RADIO GEOMETRIC MEAN GRACEFUL LABELING ON DEGREE SPLITTING OF SOME SPECIAL GRAPHS

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Abstract:

A radio geometric mean graceful labeling of a connected graph G is a bijection μ from a vertex set V(G) to {1, 2, 3, …, |V(G)|} such that for any two distinct vertices u and v of G, $d(u, v) + \left\lceil\sqrt{\mu(u)\mu(v)}\right\rceil \geq 1 + diam(G)$. A graph which admits radio geometric mean graceful labeling is called radio geometric mean graceful graph. In this paper, we investigate the radio geometric mean graceful labeling on degree splitting of some graphs.

Keywords: labeling, radio geometric mean graceful labeling, degree splitting of graph.

I. INTRODUCTION

Graphs described here is simple, undirected and connected. Let V(G) and E(G) denote the vertex set and edge set of a graph G respectively. A graph labeling is an assignment of integers to the vertices or edges or both based on certain conditions. The concept of radio labeling was introduced by Chartrand et al [1] in 2001. S. Somasundaram and R. Ponraj introduced the notion of mean labeling of graphs [4]. Radio mean labeling was introduced by R. Ponraj et al [5]. R. Ponraj and S. Somasundram developed the concept of degree splitting of graphs [7]. S. Somasundaram, S.S. Sandhya and S.P. Viji introduced the concept of Geometric mean labeling on Degree splitting of graphs [6]. C. David Raj, M. Deva Saroja and V.T. Brindha Mary determined radio mean labeling on Degree splitting of graphs [8]. C. David Raj, T. Mary Shalini and K. Rubin Mary determined radio geometric mean graceful labeling on Degree splitting of some graphs [9]. We refer Gallian for more comprehensive survey [2]. We follow Harary [3] for some standard words, expressions and symbols. The notions $DS(G)$ is the degree splitting of G, $d(u,v)$ is the distance between the vertices u and v and diam(G) is the diameter of G.

Definition 1.1: A web graph Wb_n is the graph obtained from a closed helm by adding a pendent edge to each vertex of outer cycle.

Definition 1.2: The graph Lotus inside a circle LC_n is obtained from the cycle C_n: $w_1 w_2 \dots w_n w_1$ and a star $K_{1,n}$ with central vertex u and the end vertices $u_1, u_2, \dots, u_n$ by joining each u_i to w_i and $w_{(i+1)(mod\ n)}$.

Definition 1.3: A sunflower SF_n is a graph obtained by taking a wheel with the apex vertex u and the consecutive rim vertices $u_1, u_2, \dots, u_n$ and additional vertices $v_1, v_2, \dots, v_n$ where v_i is joined by edges u_i and $u_{(i+1)(mod\ n)}$.

II. Main Result

Theorem 2.1: $DS(Wb_n)$ is a radio geometric mean graceful graph.

Proof:

Let $u,\ v_i,\ 1 \le i \le n$ be the vertices of the wheel graph in which u is the central vertex. Add a vertex u_i and join it with v_i, $1 \le i \le n$. Also join u_i with u_{i+1}, $1 \le i \le n-1$ and u_n with u_1 to form a cycle. Add a pendent vertex w_i and join it with u_i, $1 \le i \le n$. The resultant graph is Wb_n. Introduce two new vertices v, w and join them with the vertices of Wb_n of degree one and four respectively. The new resultant graph is $DS(Wb_n)$ whose vertex set is $V(DS(Wb_n)) = \{u,\ u_i,\ v_i,\ w_i,\ 1 \le i \le n\} \cup \{v,\ w\}$. Clearly, $diam(DS(Wb_n)) = 4$.

Define a bijection $\mu: V(DS(Wb_n)) \to \{1, 2, 3, \dots, |V(DS(Wb_n))|\}$ by

$\mu(u) = 1$;

$\mu(w_i) = i + 1,\ 1 \le i \le n;$

$\mu(u_i) = n + i + 1,\ 1 \le i \le n;$

$\mu(v_i) = 2n + i + 1,\ 1 \le i \le n;$

$\mu(v) = 3n + 2;$

$\mu(w) = 3n + 3.$

To check the radio geometric mean graceful condition for μ.

