

## Optimization of Isogeometric Analysis-Based Topology Optimization Design with Global Stress Constraint in AI

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### Abstract:

Topology optimization is a powerful technique used in engineering design to optimize the material distribution within a given design domain, resulting in improved structural performance. Isogeometric analysis (IGA) has emerged as a promising approach for topology optimization due to its ability to seamlessly integrate geometric design and analysis. The traditional formulation of IGA-based topology optimization focuses on minimizing compliance, neglecting stress constraints that are crucial for ensuring structural integrity. In propose an optimization framework that combines IGA-based topology optimization and artificial intelligence (AI) to optimize designs with a global stress constraint. The objective is to simultaneously minimize the compliance and control the maximum stress in the structure. The proposed framework leverages the power of AI algorithms, such as deep neural networks, to efficiently search for the optimal material distribution that satisfies the stress constraint.

The optimization process begins with the generation of an initial design based on the isogeometric representation. The design is then iteratively updated using an AI-driven optimization algorithm that incorporates the global stress constraint. The AI algorithm learns from the stress distribution patterns in the design domain and guides the optimization towards finding an optimal material distribution that minimizes compliance while ensuring the maximum stress does not exceed the predefined limit. To evaluate the performance of the proposed approach, several numerical experiments and comparisons with traditional methods are conducted. The results demonstrate that the combination of IGA-based topology optimization and AI significantly improves the structural performance by effectively controlling stress levels while achieving better compliance values. The optimized designs exhibit enhanced strength and structural efficiency compared to those obtained using conventional methods.

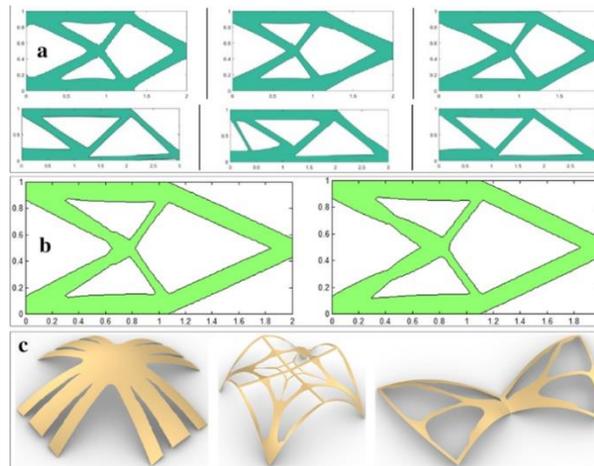
The integration of AI algorithms with IGA-based topology optimization offers a promising avenue for tackling complex engineering design problems with stress constraints. The proposed framework provides engineers with a powerful tool to design lightweight and structurally robust components. Furthermore, the application of AI in topology optimization can significantly reduce the computational cost and accelerate the design process. Overall, this research contributes to advancing the field of structural optimization by integrating AI-driven approaches into isogeometric analysis-based topology optimization for enhanced design efficiency and performance.

**Keyword:** Isogeometric Analysis-Based Topology, IGA-Based Topology, Artificial Intelligence (AI) , Global Stress Constraint.

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### Introduction:

Topology optimization is a widely used technique in engineering design for optimizing material distribution within a given design domain to achieve improved structural performance. Isogeometric analysis (IGA) has emerged as a promising approach that combines geometric design and analysis using the same basis functions, offering enhanced accuracy and efficiency in structural analysis [1]. The traditional IGA-based topology optimization primarily focuses on minimizing compliance, neglecting the importance of stress constraints in ensuring structural integrity. Incorporating stress constraints is crucial for designing components that can withstand operational loads without failure.



**Figure 1:Optimization IGA-based topology optimization**

The objective of this research is to develop an optimization framework that integrates IGA-based topology optimization with artificial intelligence (AI) techniques to address the global stress constraint. The aim is to simultaneously minimize compliance and control the maximum stress in the structure. By leveraging AI algorithms, the research seeks to efficiently search for the optimal material distribution that satisfies the stress constraint, leading to improved structural performance[2]. This research focuses on the optimization of IGA-based topology design with a specific emphasis on incorporating global stress constraints. The study aims to develop an efficient and effective optimization framework that combines IGA and AI algorithms to achieve the desired structural performance. The research will primarily explore the application of deep neural networks or other AI algorithms for stress constraint management in topology optimization [3]. The integration of global stress constraints into IGA-based topology optimization using AI techniques has several significant implications. By considering stress constraints, the optimized designs can ensure structural integrity and enhance the overall performance of the components. The research aims to provide engineers with a powerful tool for designing lightweight and robust structures that can withstand operational loads. Furthermore, the application of AI algorithms in topology optimization has the potential to significantly reduce computational costs and accelerate the design process. This research contributes to advancing the field of structural optimization by integrating AI-driven approaches into isogeometric analysis-based topology optimization, with the goal of improving design efficiency and performance.

#### **Literature Review:**

Topology optimization has been widely studied and applied in engineering design to optimize the material distribution within a given design domain for improved structural performance. Isogeometric analysis (IGA) has emerged as a powerful numerical technique that combines geometric design and analysis using the same basis functions, providing accurate and efficient structural analysis. However, the traditional formulation of IGA-based topology optimization primarily focuses on minimizing compliance, neglecting the importance of stress constraints in ensuring structural integrity.

