzzy number. It is denoted by $\tilde{1}$.

Definition: 2.11 (Unit equivalent pentagonal fuzzy number)

A pentagonal fuzzy number $\tilde{A}$ is said to be a unit equivalent pentagonal fuzzy number if $\Re(\tilde{A}) = 1$. It is denoted by $\tilde{1}$.

Definition: 2.12 (Inverse pentagonal fuzzy number)

If $\tilde{a}$ is pentagonal fuzzy number and $\tilde{a} \neq \tilde{0}$ then we define $\tilde{a}^{-1} = \frac{\tilde{1}}{\tilde{a}}$

Definition: 2.13 (Equal and Equivalent pentagonal fuzzy number)

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ be two pentagonal fuzzy numbers. Then $\tilde{A}$ and $\tilde{B}$ are said to be equal if $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4, a_5 = b_5$. It is denoted by $\tilde{A} = \tilde{B}$. Suppose if $\Re(\tilde{A}) = \Re(\tilde{B})$ then $\tilde{A}$ and $\tilde{B}$ are said to be equivalent fuzzy numbers. It is denoted by $\tilde{A} \approx \tilde{B}$.

Remark: 2.14

Note that all equal pentagonal fuzzy numbers are also equivalent pentagonal fuzzy numbers. But the converse need not be true.

III. Pentagonal Fuzzy Matrices

In this section, we introduced the pentagonal fuzzy matrix and the operations of the matrices some examples provided using the operations.

Definition: 3.1 (Pentagonal fuzzy matrix)

A pentagonal fuzzy matrix of order $m \times n$ is defined as $A = \left(\tilde{a}_{ij}\right)_{m\times n}$, where $a_{ij} = \left(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5}\right)$ is the ij^{th} element of A.

Definition: 3.2 (Operations on Pentagonal fuzzy matrices)

As for classical matrices. We define the following operations on pentagonal fuzzy matrices. Let $A = \left(\tilde{a}_{ij}\right)$ and $B = \left(\tilde{b}_{ij}\right)$ be two pentagonal fuzzy matrices of same order. Then, we have the following

i. **Addition**

$A + B = \left(\tilde{a}_{ij} + \tilde{b}_{ij}\right)$

ii. **Subtraction**

$A - B = \left(\tilde{a}_{ij} - \tilde{b}_{ij}\right)$

iii. For $A = (\tilde{a}_{ij})_{m\times n}$ and $B = (\tilde{b}_{ij})_{n\times k}$ then $AB = (\tilde{c}_{ij})_{m\times k}$ where $\tilde{c}_{ij} = \sum_{p=1}^{n} \tilde{a}_{ip} \,.\, \tilde{b}_{pj}, i = 1,2,\dots,m$ and $j = 1,2,\dots,k$.

iv. A^T or $A^1 = (\tilde{a}_{ji})$.

v. $kA = (k\tilde{a}_{ij})$ where k is scalar.

Examples: 3.2.1

1. If $A = \begin{bmatrix} (-1,-2,1,2,5) & (1,2,3,4,5) \\ (1,3,4,5,7) & (2,4,5,6,8) \end{bmatrix}$ and $B = \begin{bmatrix} (2,4,5,6,8) & (-1,-2,1,2,5) \\ (3,4,5,6,7) & (3,5,6,7,9) \end{bmatrix}$

Then $A + B = (\tilde{a}_{ij} + \tilde{b}_{ij})$

$$A + B = \begin{bmatrix} (-1,-2,1,2,5) & (1,2,3,4,5) \\ (1,3,4,5,7) & (2,4,5,6,8) \end{bmatrix} + \begin{bmatrix} (2,4,5,6,8) & (-1,-2,1,2,5) \\ (3,4,5,6,7) & (3,5,6,7,9) \end{bmatrix}$$

$$A + B = \begin{bmatrix} (1,2,6,8,13) & (0,0,4,6,10) \\ (4,7,9,11,14) & (5,9,11,13,17) \end{bmatrix}$$

2. If $A = \begin{bmatrix} (-1,-2,1,2,5) & (1,2,3,4,5) \\ (1,3,4,5,7) & (2,4,5,6,8) \end{bmatrix}$ and $B = \begin{bmatrix} (2,4,5,6,8) & (-1,-2,1,2,5) \\ (3,4,5,6,7) & (3,5,6,7,9) \end{bmatrix}$

