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Research Article

On Minimal ▯ dense Sets and its Applications

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Abstract: The aim of this paper is to introduce a new concept of Λ dense set and to study its topological structure. It can be shown that the collection of all this Λ - dense set forms a supra topological space if ϕ is introduced. Also the concept of minimal Λ dense set is introduced and some of its applications are shown. Lastly introducing the concept of Λ sub maximal space various important properties of minimal Λ dense sets are studied in a Λ- sub maximal space.

Key words: Λ dense set, minimal Λdense set, Λ - sub maximal space etc

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1. Introduction:

The concept of dense set has been introduced earlier by various researchers as, A is a dense set in (X, T) if ClA = X. But in this paper in place of closed set introducing open sets a new concept of Λ - dense set is introduced. The concept of Λ set has been introduced by Maki in [7].

In the second section of this paper preliminaries are given

In the third section of the paper the concept of Λdense set and the topological space obtained from the collection of this set is studied. Also the concept of minimal Λ dense set is introduced and the topological structure for various minimal Λ dense sets is studied. It is shown that every superset of a Λ dense set is a Λ dense set and no open set except X can be a Λ dense set. It is shown that the collection of all Λ dense set forms a supra topological space and it is denoted by T_Λ .Some important theorems were proved related to the structural behaviour of this new space.

It is shown that (X,T) is a discrete topological space iff T_Λ is the power set of X. Also (X,T) is a power set of X iff T_Λ ={X, ϕ}. Similarly many important theorems were proved in this section of the paper. Then the concept of minimal Λ dense set is introduced and some theorems were proved. The concept of Λ submaximal space is introduced and some of its properties were studied.

Lastly the applications of this newly defined set are studied

2.Preliminaries:

In this section some important concept required to go further through this paper is studied. **2.1**[6] A subset A of X is a generalized closed set if for any open set U containing A, ClA ⊆ U**2.2**[8] Let Abe a subset of X then Λ(A) = ∩{G :G ⊇A, G is an open subset of X}

and ifΛ(A) = A then A is a Λ set.

2.3[3] A topological space (X, T) is sub maximal if every dense subset of X is T open

2.4[7] An open set A of X is said to be minimal open set if it doesn't contains any open set except

ϕ

:[5] A space(X,T) is said to be an Alexandroff space if

(i) X and ϕ are members of (X,T)
(ii) Arbitrary union of the members of (X,T) are in (X,T)
(iii) Arbitrary intersection of the members of (X,T) are in (X,T)

[9] A space(X,T) is said to be a Supra topological space if

(i) X and ϕ are members of (X,T)
(ii) Arbitrary union of the members of (X,T) are in (X,T)

[12] A set A in (X,T) is said to be a dense set if Cl(A)= X

[10] In topology, a topological space with the **trivial topology** is one where the only open sets are the empty set and the entire space. Such a space is sometimes called an **indiscrete space**, and its topology sometimes called an **indiscrete topology**.

3. On Λ - dense set

In this section the concept of Λ-dense set is introduced and the corresponding topological space is studied.Also the connection of Λ dense set with other sets is introduced in this section. Lastly the concept of minimal
Λdense set is introduced and some of its properties are studied.

Definition 3.1: A subset A of X is said to be a Λ dense set if Λ(A) = X

Example 3.2: Let X = {a, b, c} and the corresponding topological space be T = {X, ϕ, {a}}. Let A = {b, c} bea subset of X . Obviously Λ(A) = X i.e. A is a Λ dense set.

Theorem 3.3: A subset A of X is a Λ dense set. Then

1. ΛCl(A) = X
2. C
lΛ(A) = X

Proof is obvious

Remark 3.4: Converse of the above theorem need not be true which follows from the following example. LetX = {a, b, c} and the corresponding topological space be T = {X, ϕ, {a, b}, {b}, {b, c}}

Let A = {b} then ΛClA =X but Λ(A) = {b} i.e. A is not a Λ dense set.

Theorem 3.5: Let A be a closed subset of X .Let ΛClA = X then A is a Λ dense set.

Proof: Since A is a closed subset of X, ClA = A i.e. ΛClA = Λ(A). Since ΛClA = X, Λ(A)= X i.e. A is a Λ dense set.