Case (i): Verify the pair $(u,\ v)$;

$$d(u,\ v) + \left\lceil \sqrt{\mu(u)\mu(v)} \right\rceil = 4 + \left\lceil \sqrt{(1)(3n+2)} \right\rceil \ge 5 = 1 + diam(DS(Wb_n)).$$

Case (ii): Verify the pair $(u,\ w)$;

$d(u,\ w) + \left\lceil\sqrt{\mu(u)\mu(w)}\right\rceil = 2 + \left\lceil\sqrt{(1)(3n+3)}\ \right\rceil \geq 5.$

Case(iii): Verify the pair $(u,\ w_i),\ 1 \leq i \leq n$;

$d(u,\ w_i) + \left\lceil\sqrt{\mu(u)\mu(w_i)}\ \right\rceil = 3 + \left\lceil\sqrt{(1)(i+1)}\right\rceil \geq 5$.

Case(iv): Verify the pair $(u,\ u_i), 1 \leq i \leq n$;

$d(u,\ u_i) + \left\lceil\sqrt{\mu(u)\mu(u_i)}\ \right\rceil = 2 + \left\lceil\sqrt{(1)(n+i+1)}\right\rceil \geq 5.$

Case(v): Verify the pair $(u,\ v_i),\ 1 \leq i \leq n$;

$d(u,\ v_i) + \left\lceil\sqrt{\mu(u)\mu(v_i)}\ \right\rceil = 1 + \left\lceil\sqrt{(1)(2n+i+1)}\right\rceil \geq 5.$

Case (vi): Verify the pair $(v,\ w)$;

$d(v,\ w) + \left\lceil\sqrt{\mu(v)\mu(w)}\right\rceil = 3 + \left\lceil\sqrt{(3n+2)(3n+3)}\ \right\rceil \geq 5.$

Case (vii): Verify the pair $(v,\ w_i),\ 1 \leq i \leq n$;

$d(v,\ w_i) + \left\lceil\sqrt{\mu(v)\mu(w_i)}\ \right\rceil = 1 + \left\lceil\sqrt{(3n+2)(i+1)}\ \right\rceil \geq 5.$

Case (viii): Verify the pair $(v,\ u_i),\ 1 \leq i \leq n$;

$d(v,\ u_i) + \left\lceil\sqrt{\mu(v)\mu(u_i)}\ \right\rceil = 2 + \left\lceil\sqrt{(3n+2)(n+i+1)}\ \right\rceil \geq 5.$

Case (ix): Verify the pair $(v,\ v_i),\ 1 \leq i \leq n$,;

$d(v,\ v_i) + \left\lceil\sqrt{\mu(v)\mu(v_i)}\ \right\rceil = 3 + \left\lceil\sqrt{(3n+2)(2n+i+1)}\ \right\rceil \geq 5.$

Case (x): Verify the pair $(w,\ w_i),\ 1 \leq i \leq n$;

$d(w,\ w_i) + \left\lceil\sqrt{\mu(w)\mu(w_i)}\ \right\rceil = 2 + \left\lceil\sqrt{(3n+3)(i+1)}\ \right\rceil \geq 5.$

Case (xi): Verify the pair $(w,\ u_i),\ 1 \leq i \leq n$;

$d(w,\ u_i) + \left\lceil\sqrt{\mu(w)\mu(u_i)}\ \right\rceil = 1 + \left\lceil\sqrt{(3n+3)(n+i+1)}\ \right\rceil \geq 5.$

Case (xii): Verify the pair $(w,\ v_i),\ 1 \leq i \leq n$;

$d(w,\ v_i) + \left\lceil\sqrt{\mu(w)\mu(v_i)}\ \right\rceil = 1 + \left\lceil\sqrt{(3n+3)(2n+i+1)}\ \right\rceil \geq 5.$

Case (xiii): Verify the pair $\left(w_i,\ u_j\right),\ 1 \leq i, j \leq n$;

$d\left(w_i,\ u_j\right)\ + \left\lceil\sqrt{\mu(w_i)\mu\left(u_j\right)}\right\rceil \geq 1 + \left\lceil\sqrt{(i+1)(n+j+1)}\ \right\rceil \geq 5.$

Case (xiv): Verify the pair $\left(w_i,\ v_j\right)$, $1 \leq i, j \leq n$;

$$d\left(w_i,\ v_j\right) + \left\lceil\sqrt{\mu(w_i)\mu(v_j)}\right\rceil \geq 2 + \left\lceil\sqrt{(i+1)(2n+j+1)}\right\rceil \geq 5.$$