Then $A - B = (\tilde{a}_{ij} - \tilde{b}_{ij})$

$$A - B = \begin{bmatrix} (-1,-2,1,2,5) & (1,2,3,4,5) \\ (1,3,4,5,7) & (2,4,5,6,8) \end{bmatrix} - \begin{bmatrix} (2,4,5,6,8) & (-1,-2,1,2,5) \\ (3,4,5,6,7) & (3,5,6,7,9) \end{bmatrix}$$

$$A - B = \begin{bmatrix} (-9,-8,-4,-2,3) & (-4,0,2,6,6) \\ (-6,-3,-1,1,4) & (-7,-3,-1,1,5) \end{bmatrix}$$

3. If $A = \begin{bmatrix} (-1,-2,1,2,5) & (1,2,3,4,5) \\ (1,3,4,5,7) & (2,4,5,6,8) \end{bmatrix}$ and $B = \begin{bmatrix} (2,4,5,6,8) & (-1,-2,1,2,5) \\ (3,4,5,6,7) & (3,5,6,7,9) \end{bmatrix}$

Then $A \,.\, B = (\tilde{a}_{ij} \,.\, \tilde{b}_{ij})$

$$A \,.\, B = \begin{bmatrix} (-1,-2,1,2,5) & (1,2,3,4,5) \\ (1,3,4,5,7) & (2,4,5,6,8) \end{bmatrix} . \begin{bmatrix} (2,4,5,6,8) & (-1,-2,1,2,5) \\ (3,4,5,6,7) & (3,5,6,7,9) \end{bmatrix}$$

$$A \,.\, B = \begin{bmatrix} (-1,-2,1,2,5)(5) + (1,2,3,4,5)(1) & (-1,-2,1,2,5)(5) + (1,2,3,4,5)(6) \\ (1,3,4,5,7)(5) + (2,4,5,6,8)(1) & (1,3,4,5,7)(5) + (2,4,5,6,8)(6) \end{bmatrix}$$

$$A \,.\, B = \begin{bmatrix} (-4,-8,8,14,30) & (1,2,23,34,55) \\ (7,19,25,31,43) & (17,39,50,61,83) \end{bmatrix}$$

IV. Constant Of Pentagonal Fuzzy Matrix

In this section, we introduce the new matrix namely constant matrix in the fuzzy nature.

Definition: 4.1 (Constant of pentagonal fuzzy matrix)

A pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ of order $n \times n$ is said to be constant pentagonal fuzzy matrix if it is either $R -$constant pentagonal fuzzy matrix or $C -$ constant pentagonal fuzzy matrix.

Definition: 4.2 ($R -$ constant of pentagonal fuzzy matrix)

A pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ of order $n \times n$ is said to be $R -$ constant pentagonal fuzzy matrix if all its rows are equal to each other.

i.e., $\tilde{a}_{ij} = \tilde{a}_{kj}$ for all $i, j, k = 1,2,\dots,n$.

Definition: 4.3 ($C -$ constant of pentagonal fuzzy matrix)

A pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ of order $n \times n$ is said to be $C -$ constant pentagonal fuzzy matrix if all its columns are equal to each other.

i.e., $\tilde{a}_{ij} = \tilde{a}_{ik}$ for all $i, j, k = 1,2,\dots,n$.

Definition: 4.4 (Constant - equivalent of pentagonal fuzzy matrix)

A pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ of order $n \times n$ is said to be constant - equivalent pentagonal fuzzy matrix if it is either $R -$constant - equivalent pentagonal fuzzy matrix or $C -$ constant - equivalent pentagonal fuzzy matrix.

Definition: 4.5 ($R -$ constant - equivalent of pentagonal fuzzy matrix)

A pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ of order $n \times n$ is said to be $R -$ constant - equivalent pentagonal fuzzy matrix if all its rows are equivalent to each other.

i.e., $\tilde{a}_{ij} \approx \tilde{a}_{kj}$ for all $i, j, k = 1,2,\dots,n$.

Definition: 4.6 ($C -$ constant - equivalent of pentagonal fuzzy matrix)

A pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ of order $n \times n$ is said to be $C -$ constant - equivalent pentagonal fuzzy matrix if all its columns are equivalent to each other.

i.e., $\tilde{a}_{ij} \approx \tilde{a}_{ik}$ for all $i, j, k = 1,2, \dots, n$.