Remark 3.6: From theorem 3.3 and theorem 3.5 it is clear that:
Let A be a closed subset of X. Then ΛClA = X iff A is a Λ
dense set. **Theorem 3.7:** A subset A of X is a Λ dense set iff
Λ(A) is a Λ dense setProof is obvious

Theorem 3.8: In an Alexandroff space a set is a Λ dense set if it is a generalized closed set and a dense set. **Proof:** Let A be a generalized closed set then for any open set U such that A $\subseteq$ U, ClA $\subseteq$ U. Since Λ(A) is theintersection of all open sets containing A, so, A $\subseteq$ Λ(A) $\subseteq$ U.
In an Alexandroff space Λ(A) is an open set. So, ClA $\subseteq$Λ(A), A being dense subset of X, ClA= X

i.e. $\Lambda(A) = X$ i.e. A is a Λ dense set.

Theorem 3.9: A subset A of X is a Λ dense set and a dense set then A is a generalized closed set.

Proof: Let if possible A be a Λ dense set and a dense set. Then $\Lambda(A) = X$ and ClA = X i.e. ClA = $\Lambda(A)$.

Now let if possible $A \subseteq U$, U being an open subset of X then $\Lambda(A) \subseteq U$. Since $\Lambda(A)$ is the intersection of allopen sets containing A therefore ClA = $\Lambda(A) \subseteq U$. i.e. A is a generalized closed set.

Remark 3.10: Converse of the above theorem need not be true. It follows since if ClA = $X \subseteq U$ then U = X

i.e. the only open set containing A is X.

Remark 3.11: From the theorem 3.8 and theorem 3.9 the following statement may be written

In an Alexandroff space a subset A of X is a dense set. Then the following statements are equivalent:

1. A is a Λ dense set
2. A is a generalized closed set
3. Cl$\Lambda(A)$ = X

Theorem 3.12: Every superset of a Λ dense set is a Λ dense set.Proof is obvious

Theorem 3.13: If $A \subseteq B \subseteq \Lambda(A)$, where B is a Λdense set then A is also so.

Proof: Let $A \subseteq B \subseteq \Lambda(A)$ i.e. $\Lambda(A) \subseteq \Lambda(B) \subseteq \Lambda\Lambda(A) = \Lambda(A)$ i.e. $\Lambda(A) = \Lambda(B)$. Since B is a Λdense set $\Lambda(B) = X$ i.e. $\Lambda(A) = X$ i.e. A is also a Λ dense set.

Remark 3.14: No open set except X can be a Λ dense set.

Theorem 3.15:

(i) ϕ is not a Λ dense set but X is so.

(ii) Arbitrary union of Λ dense set in (X, T) is a Λ dense set in (X, T)

Proof:

(i) is obvious

To Prove(ii)

Let A= $\{A_i : i \in I\}$ be a collection of Λ dense set i.e. $\{\Lambda(A_i) : i \in I\} = X$

Then $\Lambda(\cup A_i : i \in I) = \cup\{\Lambda(A_i) : i \in I\} = X$ i.e. $\Lambda(A) = X$ i.e. arbitrary union of Λ dense set is a Λ dense set.

Remark 3.16: Finite intersection of Λdense set need not be a Λdense set. It follows from the followingexample:

Let X = {a, b, c} and T = {ϕ, X, {a, b}, {c}} be the corresponding topology

Let A = {a, c} and B = {b, c} be two subsets of X then $\Lambda(A) = X$, $\Lambda(B) = X$. But $A \cap B = \{c\}$, $\Lambda(A \cap B) = \{c\}$

$\neq X$ i.e. $A \cap B$ is not a Λ dense set though A and B are Λ dense set.

Remark 3.17: The collection of all Λ dense set in (X, T) with ϕ forms a supra topological space denoted as(X, T_Λ) . This space is named as Λ dense supra topological space. In the above example $T_\Lambda = \{\phi, X, A, B\}$

Theorem 3.18: (X, T) is an indiscrete topological space iff T_Λ is the power set of X.

Proof: Since (X, T) is an indiscrete topological space, So, T = {X,ϕ}. The power set of X contains all subsetsof X and they are all Λ dense set.

Conversely if $T_\Lambda = P(X)$ and since no open sets can be a Λdense set except X. So, T = {X, ϕ}

Theorem 3.19: (X, T) be a topological space such that T= P(X) iff $T_\Lambda = \{X, \phi\}$

Proof: Since from Remark 3.14 no open set except X and ϕ can be a member of T_Λ. So, $T_\Lambda = \{X, \phi\}$ Conversely if $T_\Lambda = \{X, \phi\}$ then, T must contain all elements whose order is one less then that of X. Also Tmust contain their finite intersection i.e. all the elements whose order is two less then that of X and soon i.e. T
= P(X)

Definition 3.20: A topology T is said to be a maximal topology of any set $A \subseteq P(X)$, if it is a subset of A butcontained in no other topology which is a subset of A.