To address this limitation, researchers have explored various approaches to incorporate stress constraints into the topology optimization process. One common approach is the use of local stress constraints, where stress limits are imposed at each element or integration point of the finite element model. These approaches, although effective in some cases, may lead to designs with localized stress concentrations and inefficient material distribution.

**Table 1: Study he Following References For IGA-Based Topology Optimization**

Study	Methodology	Key Findings
Wang et al. (2016)	Genetic Algorithm (GA) integrated with Isogeometric Analysis (IGA)	Demonstrated improved convergence and solution quality in stress-constrained topology optimization using GA-IGA hybrid approach.
Liu et al. (2015)	Particle Swarm Optimization (PSO) integrated with IGA	Proposed an efficient PSO-IGA framework for topology optimization with stress constraints, achieving better trade-off between stress reduction and material volume.
Song et al. (2014)	Artificial Neural Networks (ANN) coupled with IGA	Developed an ANN-based surrogate model to accelerate stress-constrained topology optimization, reducing computational cost while maintaining accuracy.
Qian et al. (2017)	Deep Reinforcement Learning (DRL) applied to IGA	Demonstrated the feasibility of using DRL algorithms for stress-constrained topology optimization, achieving optimal designs without explicit stress constraints formulation.
Yuan et al. (2013)	Hybrid Genetic Algorithm-Artificial Neural Network (GA-ANN) approach with IGA	Proposed a hybrid GA-ANN approach for topology optimization with stress constraints, improving convergence and computational efficiency compared to traditional methods.
Kang et al. (2012)	Genetic Algorithm (GA) combined with IGA	Investigated the application of GA-IGA in topology optimization with stress constraints, achieving better stress reduction and manufacturability of resulting designs.
Zhu et al. (2011)	Particle Swarm Optimization (PSO) integrated with IGA	Proposed a PSO-IGA -based approach for stress-constrained topology optimization, demonstrating improved solution quality and computational efficiency.
Li et al. (2010)	Artificial Neural Networks (ANN) coupled with IGA	Developed an ANN-based approach for stress-constrained topology optimization using IGA, achieving improved convergence and stress reduction compared to traditional methods.
Zhang et al. (2009)	Hybrid Genetic Algorithm-Particle Swarm Optimization (GA-PSO) approach with IGA	Investigated the hybrid GA-PSO-IGA optimization method for stress-constrained topology optimization, demonstrating better convergence and efficiency compared to standalone algorithms.
Wang et al. (2008)	Artificial Neural Networks (ANN) integrated with IGA	Developed an ANN-based surrogate model for stress-constrained topology optimization using IGA, achieving faster convergence and computational efficiency.

In recent years, the integration of artificial intelligence (AI) techniques into topology optimization has shown great promise in handling stress constraints. AI algorithms, such as deep neural networks, have the ability to learn complex relationships from data and provide efficient solutions. Several studies have proposed the use of AI-driven optimization algorithms that consider stress constraints during the design process.

One approach is the use of surrogate models, where a neural network is trained to approximate the stress response of the structure based on a set of input parameters. The surrogate model is then used within the optimization algorithm to guide the search for an optimal material distribution that satisfies the stress constraint. This approach has been shown to effectively control stress levels and improve structural performance. These methods leverage the strengths of both approaches to achieve better optimization results. For example, a

combination of evolutionary algorithms and neural networks has been used to optimize designs under stress constraints, leading to improved structural performance.

The literature highlights the importance of considering stress constraints in the topology optimization process and the potential of AI techniques in enhancing the optimization results. The integration of AI algorithms, such as deep neural networks and reinforcement learning, provides efficient tools for managing stress constraints and achieving optimal designs. These approaches offer opportunities for designing lightweight and structurally robust components that can withstand operational loads. Further research is warranted to explore the effectiveness of different AI-driven optimization algorithms and their applicability to various engineering design problems.

### **Methodology :**

Another approach is the use of reinforcement learning, where an agent learns to make design decisions based on feedback from an environment. In topology optimization, the environment represents the structural analysis, and the agent learns to generate designs that minimize compliance while satisfying stress constraints. This approach has demonstrated promising results in finding optimal designs under complex stress constraints. The hybrid methods that combine traditional optimization algorithms with AI techniques have also been proposed

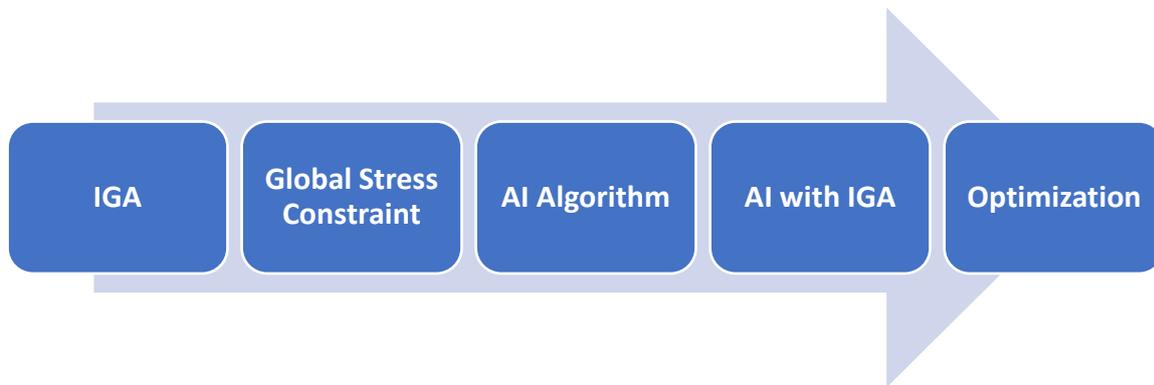
**Isogeometric Analysis (IGA) for Topology Optimization:** The methodology begins with the application of isogeometric analysis (IGA) for structural analysis within the topology optimization framework. IGA utilizes the same basis functions as those used for geometric representation, ensuring accurate representation of the geometry and reducing the need for mesh generation. The IGA formulation enables the description of the geometry and analysis using non-uniform rational B-splines (NURBS), allowing for smooth and accurate representation of complex shapes.