Case (xv): Verify the pair $\left(u_i,\ v_j\right)$, $1 \leq i, j \leq n$;

$$d\left(u_i,\ v_j\right) + \left\lceil\sqrt{\mu(u_i)\mu(v_j)}\right\rceil \geq 1 + \left\lceil\sqrt{(n+i+1)(2n+j+1)}\right\rceil \geq 5.$$

Case (xvi): Verify the pair $\left(u_i,\ u_j\right)$, $1 \leq i \leq n-1,\ \ i+1 \leq j \leq n$;

$$d\left(u_i,\ u_j\right) + \left\lceil\sqrt{\mu(u_i)\mu(u_j)}\right\rceil \geq 1 + \left\lceil\sqrt{(n+i+1)(n+j+1)}\right\rceil \geq 5.$$

Case (xvii): Verify the pair $\left(v_i,\ v_j\right)$, $1 \leq i \leq n-1,\ \ i+1 \leq j \leq n$;

$$d\left(v_i,\ v_j\right) + \left\lceil\sqrt{\mu(v_i)\mu(v_j)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+i+1)(2n+j+1)}\right\rceil \geq 5.$$

Case (xviii): Verify the pair $\left(w_i,\ w_j\right)$, $1 \leq i \leq n-1,\ \ i+1 \leq j \leq n$

$$d\left(w_i,\ w_j\right) + \left\lceil\sqrt{\mu(w_i)\mu(w_j)}\right\rceil = 2 + \left\lceil\sqrt{(i+1)(j+1)}\right\rceil \geq 5.$$

Thus all the pair of vertices satisfies the radio geometric mean graceful condition.

Hence DS (Wb_n) is a radio geometric mean graceful graph.

Example 2.2:

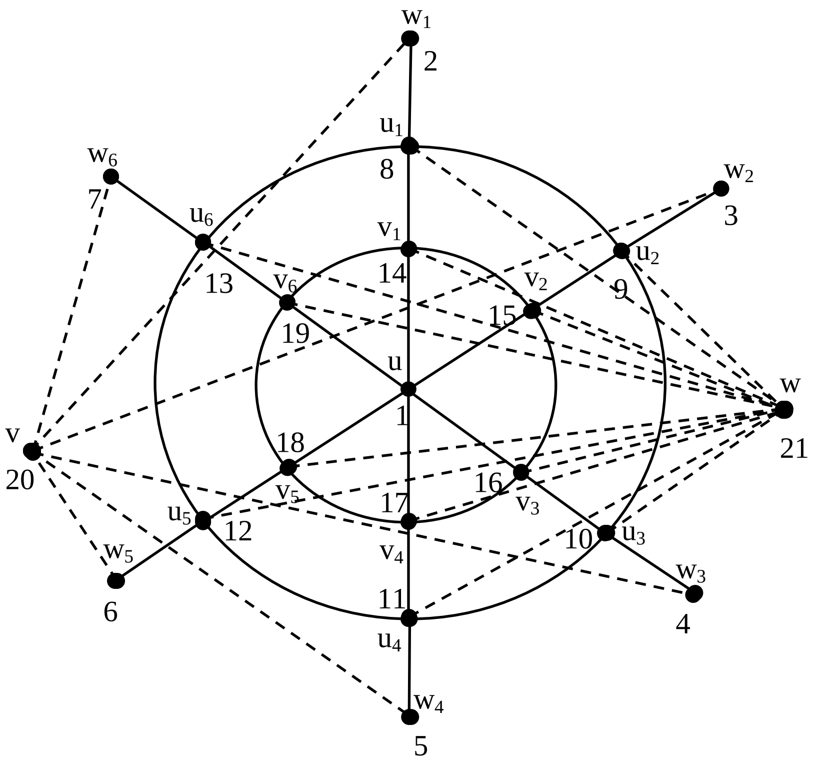

Fig 1. Radio Geometric Mean Graceful Labeling of DS(Wb_6).

Theorem 2.3: $DS(LC_n)$ is a radio geometric mean graceful graph.