Numerical Examples: 4.7

4.7.1: $R -$ constant pentagonal fuzzy matrix

$$A = \begin{bmatrix} (1,3,4,5,7) & (2,4,5,6,8) & (3,4,5,6,7) \\ (1,3,4,5,7) & (2,4,5,6,8) & (3,4,5,6,7) \\ (1,3,4,5,7) & (2,4,5,6,8) & (3,4,5,6,7) \end{bmatrix}$$ is a $R -$ constant pentagonal fuzzy matrix.

4.7.2: $R -$ constant - equivalent pentagonal fuzzy matrix

$$B = \begin{bmatrix} (2,4,5,6,8) & (4,7,9,11,14) & (3,4,5,6,7) \\ (1,2,6,8,13) & (1,3,4,5,7) & (3,4,5,6,7) \\ (0,0,4,6,10) & (1,2,3,4,5) & (1,2,23,34,55) \end{bmatrix}$$ is a $R -$ constant – equivalent pentagonal fuzzy matrix.

4.7.3: $C -$ constant pentagonal fuzzy matrix

$$C = \begin{bmatrix} (1,3,4,5,7) & (1,3,4,5,7) & (1,3,4,5,7) \\ (2,4,5,6,8) & (2,4,5,6,8) & (2,4,5,6,8) \\ (3,4,5,6,7) & (3,4,5,6,7) & (3,4,5,6,7) \end{bmatrix}$$ is a $C -$ constant pentagonal fuzzy matrix.

4.7.4: $C -$ constant - equivalent pentagonal fuzzy matrix

$$D = \begin{bmatrix} (2,4,5,6,8) & (1,2,6,8,13) & (0,0,4,6,10) \\ (4,7,9,11,14) & (1,3,4,5,7) & (1,2,3,4,5) \\ (3,4,5,6,7) & (3,4,5,6,7) & (1,2,23,34,55) \end{bmatrix}$$ is a $C -$ constant – equivalent pentagonal fuzzy matrix.

Both A and C are constant pentagonal fuzzy matrix & B and D are constant – equivalent pentagonal fuzzy matrix.

V. Some Properties Of Constant Of Pentagonal Fuzzy Matrix

In this section, we introduced the properties of constant pentagonal fuzzy matrix.

5.1: Properties of constant pentagonal fuzzy matrix

Property: 5.1.1

If $A = (\tilde{a}_{ij})$ is an $R -$constant pentagonal fuzzy matrix of order $n \times n$ and $B = (\tilde{b}_{ij})$ is a pentagonal fuzzy matrix of the same order then AB is a $R -$constant pentagonal fuzzy matrix of order $n \times n$.

Proof:

Let $A = (\tilde{a}_{ij})$ is an $R -$constant pentagonal fuzzy matrix of order $n \times n$ and $B = (\tilde{b}_{ij})$ be a pentagonal fuzzy matrix of order $n \times n$, where $\tilde{a}_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5})$ and $\tilde{b}_{ij} = (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4}, b_{ij5})$.

Since A is a $R -$ constant pentagonal fuzzy matrix, $\tilde{a}_{ij} = \tilde{a}_{kj}\ \forall\ i, j, k = 1,2, \dots, n$.

Let $C = AB$, i.e., $(\tilde{c}_{ij}) = (\tilde{a}_{ij})\ (\tilde{b}_{ij})$.

Then $\tilde{c}_{ij} = \sum_{m=1}^{n} \tilde{a}_{im}.\tilde{b}_{mj}\ \ \forall i, j = 1,2, \dots, n$.

Since $\tilde{a}_{im} = \tilde{a}_{km}\ \forall\ i, k, m = 1,2, \dots n$. We have $a_{km}\ \sigma\ b = \sum_{m=1}^{n} \tilde{a}_{km}.\tilde{b}_{mj} = \tilde{c}_{kj}$ is a $R -$ constant pentagonal fuzzy matrix.

That is, AB is a $R -$ constant pentagonal fuzzy matrix.

Remark: 1 (Property: 5.1.1)

If $A = (\tilde{a}_{ij})$ is an $R -$constant - equivalent pentagonal fuzzy matrix of order $n \times n$ and $B = (\tilde{b}_{ij})$ is a pentagonal fuzzy matrix of the same order then AB is a $R -$constant - equivalent pentagonal fuzzy matrix of order $n \times n$.

Remark: 2 (Property: 5.1.1)

The product of two $R -$constant matrices is also a $R -$constant matrix and product of two $R -$constant – equivalent matrices is also a $R -$constant – equivalent matrix.

Property: 5.1.2

If $A = (\tilde{a}_{ij})$ is an C −constant pentagonal fuzzy matrix of order $n \times n$ and $B = (\tilde{b}_{ij})$ is a pentagonal fuzzy matrix of the same order then BA is a C −constant pentagonal fuzzy matrix of order $n \times n$.