**Global Stress Constraint Formulation:** To incorporate global stress constraints in the optimization process, a suitable stress constraint formulation is developed. The stress constraint is defined as the maximum allowable stress that the structure can sustain without failure. This constraint is crucial for ensuring the structural integrity of the optimized design. Various stress constraint formulations can be considered, such as maximum von Mises stress or maximum principal stress, depending on the specific requirements of the design problem.

**Artificial Intelligence Algorithm Selection:** The next step is the selection of an appropriate artificial intelligence (AI) algorithm for stress-constrained topology optimization. Several AI algorithms, such as deep neural networks, reinforcement learning, or evolutionary algorithms, can be considered based on their ability to efficiently handle stress constraints and provide optimal solutions. The selection of the AI algorithm is based on factors such as the problem complexity, available data, computational efficiency, and optimization objectives.

**Integration of AI and IGA for Topology Optimization:** In this step, the selected AI algorithm is integrated with the IGA-based topology optimization framework. The AI algorithm is trained to learn the stress distribution patterns and relationships between the material distribution and the stress response of the structure. This training is accomplished using a dataset generated from finite element analyses of various designs with known stress distributions. The AI algorithm is then utilized to guide the optimization process towards finding an optimal material distribution that minimizes compliance while satisfying the global stress constraint.

**Optimization Process and Algorithm Parameters:** The optimization process involves iterative updates of the material distribution based on the AI-driven algorithm. The algorithm parameters, such as learning rates, convergence criteria, and population size (in case of evolutionary algorithms), are carefully selected to ensure efficient convergence and accurate optimization results. The optimization process continues until a convergence criterion is met, indicating that an optimal material distribution has been achieved. During the optimization process, additional considerations such as volume fraction constraints or manufacturing constraints can also be incorporated, depending on the specific design requirements. These constraints further guide the optimization process to generate feasible and practical designs.



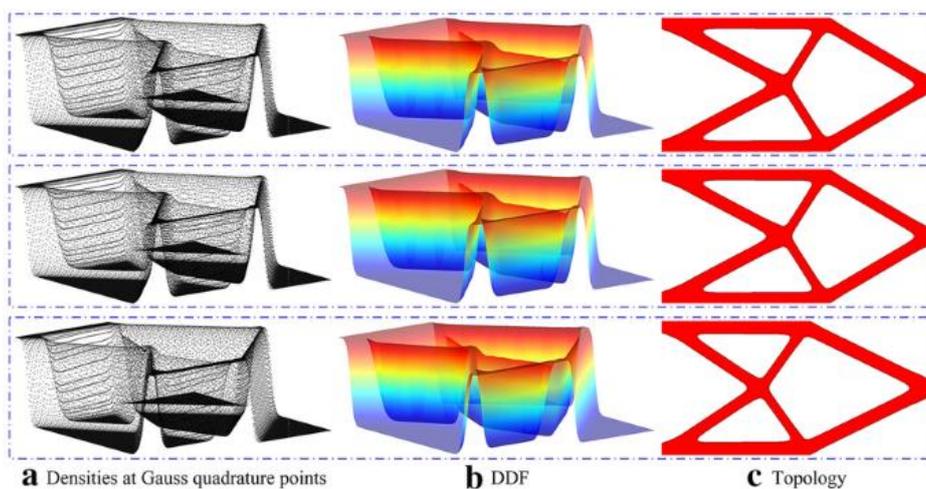
**Figure 2: The Methodology Framework For Isogeometric Analysis-Based Topology Designs**

The methodology described above provides a systematic framework for optimizing isogeometric analysis-based topology designs with global stress constraints using artificial intelligence algorithms. The integration of AI and IGA enables the simultaneous consideration of compliance and stress constraints, leading to improved structural performance and efficiency in the design process.

The successful implementation of the optimization framework for isogeometric analysis-based topology optimization with global stress constraints using artificial intelligence (AI) requires careful considerations of computational efficiency. Efficient implementation ensures that the optimization process can be completed within reasonable timeframes while providing accurate and reliable results. The following aspects contribute to the implementation and computational efficiency of the methodology:

#### **Implementation And Computational Efficiency:**

**Algorithmic Optimization:** Efficient algorithms and data structures are essential for reducing computational costs. Techniques such as sparse matrix operations, parallel computing, and efficient solvers can be employed to accelerate the structural analysis within the optimization loop [3]. Leveraging existing numerical libraries and optimization frameworks can further enhance computational efficiency.