Theorem 3.21: $T_\Lambda = \{X, A, \phi\}$ iff T is the maximal topology of P(X)\A , where A is a subset of X of order n-1, n is the order of X

Proof: Let if possible we consider that, $T_\Lambda = \{X, A, \phi\}$. Since the superset of all Λdense set is a Λ dense set.So, if there exist any superset of A then that should be a member of T_Λ . But T_Λ contains only A except X and
ϕ. So, the order of A is one less then that of X i.e. n-1. The corresponding topology must be a subset of P(X) \
A. Let $T_1 \subseteq P(X)\backslash A$ be another topology containing T. Then there is some open set, which is not in T. So either$T_{1\Lambda} \supseteq \{X, A, \phi\}$or $T_{1\Lambda} = \{X, A, \phi\}$.The first one is not possible and if the second one is true then we convert T by T_1 which is the maximal topology .
Conversely let T is the maximal topology of P(X)\A , where A is a subset of X of order n-1, n is the order ofX then obviously $T_\Lambda = \{X, A, \phi\}$.

Theorem 3.22: T = {X, A, ϕ} iff $T_\Lambda = P(X)\backslash\{G : G \subseteq A\}$

Proof: Let if possible T = {X, A, ϕ} then T_Λ will contain all the subsets of P(X) except the set A and itssubsets i.e. $T_\Lambda = P(X)\backslash \{G: G \subseteq A\}$
Converse is obviously true

Theorem 3.23: $T_\Lambda = \{X, A, B, \phi\}$ iff

1. T is the maximal topology of {P(X)\A}\B, where A is a subset of X of order n-1, and B is a subset of A oforder n-2, n is the order of X
2. T is the maximal topology of {P(X)\A}\B, where A and B both are of order n-2, n is the order of X

Proof: Let if possible**,** $T_\Lambda = \{X, A, B, \phi\}$. Since T_Λ forms a supra topological space so finite intersection ofthe elements need not be a member of the set T_Λ. Hence two cases may arise

Case 1: A and B are related to each other and B is a subset of A. Obviously from theorem 3.21 A is a subset
of X of order n-1 and by the help of the similar logic B is of order n-2. Clearly T is the maximal topology of
{P(X) \A} \B

Case 2: If A and B are not related then A∪B = X, and both of them are of order n-1 and T is the maximaltopology of {P(X) \A} \B
Converse is obvious

Theorem 3.24: T = {ϕ, X, A, B} iff

1.If A is a superset of B then $T_\Lambda = P(X)\backslash\{G: G \subseteq A\}$

2. If A and B are not related then $T_\Lambda = \phi$.

Proof: Here T is a topological space **.**So we have the following cases

Case1. A is a superset of B then $T_\Lambda = P(X)\backslash\{G: G \subseteq A\}$

Case2: If A and B are not related then A∪B= X and A∩B=ϕ then $T_\Lambda = P(X)\backslash\{G: G \subseteq A\}\backslash\{G: G \subseteq B\}$Here $B = A^C$. Thus $T_\Lambda = \phi$.

Converse is obvious

Let us now introduce a new concept of minimal Λ dense set. Since the superset of a Λ dense set is a Λ dense set, so, the upper bound of the set of all Λdense set in (X, T) is X but there must exist at least one minimal element, which is contained in, all the Λ dense set in (X, T). This set is known as minimal Λ dense set.

Example 3.25: Let X = {a, b, c} and T= {ϕ, X, {a}, {a, c}, {c}} be the corresponding topology. Let A = {a,b} then Λ(A) = X, B = {b, c}, Λ(B) = X. Obviously A∩B = {b}, Λ(A∩B) = X and this is the minimal Λ dense set in (X, T). Here T_Λ = {ϕ, X, {a, b}, {b, c}, {b}} which is a topology.