Proof:

Let $u_1, u_2, \ldots.., u_n$ be the vertices of C_n and a star $K_{1,n}$ with central vertex v and the pendent vertices $v_1, v_2, \ldots\ldots, v_n$ by joining each v_i to u_i and $u_{(i+1)(mod\ n)}$. The graph thus obtained is LC_n. Introduce two new vertices u, w and join them with the vertices of LC_n of degree four and three respectively. The resultant graph is $DS(LC_n)$ whose vertex set is $V(DS(LC_n)) = \{v,\ u_i,\ v_i,\ 1 \le i \le n\} \cup \{u,\ w\}$. Clearly, $diam(DS(LC_n)) = 3$.

Define a bijection $\mu: V\big(DS(LC_n)\big) \to \big\{1, 2, 3, \ldots.,\ \big|V\big(DS(LC_n)\big)\big|\big\}$ by

$\mu(v_i) = i,\ 1 \le i \le n;$

$\mu(u_i) = n + i,\ 1 \le i \le n;$

$\mu(u) = 2n + 1\ ;$

$\mu(w) = 2n + 2;$

$\mu(v) = 2n + 3.$

To check the radio geometric mean graceful condition for μ.

Case(i): Verify the pair $(u,\ v_i),\ 1 \le i \le n;$

$$d(u,\ v_i) + \left\lceil\sqrt{\mu(u)\mu(v_i)}\,\right\rceil = 2 + \left\lceil\sqrt{(2n+1)(i)}\right\rceil \ge 4 = 1 + diam(DS(LC_n)).$$

Case(ii): Verify the pair $(u,\ u_i),\ 1 \le i \le n;$

$$d(u,\ u_i) + \left\lceil\sqrt{\mu(u)\mu(u_i)}\,\right\rceil = 1 + \left\lceil\sqrt{(2n+1)(n+i)}\right\rceil \ge 4.$$

Case (iii): Verify the pair $(u,\ w);$

$$d(u,\ w) + \left\lceil\sqrt{\mu(u)\mu(w)}\right\rceil = 3 + \left\lceil\sqrt{(2n+1)(2n+2)}\,\right\rceil \ge 4.$$

Case (iv): Verify the pair $(u,\ v);$

$$d(u,\ v) + \left\lceil\sqrt{\mu(u)\mu(v)}\right\rceil = 3 + \left\lceil\sqrt{(2n+1)(2n+3)}\,\right\rceil \ge 4.$$

Case (v): Verify the pair $(w,\ u_i)$, $1 \le i \le n;$

$$d(w,\ u_i) + \left\lceil\sqrt{\mu(w)\mu(u_i)}\,\right\rceil = 2 + \left\lceil\sqrt{(2n+2)(n+i)}\,\right\rceil \ge 4.$$

Case (vi): Verify the pair $(w,\ v_i)$, $1 \le i \le n;$

$$d(w,\ v_i) + \left\lceil\sqrt{\mu(w)\mu(v_i)}\,\right\rceil = 1 + \left\lceil\sqrt{(2n+2)(i)}\,\right\rceil \ge 4.$$

Case (vii): Verify the pair $(v,\ w);$

$d(v,\ w) + \left\lceil \sqrt{\mu(v)\mu(w)} \right\rceil = 2 + \left\lceil \sqrt{(2n+3)(2n+2)} \right\rceil \geq 4.$

Case (viii): Verify the pair $(v,\ u_i)$, $1 \leq i \leq n$;

$d(v,\ u_i) + \left\lceil \sqrt{\mu(v)\mu(u_i)} \right\rceil = 2 + \left\lceil \sqrt{(2n+3)(n+i)} \right\rceil \geq 4.$

Case (ix): Verify the pair $(v,\ v_i)$, $1 \leq i \leq n$;

$d(v,\ v_i) + \left\lceil \sqrt{\mu(v)\mu(v_i)} \right\rceil = 1 + \left\lceil \sqrt{(2n+3)(i)} \right\rceil \geq 4.$

Case (x): Verify the pair $\left(u_i,\ v_j\right)$, $1 \leq i, j \leq n$;

$d\left(u_i,\ v_j\right) + \left\lceil \sqrt{\mu(u_i)\mu(v_j)} \right\rceil \geq 1 + \left\lceil \sqrt{(n+i)(j)} \right\rceil \geq 4.$

Case (xi): Verify the pair $\left(u_i,\ u_j\right)$, $1 \leq i \leq n-1,\ i+1 \leq j \leq n$;