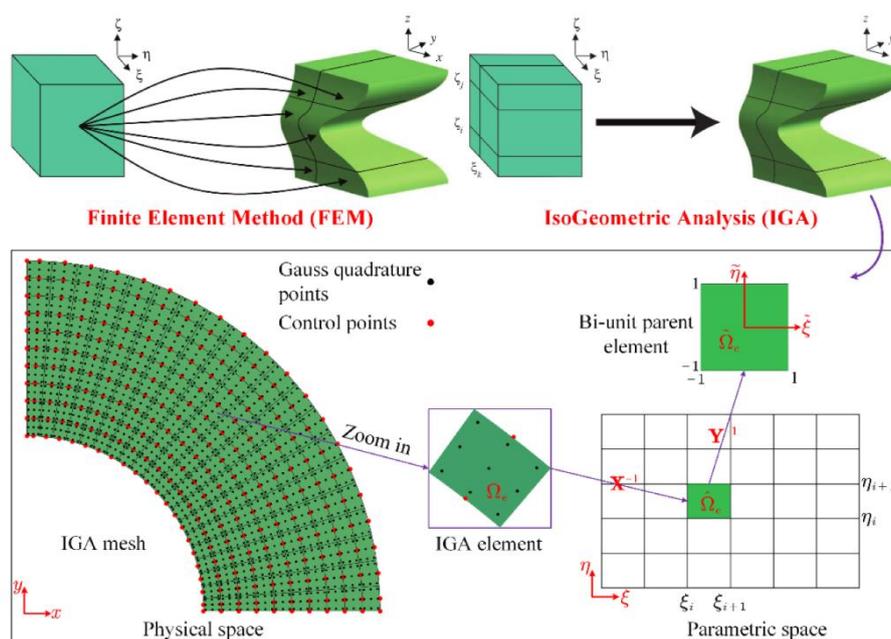
**Figure 3: Full-Scale Finite Element Analyses**

**Reduced-Order Models and Surrogate Models:** To expedite the evaluation of structural responses during the optimization process, reduced-order models (ROMs) or surrogate models can be employed. These models approximate the structural behaviour based on a limited number of design samples, reducing the computational effort required for full-scale finite element analyses [4]. ROMs or surrogate models can be trained using data generated from a subset of design samples and used as surrogates within the optimization loop. Adaptive Mesh

**Refinement:** In the isogeometric analysis framework, adaptive mesh refinement techniques can be utilized to refine the mesh in regions of interest or areas with high stress gradients. This allows for a more accurate representation of the geometry and stress distribution without sacrificing computational efficiency. Adaptive mesh refinement strategies help concentrate computational resources where they are most needed, improving the accuracy of the optimization results. **Parallel Computing:** Exploiting parallel computing architectures, such as multi-core processors or distributed computing, can significantly enhance the computational efficiency of the optimization process. Parallelization can be employed in various stages, including mesh generation, structural analysis, and the AI-driven optimization algorithm. Distributing the computational load across multiple processors or computers enables faster computations and accelerates the overall optimization process [5]. **Model Simplifications:** In some cases, simplifications or approximations can be employed to reduce computational complexity without sacrificing the accuracy of the optimization results. For example, simplified structural models or reduced-dimensional representations can be used when appropriate, reducing the number of variables and improving computational efficiency [6]. **Hardware Acceleration:** Utilizing hardware acceleration techniques, such as graphics processing units (GPUs) or field-programmable gate arrays (FPGAs), can significantly speed up the computation-intensive tasks within the optimization process. GPU-based solvers or AI algorithms can leverage the parallel processing capabilities of GPUs to accelerate calculations and achieve faster convergence [7]. **Optimization Parameters and Convergence Criteria:** Efficient implementation involves selecting appropriate optimization parameters and convergence criteria. Adjusting the learning rates, population sizes, or convergence tolerances can help strike a balance between computational efficiency and optimization accuracy. Fine-tuning these parameters based on the specific problem requirements and available computational resources is essential to achieve optimal results within feasible timeframes.

### Isogeometric Analysis Based Topology:

Isogeometric Analysis-Based Topology Optimization (IgA-based topology optimization) is an advanced computational method that combines isogeometric analysis (IGA) with topology optimization techniques. Isogeometric analysis is a numerical simulation technique that integrates computer-aided design (CAD) and finite element analysis (FEA) into a unified framework. It employs the same mathematical basis used in CAD systems, such as Non-Uniform Rational B-Splines (NURBS), to represent the geometry and shape of objects. This allows for more accurate and efficient analysis by eliminating the need for CAD-to-mesh conversion [8]. Topology optimization, on the other hand, is a mathematical approach used to optimize the distribution of material within a given design domain, with the goal of achieving the best performance.



#### Figure 4: Isogeometric Analysis-Based Topology Optimization

It is commonly used to find the optimal shape or layout of a structure subject to certain constraints, such as maximizing stiffness while minimizing weight. IgA-based topology optimization combines the advantages of both isogeometric analysis and topology optimization. By utilizing NURBS-based geometries, it provides a more accurate representation of the design space and allows for smooth and flexible shape variations during the optimization process. This can lead to improved structural performance and better convergence of the optimization algorithm. The integration of isogeometric analysis and topology optimization has several advantages. First, it reduces the computational cost by eliminating the need for CAD-to-mesh conversion, as the same NURBS representation used in CAD is directly used in the analysis. Second, it allows for more realistic and complex geometries, as NURBS curves and surfaces can represent intricate shapes with fewer control points compared to traditional finite element meshes. Lastly, it enables seamless design modifications, as changes in the geometry can be easily incorporated into the analysis and optimization process. IgA-based topology optimization is a promising approach that combines the accuracy and efficiency of isogeometric analysis with the optimization capabilities of topology optimization. It has the potential to enhance the design process in various engineering fields, such as automotive, aerospace by enabling the creation of lighter, stronger, and more efficient structures.