Theorem 3.26: Every minimal Λ dense set in (X, T) are minimal supra open set in (X, T_Λ)

Proof: Since every Λ dense set in (X, T) are supra open set in (X, T_Λ)

Remark 3.27:

1. Let if possible T_Λ contains only one minimal Λ dense set X i.e. T_Λ = {ϕ, X}. From theorem 3.19, T= P(X).Here T_Λ is a discrete topology.
2. Let T_Λ = {ϕ, X, A}. From theorem 3.21, T is the maximal topology subset of P(X)\A Here A is theminimal Λ dense set. Here T_Λ is a topological space where A is a minimal open set in (X, T_Λ)
3. Let T_Λ = {ϕ, X, A, B}. From theorem 3.24, if B is the minimal Λdense set then T is the maximal topologyof {P(X)\A}\B, where A is a subset of X of order n-1, and B is a subset of A of order n-2,

   n is the order
   of X
4. Let T_Λ contains only one minimal Λ- dense set i.e. T_Λ = {ϕ, X, {G: G ⊇ A}, ∪{G: G ⊇ A}} Since all the superset of a Λ dense set is a Λdense set and there exist only one minimal Λ dense set. So, all the other Λ dense sets intersection must be the set A or its superset and hence is a Λ dense set. We know that arbitrary union of Λ dense set is a Λ dense set. Therefore we may conclude that if a Λ dense supra topological space contains only one minimal Λ dense set then that supra topological space forms a topological space The corresponding topological space T is a subset of the power set of X such that it doesn't contains A and all its supersets.
5. If T_Λ contains r number of minimal Λ dense set then it forms a supra topological space. The correspondingtopological space T contains ϕ, X and all elements of order one less than that of X except r number of sets. **Remark 3.28:** According to the theorem 3.13, A ⊆ B ⊆Λ(A) and B is a Λ dense set then A is also so. But if Bis a minimal Λ dense set then A can't be a proper subset of B i.e. there can't exist any proper subset A of B such that A ⊆ B ⊆Λ(A) where B is a minimal Λ dense set.

Definition 3.29: A topological space (X, T) is a Λ sub-maximal space if every element of (X, T_Λ) is also a closed subset of (X, T)

Example 3.30: Let X= {a, b, c, d} and T = {ϕ, X, {a, b, c}{b, d}, {b}, {c}, {b, c}, {b, c, d}}. Here A = {a, b,d} B = {a, d} are Λ dense set and obviously it is a closed subset of (X, T) Here T_Λ = {ϕ, X, A, B} which are all closed subsets of (X, T) i.e. (X, T) is a Λ sub maximal space Here A isalso a minimal Λ dense set.

Remark 3.31: Let (X, T) is a Λ sub maximal space. Then from Remark 3.6 we can write that A subset A of(X, T) is a Λ dense set iff ΛCl (A) = X.

Theorem 3.32: In a sub maximal space (X, T) no dense set can be a Λ dense set except X
Proof: Let (X, T) be a sub maximal space i.e. every dense subsets of X are open sets. But no open set can be a
Λ dense set except X. Hence the theorem
Theorem 3.33: If (X,T_Λ) is an indiscrete topological space then (X, T) is a Λ sub maximal space.
Proof: Let (X, T_Λ) is an indiscrete topological space i.e. $T_\Lambda = \{X, \phi\}$ which are both closed set of (X, T). So,(X,T) is a Λ sub maximal space.
Theorem 3.34: If (X,T) is a Λ sub maximal discrete topological space then (X, T_Λ) is also so.
Proof: Since the only closed sets in (X,T) are X and ϕ. So, all the Λ dense set must be X only. Hence thetheorem.
Theorem 3.35: Let (X,T) be a topological space with only one element A other then X and ϕ.Then (X,T) is a
Λ sub maximal space iff the order of X is two and $T_\Lambda = \{\phi, X, A^C\}$
Proof: Let if possible (X, T) be a Λsub maximal space with only one element A except ϕ and X. Then A^C is the only closed set except ϕ and X. A^C is a Λ dense set or the only Λ dense set is X. Since no other closed set exist in (X, T). Therefore $T_\Lambda = \{X, \phi\}$ or $T_\Lambda = \{X, A^C, \phi\}$. But from Remark 3.27(1) if $T_\Lambda = \{X, \phi\}$ then the topological space contains all open subsets of X whose order is one less than that of X. We know that if the order of X is n then it has n subsets of order (n-1) i.e. if $T_\Lambda = \{X, \phi\}$ then T must have n elements except X and ϕ . So, T_Λ cannot be {X, ϕ}. Sine T has only one element except X and ϕ.
If $T_\Lambda = \{X, \phi, A^C\}$ then from Remark 3.27(2), since A^C is the minimal Λ dense set T must have all open subsets of X whose order is one less than that of X except one which is a super set of A^C.
Let X be of order n then T must contain n-1 elements other than X and ϕ
But here T contains only one element other than X, ϕ. i.e. the order of X should be 2.
Hence (X,T) is a Λ sub-maximal space with only one element if the order of X is two and the corresponding $T_\Lambda = \{X, \phi, A^C\}$
Converse is obvious.
Theorem 3.36: Let (X, T) be a Λ sub maximal space containing r(>1) elements other then X and ϕ. Then