$d\left(u_i,\ u_j\right) + \left\lceil \sqrt{\mu(u_i)\mu\left(u_j\right)} \right\rceil \geq 1 + \left\lceil \sqrt{(n+i)(n+j)} \right\rceil \geq 4.$

Case (xii): Verify the pair $\left(v_i,\ v_j\right)$, $1 \leq i \leq n-1,\ i+1 \leq j \leq n$;

$d\left(v_i,\ v_j\right) + \left\lceil \sqrt{\mu(v_i)\mu(v_j)} \right\rceil = 2 + \left\lceil \sqrt{(i)(j)} \right\rceil \geq 4.$

Thus all the pair of vertices satisfies the radio geometric mean graceful condition.

Hence $DS(LC_n)$ is a radio geometric mean graceful graph.

Example 2.4:

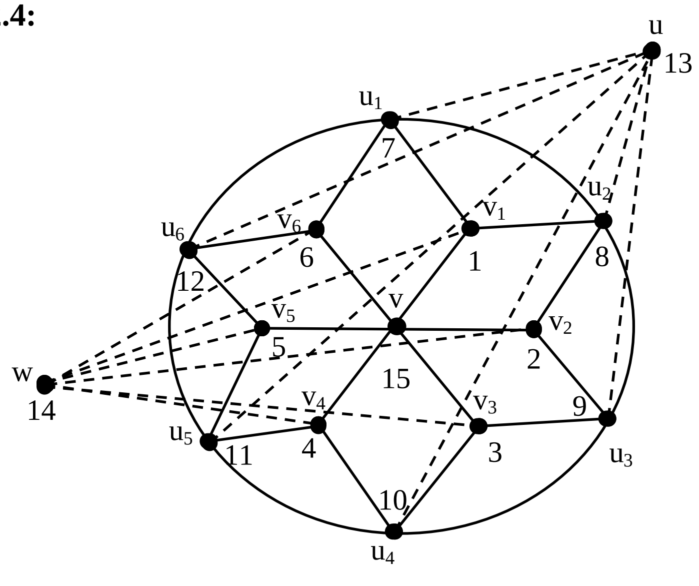

Fig 2. Radio Geometric Mean Graceful Labeling of DS(LC_6).

Theorem 2.5: $DS(SF_n)$ is a radio geometric mean graceful graph.

Proof:

Let u, u_i, $1 \le i \le n$ be the vertices of a wheel graph in which u is the central vertex. Add a vertex v_i and join it with u_i, u_{i+1}, $1 \le i \le n-1$ and also join v_n with u_n and u_1. The resultant graph is SF_n. Introduce two new vertices w_1, w_2 and join them with the vertices of SF_n of degree five and two respectively. The new resultant graph is $DS(SF_n)$ whose vertex set is $V(DS(SF_n)) = \{u, u_i, v_i, 1 \le i \le n\} \cup \{w_1, w_2\}$. Clearly, $diam(DS(SF_n)) = 3$.

Define a bijection $\mu: V\left(DS(SF_n)\right) \to \left\{1, 2, 3, \ \dots, \ \left|V\left(DS(SF_n)\right)\right|\right\}$ by

$\mu(w_i) = i, \ 1 \le i \le 2;$

$\mu(v_i) = i + 2, \ 1 \le i \le n;$

$\mu(u_i) = n + i + 2, \ 1 \le i \le n;$

$\mu(u) = 2n + 3.$

To check the radio geometric mean graceful condition for μ.

Case(i): Verify the pair $(u, \ w_i)$, $1 \le i \le 2$;

$$d(u, \ w_i) + \left\lceil \sqrt{\mu(u)\mu(w_i)} \right\rceil \ge 2 + \left\lceil \sqrt{(2n+3)(i)} \right\rceil \ge 4 = 1 + \ diam(DS(SF_n)).$$

Case(ii): Verify the pair $(u, \ u_i), 1 \le i \le n$;

$$d(u, \ u_i) + \left\lceil \sqrt{\mu(u)\mu(u_i)} \right\rceil = 1 + \left\lceil \sqrt{(2n+3)(n+i+2)} \right\rceil \ge 4.$$