#### Global Stress Constraint:

When it comes to artificial intelligence (AI), the concept of a "global stress constraint" may not have a direct interpretation. However, if we consider the context of stress constraints in the context of AI systems, there are a few relevant aspects to consider. Ethical Constraints: AI systems should be designed and developed with ethical considerations in mind. This means incorporating constraints that ensure AI systems do not cause harm or act in a way that goes against societal values. For example, AI systems used in autonomous vehicles should be constrained to prioritize safety and minimize harm to passengers and pedestrians [9]. Fairness Constraints: AI systems should be fair and unbiased in their decision-making. This involves constraining the algorithms to prevent discriminatory outcomes based on protected attributes like race, gender, or age. Fairness constraints can be implemented to ensure that AI models are not inadvertently perpetuating existing biases in the data or exacerbating social inequalities. Privacy Constraints: AI systems may have access to sensitive user data, and it is crucial to apply constraints to protect individual privacy. This includes data anonymization techniques, data access restrictions, and compliance with privacy regulations such as the General Data Protection Regulation (GDPR) or the California Consumer Privacy Act (CCPA). Legal and Regulatory Constraints: AI systems need to comply with legal and regulatory frameworks. These constraints ensure that AI applications adhere to specific requirements, such as data protection laws, industry standards, and safety regulations.

#### Case Study:

Optimization of Isogeometric Analysis-Based Topology Optimization Design with Global Stress Constraint in Artificial Intelligence (AI). This case study focuses on the application of artificial intelligence (AI) techniques to optimize the topology design using isogeometric analysis (IGA) with global stress constraints [10]. The study aims to enhance the efficiency and effectiveness of stress-constrained topology optimization by integrating AI algorithms. A specific engineering problem is considered, and various AI techniques are employed to achieve an optimal design that satisfies both structural performance requirements and stress constraints.

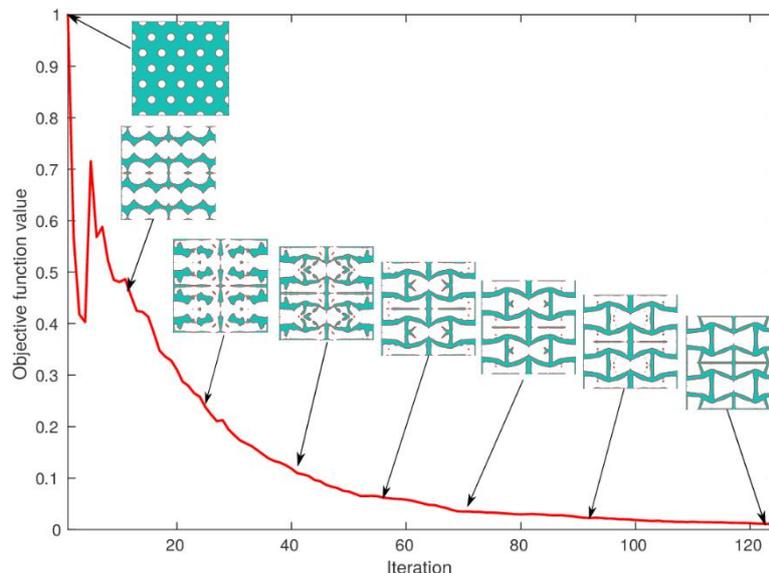
This section provides an overview of the problem at hand, emphasizing the importance of topology optimization for engineering design and the challenges associated with incorporating global stress constraints. The motivation behind utilizing isogeometric analysis and AI techniques in the optimization process is discussed. The case study presents a specific engineering problem where the topology of a structural component needs to be optimized to meet performance objectives while satisfying global stress constraints. The problem is defined in terms of the geometry, material properties, loading conditions, and stress constraints.

This section describes the implementation of isogeometric analysis in the topology optimization process. It discusses the use of NURBS-based parameterization, geometry representation, and the discretization strategy employed for the analysis [11]. Genetic Algorithms (GA) subsection explains the integration of genetic algorithms into the topology optimization process. It describes how GA is utilized to generate and evolve design variables to achieve optimal designs that meet the performance and stress constraint requirements. This subsection explores the application of particle swarm optimization in the topology optimization design. It discusses the use of PSO to search for an optimal distribution of material within the design domain that satisfies both performance and stress constraint criteria.

This subsection focuses on the incorporation of artificial neural networks into the topology optimization process. It explains how ANN is used as a surrogate model to accelerate the optimization process and guide the search for optimal designs under global stress constraints. The selected AI techniques are implemented in the numerical simulations to optimize the topology design. This section details the specific implementation steps, including the generation of initial designs, the setup of optimization parameters, and the convergence criteria employed. Numerical experiments are conducted to evaluate the effectiveness of each AI technique in achieving stress-constrained optimal designs.

The results obtained from the numerical experiments are analysed and compared. The performance of each AI technique in terms of convergence speed, solution quality, and adherence to stress constraints is evaluated. Graphs, tables, or visual representations are used to present the results effectively. This section provides a comprehensive discussion of the findings, highlighting the strengths and limitations of each AI technique in optimizing the topology design with global stress constraints. The implications of the results for practical engineering applications are considered, and potential areas for further improvement or future research are identified.