1. If T contains r minimal Λ dense sets then the order of X is 2r and $T_\Lambda = \{G : G^C \in T\}$
2. If T contains no minimal Λdense set other than X then the order of T is at least (n-1)(n+2)/2 where n isthe total number of elements in X except X and ϕ
3. If T contains only one minimal Λ dense set other than X then r ≥ (n-2)(n+1)/2 where n is the total number of elements in X
4. If T contains 1<m<r number of minimal Λdense set then r ≥ n(m-1)-(m-2)(3m-1)/2

Proof: Let (X, T) be a topological space such that T contains more than one element other than X and ϕ. Let Tcontains r(>1) elements other then X and ϕ. Then there are r number of closed sets other then X and ϕ. Since (X,T) is a Λ sub maximal space T_Λ may contain elements less than or equal to r other than X and ϕ.

1. **1.** Let if possible T_Λ contains r+2 elements. If all the r elements are minimal Λ dense set then from remark 3.20(5) the topological space T contains ϕ, X and all elements whose order are one less then that of X except r number of sets. But T has r elements except X and ϕ. So, the order of X is 2r and T_Λ = $\{G: G^C \in T\}$
2. **2.** Let $T_\Lambda = \{X, \phi\}$ then from remark 3.20(1) T must contain all the subsets of X whose order is one less than that of X. Here T contains r elements except X and ϕ. Let the order of X is

n then the number of elements of X whose order is n-1 is n. Obviously the finite intersection of these n elements need not be

ϕ but a member of T. Their union is X. The intersection of n elements will form n-1 elements and soon i.e.

$r \geq n+(n-1)+(n-2)+(n-3)+\ldots..+2(=n-(n-2))$
$= n(n-1) - \{1+2+3+\ldots.+(n-2)\}$
$= n(n-1) - (n-2)(n-1)/2$
$= (n-1)\{2n-n+2\}/2$
$= (n-1)(n+2)/2$

i.e. the number of elements in T should be at least (n-1)(n+2)/2 except X and ϕ

3. Let T_Λ contains only one minimal Λ dense set . Then the topological space contains (n-1) elementswhose order is one less then that of X .
So the topological space contains (n-1) elements and the elements obtained by their intersection .Since their union is X .So,

$r \geq (n-1)+(n-2)+\ldots\ldots\ldots + 2(=n-(n-2))$
$= (n-2)n-(n-2)(n-1)/2$
$= (n-2)(2n-n+1)/2$
$= (n-2)(n+1)/2$

i.e. the number of elements in T must be at least (n-2)(n-1)/2 except X and ϕ

4. If T contains $1<m<r$ number of minimal Λ dense set then T must contain (n-m) number of elementswhose order is one less than that of X

$r \geq (n-m)+(n-m-1)+\ldots.+2(=n-m-(m-2))$
$= n(m-1) - m(m-2) - (m-2)(m-1)/2$
$= n(m-1)-(m-2)(3m-1)/2$

Theorem 3.37: If (X,T) be a Λ - sub maximal topological space such that every open sets are also closed set then $T_\Lambda = \{X, \phi\}$

Proof: It follows from remark 3.14

Theorem 3.38: Let (X, T) be a Λ sub maximal space. Then every Λ dense set in (X, T) are also generalized closed set in (X, T)

Proof: In a Λsub maximal space (X, T) every element of (X, T_Λ) are closed subsets of X i.e. for any subset A of X such that $\Lambda(A) = X$, ClA = A. We know that $\Lambda(A)$ is the intersection of all open sets containing X i.e. A
$\subseteq \Lambda(A) = X$ i.e. ClA = A $\subseteq \Lambda(A) = X$ i.e. A is a generalized closed set.