Case(iii): Verify the pair $(u, \ v_i), 1 \le i \le n$;

$$d(u, v_i) + \left\lceil \sqrt{\mu(u)\mu(v_i)} \right\rceil = 2 + \left\lceil \sqrt{(2n+3)(i+2)} \right\rceil \ge 4.$$

Case (iv): Verify the pair $\left(u_i, \ w_j\right)$, $1 \le i \le n, \ 1 \le j \le 2$;

$$d\left(u_i, w_j\right) + \left\lceil \sqrt{\mu(u_i)\mu(w_j)} \right\rceil \ge 1 + \left\lceil \sqrt{(n+i+2)(j)} \right\rceil \ge 4.$$

Case (v): Verify the pair $\left(u_i, \ v_j\right)$, $1 \le i, \ j \le n$;

$$d\left(u_i, v_j\right) + \left\lceil \sqrt{\mu(u_i)\mu(v_j)} \right\rceil \ge 1 + \left\lceil \sqrt{(n+i+2)(j+2)} \right\rceil \ge 4.$$

Case (vi): Verify the pair $\left(v_i, \ w_j\right)$, $1 \le i \le n, \ 1 \le j \le 2$;

Subcase(i): $j = 1$

$$d\left(v_i, w_j\right) + \left\lceil \sqrt{\mu(v_i)\mu(w_j)} \right\rceil = 2 + \left\lceil \sqrt{(i+2)(j)} \right\rceil \ge 4.$$

Subcase(i): $j = 2$

$d(v_i, w_j) + \left\lceil \sqrt{\mu(v_i)\mu(w_j)} \right\rceil = 1 + \left\lceil \sqrt{(i+2)(2)} \right\rceil \geq 4$

Case (vii): Verify the pair (w_1, w_2) ;

$d(w_1, w_2) + \left\lceil \sqrt{\mu(w_1)\mu(w_2)} \right\rceil = 3 + \left\lceil \sqrt{(1)(2)} \right\rceil \geq 4.$

Case (viii): Verify the pair (u_i, u_j) , $1 \leq i \leq n-1, \ i+1 \leq j \leq n$;

$d(u_i, u_j) + \left\lceil \sqrt{\mu(u_i)\mu(u_j)} \right\rceil \geq 1 + \left\lceil \sqrt{(n+i+2)(n+j+2)} \right\rceil \geq 4.$

Case (ix): Verify the pair (v_i, v_j), $1 \leq i \leq n-1, \ i+1 \leq j \leq n$;

$d(v_i, v_j) + \left\lceil \sqrt{\mu(v_i)\mu(v_j)} \right\rceil = 2 + \left\lceil \sqrt{(i+2)(j+2)} \right\rceil \geq 4.$

Thus all the pair of vertices satisfies the radio geometric mean graceful condition.

Hence $DS(SF_n)$ is a radio geometric mean graceful graph.

Example 2.6:

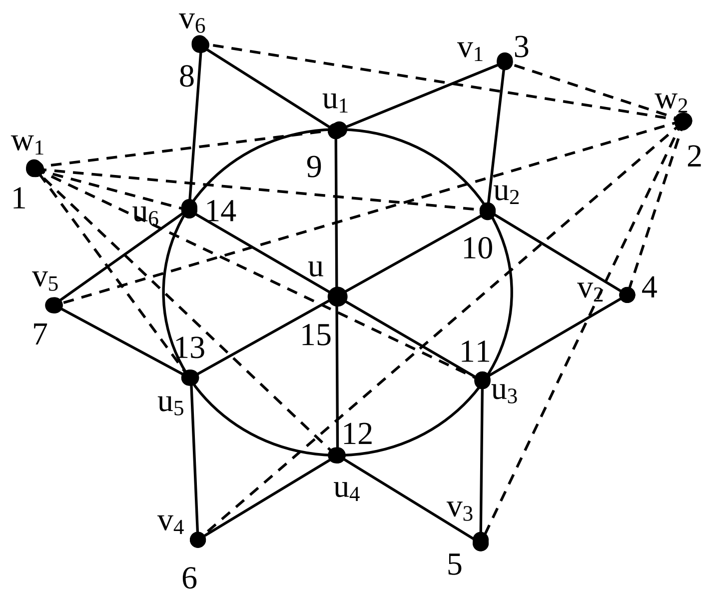

Fig 3. Radio Geometric Mean Graceful Labeling of DS(SF_6).

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