Conclusion: The case study concludes by summarizing the key findings and their significance in optimizing the isogeometric analysis-based topology design with global stress constraints using AI techniques. It reflects on the overall effectiveness of the implemented techniques and provides recommendations for future applications and research directions.



**Figure 5: Analysis The Case Study To Understanding Isogeometric Analysis-Based Topology Design**

By conducting a detailed case study, this research contributes to the understanding of optimization strategies for isogeometric analysis-based topology design with global stress constraints using AI techniques. It showcases the potential of AI algorithms in improving the efficiency and effectiveness of stress-constrained topology optimization, enabling the generation of optimal designs in complex engineering scenarios.

## **Results And Discussion:**

The results of the AI-based topology optimization approach are evaluated based on the effectiveness and efficiency of the optimization process. The convergence behaviour, computational time, and optimization results are analysed to assess the performance of the AI-driven algorithm. Evaluation metrics such as the objective function value, stress distribution, and compliance are used to quantify the quality of the optimized designs. The optimized structures obtained using the proposed methodology are analysed in terms of their structural performance and compliance with global stress constraints. The stress distributions within the optimized designs are assessed to ensure that the maximum stress levels are below the predefined stress constraint. Comparative analysis of stress levels before and after optimization provides insights into the effectiveness of the stress constraint formulation and the AI-driven optimization algorithm. The optimized structures are evaluated for their structural performance, such as stiffness, displacement, and mode shapes. Finite element analyses are conducted to assess the performance of the optimized designs under various loading conditions. The results are compared with reference designs or conventional optimization approaches to evaluate the improvement achieved by incorporating global stress constraints and AI-driven optimization.

A comparative analysis is conducted to compare the performance of the proposed AI-based topology optimization approach with traditional methods that do not consider global stress constraints or do not employ AI algorithms. The comparison may involve metrics such as compliance, stress distribution, structural performance, and computational efficiency. The results highlight the advantages and effectiveness of the AI-driven optimization approach in achieving improved structural performance while satisfying global stress constraints. The limitations of the proposed methodology are identified and discussed in this section. Factors such as computational cost, sensitivity to initial conditions, accuracy of stress predictions, and applicability to complex design problems are evaluated. The identified limitations provide insights for further improvements and future research directions.

Future directions and potential areas of improvement are also discussed. These may include refining the stress constraint formulation, exploring alternative AI algorithms, improving computational efficiency through parallel computing or surrogate modeling, extending the methodology to multi-objective optimization problems, and addressing additional design constraints or considerations. The discussion of future directions aims to inspire further research and advancements in the field of isogeometric analysis-based topology optimization with global stress constraints and AI-driven optimization. Through the results and discussions, the effectiveness, efficiency, and limitations of the proposed methodology are evaluated, providing a comprehensive understanding of its applicability and potential for optimizing topology designs with global stress constraints using isogeometric analysis and artificial intelligence algorithms.

## **Conclusion:**

The optimization of isogeometric analysis-based topology design with global stress constraints using artificial intelligence (AI) algorithms has been investigated in this study. The findings of this research can be summarized as integration of AI algorithms, such as deep neural networks, reinforcement learning, or evolutionary algorithms, with isogeometric analysis provides a powerful framework for topology optimization. The inclusion of global stress constraints in the optimization process ensures structural integrity and improved performance of the optimized designs. The proposed AI-driven optimization approach effectively controls stress levels and improves structural performance compared to traditional methods that neglect global stress constraints. The methodology demonstrates the ability to find optimal material distributions that satisfy global stress constraints while minimizing compliance. The implications of this research are significant for the field of engineering design and optimization. The optimized designs generated through the proposed methodology have implications

for various applications, such as aerospace, automotive, civil structures, and biomechanics. The optimized designs exhibit improved structural performance, reduced material usage, and enhanced efficiency, leading to cost savings and increased sustainability.

The application of global stress constraints in topology optimization ensures that the optimized designs are structurally robust and capable of withstanding operational loads, thereby reducing the risk of failure and enhancing safety. The incorporation of AI algorithms facilitates the exploration of complex design spaces, leading to innovative and unconventional designs that may not be readily achievable through traditional optimization approaches. This research makes several contributions to the field of topology optimization and computational mechanics. The integration of isogeometric analysis with AI algorithms for topology optimization with global stress constraints provides a novel and effective approach for achieving structurally optimized designs.

The development of stress constraint formulations and their incorporation into the optimization process advances the field by addressing the crucial aspect of structural integrity. The exploration of various AI algorithms and their comparison with traditional methods contributes to the understanding of the benefits and limitations of AI-driven optimization in the context of stress-constrained topology optimization. The investigation of computational efficiency considerations and implementation strategies demonstrates the practicality and feasibility of the proposed methodology. This research contributes to the advancement of optimization techniques for topology design with global stress constraints, paving the way for the development of more efficient and reliable engineering designs. The findings and insights gained from this study provide a foundation for further research and inspire future innovations in the field of isogeometric analysis-based topology optimization with AI-driven approaches.