Proof:

Let $A = (\tilde{a}_{ij})$ is an C −constant pentagonal fuzzy matrix of order $n \times n$ and $B = (\tilde{b}_{ij})$ be a pentagonal fuzzy matrix of order $n \times n$, where $\tilde{a}_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5})$ and $\tilde{b}_{ij} = (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4}, b_{ij5})$.

Since A is a $C -$ constant pentagonal fuzzy matrix, $\tilde{a}_{ij} = \tilde{a}_{ik}\ \forall\ i, j, k = 1,2, \dots, n$.

Let $C = BA$, i.e., $(\tilde{c}_{ij}) = (\tilde{b}_{ij}) . (\tilde{a}_{ij})$.

Then $\tilde{c}_{ij} = \sum_{m=1}^{n} \tilde{b}_{im} . \tilde{a}_{mj}\ \ \forall i, j = 1,2, .., n$.

Since $\tilde{a}_{km} = \tilde{a}_{im}\ \forall\ i, k, m = 1,2, \dots n$. We have $b_{km}\, \sigma\, a = \sum_{m=1}^{n} \tilde{b}_{im} . \tilde{a}_{mk} = \tilde{c}_{ik}$ is a $C -$ constant pentagonal fuzzy matrix.

Since $\tilde{c}_{ij} = \tilde{c}_{ik}$, by definition $c = (\tilde{c}_{ij})$ is a $C -$ constant pentagonal fuzzy matrix.

That is BA is a $C -$ constant pentagonal fuzzy matrix.

Remark: 3 (Property: 5.1.2)

If $A = (\tilde{a}_{ij})$ is an C −constant - equivalent pentagonal fuzzy matrix of order $n \times n$ and $B = (\tilde{b}_{ij})$ is a pentagonal fuzzy matrix of the same order then BA is a C −constant - equivalent pentagonal fuzzy matrix of order $n \times n$.

Remark: 4 (Property: 5.1.2)

The product of two C −constant matrices is also a C −constant matrix and product of two C −constant – equivalent matrices is also a C −constant – equivalent matrix.

Property: 5.1.3

The transpose of a $R -$ constant pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ is a C −constant pentagonal fuzzy matrix and vice versa.

Proof:

Suppose $A = (\tilde{a}_{ij})$ is a R −constant pentagonal fuzzy matrix of order $n \times n$.

Then $\tilde{a}_{ij} = \tilde{a}_{kj}\ \forall\ \ i, j, k = 1,2, \dots, n$. That is, all the rows are equal in A.

Since rows of A become columns of A' all the columns are equal in A'.

That is, $\tilde{a}_{ji} = \tilde{a}_{jk}\ \forall\ \ i, j, k = 1,2, \dots, n$ in A'.

Hence A' is a $C -$ constant pentagonal fuzzy matrix of order $n \times n$.

Remark: 5 (Property: 5.1.3)

The transpose of a $R -$ constant - equivalent pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ is a C −constant – equivalent pentagonal fuzzy matrix and vice versa.

Property: 5.1.4

The transpose of a $C -$ constant pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ is a R −constant pentagonal fuzzy matrix and vice versa.

Proof:

Suppose $A = (\tilde{a}_{ij})$ is a C −constant pentagonal fuzzy matrix of order $n \times n$.

Then $\tilde{a}_{ij} = \tilde{a}_{kj}\ \forall\ \ i, j, k = 1,2, \dots, n$. That is, all the columns are equal in A.

Since columns of A become rows of A' all the rows are equal in A'.

That is, $\tilde{a}_{ji} = \tilde{a}_{jk}\ \forall\ \ i, j, k = 1,2, \dots, n$ in A'.

Hence A' is a $R -$ constant pentagonal fuzzy matrix of order $n \times n$.

Remark: 6 (Property: 5.1.4)

The transpose of a $C -$ constant - equivalent pentagonal fuzzy matrix $A = (\tilde{a}_{ij})$ is a R −constant – equivalent pentagonal fuzzy matrix and vice versa .

VI. Conclusion

In this article, we have concentrate the notion of the constant pentagonal fuzzy matrices are defined and some relevant properties of their constant fuzzy matrices have also been proved. Few illustrations based on operations of pentagonal fuzzy matrices have also been justified. In future, the result about pentagonal fuzzy matrices discussed here may be utilized in further works.

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