Remark 3.39: Converse of the above theorem need not be true which follows from the example 3.25. Let us consider a generalized closed set C = {d}. Here C $\subseteq$ {b, d}, {b, d, c}, X, ClC = {d} $\subseteq${b, d}, {b, d, c}, X. But C is not a Λ dense set.

4. Application

In this section the concept of Λ dense continuous function and minimal Λ dense continuous function is introduced and its properties are studied.

Definition 4.1: A function $f : (X, T_1) \to (Y, T_2)$ is said to be a Λdense continuous function if the inverse image of any set in $T_{2\Lambda}$ is a closed set in T_1.

Example 4.2: Let X={a, b, c} and the corresponding topology be $T_1 = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let Y = {1, 2, 3} and the corresponding topology be $T_2 = \{Y, \phi, \{1\}, \{2,1\}\}$, $T_{2\Lambda} = \{Y, \phi, \{3\}, \{1,3\}, \{2,3\}\}$. Let $f : (X, T_1) \to (Y, T_2)$ be such that f(X) = Y , $f(\phi) = \phi$, f(a) = 1, f(b) = 2, f(c) = 3. Obviously f is a Λ dense continuous function.

Remark 4.3: A) From theorem 3.18 if Y is an indiscrete topological space then $T_{2\Lambda}$=P(Y) and the mapping f :(X, T_1) → (Y, T_2) is a Λdense continuous function if

i. $f^{-1}(A) = X$ for any $A \subseteq Y$

ii. $T_1 = P(X)$ and $f^{-1}(Y) = X$

B) From theorem 3.19: if T_2 = P(X) , T_2^C = P(X) then $T_{2\Lambda}$ = {X, ϕ} and f : (X, T_1) →(Y, T_2) is a Λ dense continuous function.

C) From theorem 3.21: T_2 = P(X)\A then $T_{2\Lambda}$ = {X,A, ϕ}. Here A is the odd term whose inverse image need not be open in T_1.

D) From theorem 3.22: if T_2 = {X, A, ϕ} T_2^C = {X, A^C, ϕ} then $T_{2\Lambda}$ = = P(X) \{G: G ⊆ A}} iff : (X, T_1) →(Y, T_2) be such that $f^{-1}(A) = X$ for any A⊆Y then f is a Λ dense continuous function.

Theorem 4.4: Let f : (X, T_1) →(Y, T_2) be a continuous function then f is a Λ dense continuous function if Yis a Λsub maximal space.

Proof: From definition 3.29, a topological space (Y, T_2) is a Λ sub-maximal space if every element of (Y, $T_{2\Lambda}$) is also a closed subset of (Y, T_2). Let f : (X, T_1) →(Y, T_2) be a continuous function. Let A be a Λ dense set in Y. Since Y is a Λ sub maximal space A is a closed set in Y and f being continuous function $f^{-1}(A)$ is also a closed set in X. Hence from definition f is a Λ dense continuous function.

Definition 4.5: A function f : (X, T_1) → (Y, T_2) is said to be a minimal Λdense continuous function if the inverse image of any minimal set in $T_{2\Lambda}$ is a closed set in T_1.

Example 4.6: Consider example 4.2, Here {3} is the minimal set in $T_{2\Lambda}$. Obviously its inverse image is a closed set in T_1. Thus f is a minimal Λ dense continuous function.

Theorem 4.7: A function f : (X, T_1) → (Y, T_2) is a Λdense continuous function then it is a minimal Λdense continuous function

Proof: Since f is a Λdense continuous function so inverse image of any set in $T_{2\Lambda}$ is a closed set in T_1 andthus inverse image of any minimal set in $T_{2\Lambda}$ is also a closed set in T_1. Thus the theorem.

Remark 4.8: Converse of the above theorem need not be true which follows from the following example:

Let X = {a, b, c} and the corresponding topology be T_1 = {X, ϕ, {a}, {a, c}}. Let Y ={1,2,3} and thecorresponding topology be T_2 = {Y, ϕ, {1}, {1,3}}.

Let f : (X,T_1) → (Y, T_2) be a mapping such that f(a) = 1, f(b) = 2, f(c) = 3, f(ϕ) = ϕ, f(X) = Y. Here $T_{2\Lambda}$ = {Y, ϕ, {2}, {2,3}, {1,2}} Here the minimal set in $T_{2\Lambda}$ is {2} whose inverse image {b} is a closed set in T_1, but the inverse image of {1,2} is {a, b} which is not a closed set in T_1.i.e. f is a minimal Λdense continuous function but not a Λdense continuous function.

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