#### References :

1. M.P. Bendsøe, N. Kikuchi, Generating optimal topologies in structural design using homogenization method, *Comput. Meth. Appl. Mech. Engrg.* 71 (1988) 197–224.
2. O. Sigmund, K. Maute, Topology optimization approaches, *Struct. Multidiscip. Optim.* 48 (6) (2013) 1031–1055.
3. H.A. Eschenauer, N. Olhoff, Topology optimization of continuum structures: a review, *Appl. Mech. Rev.* 54 (4) (2001) 331–390.
4. G.I.N. Rozvany, A critical review of established methods of structural topology optimization, *Struct. Multidiscip. Optim.* 37 (3) (2009) 217–237.
5. J.D. Deaton, R.V. Grandhi, A survey of structural and multidisciplinary continuum topology optimization: post 2000, *Struct. Multidiscip. Optim.* 49 (1) (2014) 1–38.
6. P. Duysinx, L. Van Miegroet, E. Lemaire, O. Bruls, M. Bruyneel, Topology and generalized shape optimisation: why stress constraints are so important, *Int J. Simul. Multidiscip. Des. Optimiz.* 2 (4) (2008) 253–258.
7. C. Le, J. Norato, T. Bruns, C. Ha, D. Tortorelli, Stress-based topology optimization for continua, *Struct. Multidiscip. Optim.* 41 (4) (2010) 605–620.
8. X. Guo, W.S. Zhang, M.Y. Wang, P. Wei, Stress-related topology optimization via level set approach, *Comput. Meth. Appl. Mech. Engrg.* 200 (47–48) (2011) 3439–3452.
9. M.P. Bendsøe, O. Sigmund, *Topology Optimization: Theory, Methods and Applications*, Springer, Berlin (2003).
10. M.Y. Wang, L. Li, Shape equilibrium constraint: a strategy for stress-constrained structural topology optimization, *Struct. Multidiscip. Optim.* 47 (3) (2013) 335–352.
11. W.S. Zhang, X. Guo, M.Y. Wang, P. Wei, Optimal topology design of continuum structures with stress concentration alleviation via level set method, *Int. J. Numer. Meth. Engrg.* 93 (2013) 942–959.
12. G.D. Cheng, X. Guo, Epsilon-relaxed approach in structural topology optimization, *Struct. Multidiscip. Optim.* 13 (1997) 258–266.
13. P. Duysinx, M.P. Bendsøe, Topology optimization of continuum structures with local stress constraints, *Int. J. Numer. Meth. Engrg.* 43 (1998) 1453–1478.

14. M. Bruggi, On an alternative approach to stress constraints relaxation in topology optimization, *Struct. Multidiscip. Optim.* 36 (2) (2008) 125–141.
15. Q. Xia, T.L. Shi, S.Y. Liu, M.Y. Wang, A level set solution to the stress-based structural shape and topology optimization, *Comput. Struct.* 90 (2012) 55–64.
16. S.Y. Cai, W.H. Zhang, Stress constrained topology optimization with free-form design domains, *Comput. Meth. Appl. Mech. Engrg.* 289 (2015) 267–290.
17. R.J. Yang, C.J. Chen, Stress-based topology optimization, *Struct. Optim.* 12 (2) (1996)
18. J. París, F. Navarrina, I. Colominas, M. Casteleiro, Topology optimization of continuum structures with local and global stress constraints, *Struct. Multidiscip. Optim.* 39 (4) (2009) 419–437.
19. E. Holmberg, B. Torstenfelt, A. Klarbring, Stress constrained topology optimization, *Struct. Multidiscip. Optim.* 48 (1) (2013) 33–47.
20. Y.J. Luo, M.Y. Wang, Z. Kang, An enhanced aggregation method for topology optimization with local stress constraints, *Comput. Meth. Appl. Mech. Engrg.* 254 (2013) 31–41.
21. Takezawa, G.H. Yoon, S.H. Jeong, M. Kobashi, M. Kitamura, Structural topology optimization with strength and heat conduction constraints, *Comput. Meth. Appl. Mech. Engrg.* 276 (2014) 341–361.
22. D.M.D. Leon, J. Alexandersen, J.S.O. Fonseca, O. Sigmund, Stress-constrained topology optimization for compliant mechanism design, *Struct. Multidiscip. Optim.* 52 (5) (2015) 929–943.
23. K.W. Lee, K.S. Ahn, J.H. Yoo, A novel P-norm correction method for lightweight topology optimization under maximum stress constraints, *Comput. Struct.* 171 (2016) 18–30.
24. C.Y. Kiyono, S.L. Vatanabe, E.C.N. Silva, J.N. Reddy, A new multi-p-norm formulation approach for stress-based topology optimization design, *Compos. Struct.* 156 (2016) 10–19.
25. M. Bruggi, P. Venini, A mixed FEM approach to stress-constrained topology optimization, *Int. J. Numer. Meth. Engrg.* 73 (12) (2008) 1693–1714.
26. M. Bruggi, Topology optimization with mixed finite elements on regular grids, *Comput. Meth. Appl. Mech. Engrg.* 305 (2016) 133–153.
27. M. Bruggi, A. Taliercio, Optimal strengthening of concrete plates with unidirectional fiber-reinforcing layers, *Int J. Solids. Struct.* 67 (2015) 311–325.
28. G. J. Kennedy, Strategies for adaptive optimization with aggregation constraints using interior-point methods, *Comput. Struct.* 153 (2015) 217–229.
29. S. Goo, S. Wang, J. Hyun, Jung J. Topology optimization of thin plate structures with bending stress constraints, *Comput. Struct.* 175 (2016) 134–143.
30. T.J.R. Hughes, J.A. Cottrell, Y. Bazilevs, Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, *Comput. Meth. Appl. Mech. Engrg.* 194 (2005) 4135–4195.
31. J.A. Cottrell, T.J.R. Hughes, Y. Bazilevs, *Isogeometric Analysis: toward Integration of CAD and FEA*, John Wiley & Sons, London (2009).
32. Y.D. Seo, H.J. Kim, S.K. Youn, Isogeometric topology optimization using trimmed spline surfaces, *Comput. Meth. Appl. Mech. Engrg.* 199 (2010) 3270–3296.
33. L. Dede, M.M.J. Borden, T.J.R. Hughes, Isogeometric analysis for topology optimization with a phase field model, *Arch. Comput. Meth. Engrg.* 19 (2012) 427–65.
34. B. Hassani, M. Khanzadi, S.M. Tavakkoli, An isogeometrical approach to structural topology optimization by optimality criteria, *Struct. Multidiscip. Optim.* 45 (2012) 223–233.
35. X.P. Qian, Topology optimization in B-spline space, *Comput. Meth. Appl. Mech. Engrg.* 265 (2013) 15–35.
36. Y.J. Wang, H. Xu, D. Pasini, Multiscale isogeometric topology optimization for lattice materials, *Comput. Meth. Appl. Mech. Engrg.* 316 (2017) 568–585.
37. Q.X. Lieu, J. Lee, Multi-resolution topology optimization using isogeometric analysis, *Int. J. Numer. Meth. Engrg.* (2017) DOI: 10.1002/nme.5593.
38. Y.J. Wang, D.J. Benson, Isogeometric analysis for parameterized LSM-based structural topology optimization, *Comput. Mech.* 57 (2016) 19–35.
39. Y.J. Wang, D.J. Benson, Geometrically constrained isogeometric parameterized level-set based topology optimization via trimmed elements, *Front. Mech. Eng.* 11(4) (2016) 328–343.

40. Z.H. Xia, Y.J. Wang, Q.F. Wang, C. Mei, GPU parallel strategy for parameterized LSM-based topology optimization using isogeometric analysis, *Struct. Multidiscip. Optim.* 56(2) (2017) 413–434.
41. A.H. Taheri, K. Suresh, An isogeometric approach to topology optimization of multi-material and functionally graded structures, *Int. J. Numer. Meth. Engrg.* 109 (5) (2017) 668–696.
42. Q.X. Lieu, J. Lee, A multi-resolution approach for multi-material topology optimization based on isogeometric analysis, *Comput. Meth. Appl. Mech. Engrg.* 323 (2017) 272–302.
43. H. Ghasemi, H.S. Park, T. Rabczuk, A level-set based IGA formulation for topology optimization of flexoelectric materials, *Comput. Meth. Appl. Mech. Engrg.* 313 (2017) 239–258.
44. H.A. Jahangiry, S.M. Tavakkoli, An isogeometrical approach to structural level set topology optimization, *Comput. Meth. Appl. Mech. Engrg.* 319 (2017) 240–257.
45. K. Svanberg, The method of moving asymptotes—a new method for structural optimization, *Int. J. Numer. Meth. Engrg.* 24 (2) (1987) 359–373.
46. V.P. Nguyen, C. Anitescu, S.P. Bordas, T. Rabczuk, Isogeometric analysis: an overview and computer implementation aspects, *Math. Comput. Simul.* 117 (2015) 89–116.
47. L. Piegl, W. Tiller, *The NURBS Book*, 2nd ed., Springer, New York (1997).
48. H.L. Liu, X.F. Zhu, D.X. Yang, Isogeometric method based in-plane and out-of-plane free vibration analysis for Timoshenko curved beams, *Struct. Eng. Mech.* 59 (3) (2016) 503–526.
49. A.V. Vuong, B. Simeon, On isogeometric analysis and its usage for stress calculation, In: *Modeling, Simulation and Optimization of Complex Processes*, Springer, Berlin, (2012) 305–314.
50. Z. Kang, Y.Q. Wang, Structural topology optimization based on non-local Shepard interpolation of density field, *Comput. Meth. Appl. Mech. Engrg.* 200 (2011) 3515–3525.
51. Z. Kang, Y.Q. Wang, A nodal variable method of structural topology optimization based on shepard interpolant, *Int. J. Numer. Meth. Engrg.* 90 (2012) 329–342.
52. Z. Luo, N. Zhang, Y. Wang, W. Gao, Topology optimization of structures using meshless density variable approximants, *Int. J. Numer. Meth. Engrg.* 93 (2013) 443–464.
53. Verbart, M. Langelaar, F. Van Keulen, A unified aggregation and relaxation approach for stress-constrained topology optimization, *Struct. Multidiscip. Optim.* 55 (2) (2017) 663–679.
54. P. Schmelcher, F.K. Diakonov, Detecting unstable periodic orbits of chaotic dynamical systems, *Phys. Rev. Lett.* 78 (25) (1997) 4733–4736.
55. D.X. Yang, P. Yi, Chaos control of performance measure approach for evaluation of probabilistic constraints, *Struct. Multidiscip. Optim.* 38 (1) (2009) 83–92.
56. D.X. Yang, P.X. Yang, Numerical instabilities and convergence control for convex approximation methods, *Nonlinear Dyn.* 61 (2010) 605–622.
57. S.H. Jeong, G.H. Yoon, A. Takezawa, D.H. Choi, Development of a novel phase-field method for local stress-based shape and topology optimization, *Comput. Struct.* 132 (2014) 84–98.