SEPERATION AXIOM ON SOFT JA CLOSED SETS

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Abstract:

Munshi introduced g-regular and g-normal spaces using g-Closed sets in topological spaces. In this section some new spaces namely Soft JA -regular and Soft JA -normal spaces are introduced and some of their characterizations are obtained.

AMS subject classifications: 06d72, 51A05

Key words and phrases: *Soft JA Regular spaces, Soft JA Normal spaces, Soft JA continuous, Soft JA Hausdorff space.*

1. Introduction

Molodtsov initiated the concept of soft sets in [1]. Maji et al. defined some operations on soft sets in [2]. Functions and of course open functions stand among the most important notions in the whole Mathematical Science. Many different forms of open functions have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of Mathematical and related sciences. Munshi introduced g-regular and g-normal spaces using g-Closed sets in topological spaces.

In this section some new spaces namely Soft JA -regular and Soft JA -normal spaces are introduced and some of their characterizations are obtained.

2. PRELIMINARIES:

Definition 2.1: Let τ be a collection of soft sets over X with the fixed set E of parameters. Then τ is called a **Soft Topology** on X if

i. $\tilde{\phi}, \tilde{X}$ belongs to τ.

ii. The union of any number of soft sets in τ belongs to τ.

iii. The intersection of any two soft sets in τ belongs to τ. The triplet (X, τ, E) is called **Soft Topological Space** over X. The members of τ are called **Soft Open** sets in X and complements of them are called **Soft Closed** sets in X.

Definition 2.2: Let (X, τ, E) be a topological space. A subset A of the space X is said to be

1. **Soft JP closed set** [4,5] if $sCl(A,E) \tilde{\subseteq} Int(U,E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft $\hat{g}$-open.
2. **Soft J closed set** [6] if $s^{*}Cl(A,E) \tilde{\subseteq} Int(U,E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft $\hat{g}$-open. The set of all Soft J closed sets are denoted by $SJ(X, \tau, E)$.
3. **Soft Jc closed set** [7] if $\alpha Cl(A,E) \tilde{\subseteq} Int(U,E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft $\hat{g}$-open.
4. **Soft JA closed set** [8] if $pCl(A,E) \tilde{\subseteq} Int(U,E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft $\hat{g}$-open.

3. Soft JAT_0 and Soft JAT_1 spaces

Definition 3.1: A Soft topological space (X, τ, E) is known to be **Soft JAT_0 space** if for each pair of distinct Soft points $(x,e), (y,e)$ in $\tilde{X}$ if there exists a Soft JA open set (A,E) or (B,E) in (X, τ, E) such that either $(x,e) \tilde{\in} (A,E), (y,e) \tilde{\notin} (A,E)$ or $(x,e) \tilde{\notin} (B,E), (y,e) \tilde{\in} (B,E)$.

Example 3.2: $X = \{x_1, x_2\}, E = \{e_1, e_2\}$ with the Soft topology $\tau = \{\tilde{\phi}, \tilde{X}, (F,E)\}$ such that $F(e_1) = \{x_1, x_2\}, F(e_2) = \{x_1\}$ then Precisely (X, τ, E) is Soft JAT_0 space.

Theorem 3.3: A Soft Subspace of a Soft JAT_0 space is a Soft JAT_0 space.

Proof: Let (Y, τ, E) be a Soft subspace of a Soft JAT_0 space (X, τ, E) and let $(x, e), (y, e)$ be two distinct Soft points in $\tilde{Y}$. Since those Soft points are also in $\tilde{X}$, by hypothesis there exists a Soft JA open set (A, E) such that it contains (x, e) but not (y, e). Then $(A, E) \tilde{\cap} \tilde{Y}$ be another Soft JA open set which contains (y, e) but not (x, e). Hence (Y, τ, E) is also a Soft JAT_0 space.

Proposition 3.4: A Soft topological space (X, τ, E) is a Soft JAT_0 space if and only if Soft JA Closures of any two Soft singleton sets are disjoint.

Proof: The argument is evident.

Definition 3.5: A Soft topological space (X, τ, E) is known to be a **Soft J$\boldsymbol{AT_1}$ space** if for each pair of distinct Soft points $(x, e), (y, e)$ in $\tilde{X}$ there exists Soft JA open sets (A, E) and (B, E) in (X, τ, E) such that $(x, e) \tilde{\in} (A, E), (y, e) \tilde{\notin} (A, E)$ and $(x, e) \tilde{\notin} (B, E), (y, e) \tilde{\in} (B, E)$.

Example 3.6: $X = \{x_1, x_2\}, E = \{e_1, e_2\}$ with the Soft topology $\tau = \{\tilde{\phi}, \tilde{X}, (P, E), (Q, E)\}$ such that $P(e_1) = \{x_1\}, P(e_2) = \phi, Q(e_1) = \{x_1\}, Q(e_2) = \{x_2\}$ then precisely (X, τ, E) is Soft JAT_1 space.

Theorem 3.7: Every Soft JAT_1 space is a Soft JAT_0 space.

Proof: The argument is evident.

Remark 3.8: It is observed from the subsequent illustration that the reverse implication of the above theorem is incorrect.

Example 3.9: $X = \{x_1, x_2\}, E = \{e_1, e_2\}$ with the Soft topology $\tau = \{\tilde{\phi}, \tilde{X}, (H, E)\}$ such that $H(e_1) = \{x_1, x_2\}, H(e_2) = \{x_1\}$. Then, Precisely (X, τ, E) is Soft JAT_0 space but not Soft JAT_1 space.

Theorem 3.10: If every Soft point (x, e) in $\tilde{X}$ is Soft JA Closed set then (X, τ, E) is a Soft JAT_1 space.

Proof: Let $(x,e),(y,e)$ be Soft points in $\tilde{X}$ such that $x \neq y$. By the hypothesis $(x,E),(y,E)$ are Soft JA Closed sets. Then $(x,E) \tilde{\cap} (y,E) = \tilde{\phi}$ and $(x,E) \tilde{\subseteq} ((y,E))^C$ and $(y,E) \tilde{\subseteq} ((x,E))^C$. Also $((x,E))^C$ and $((y,E))^C$ are Soft JA open sets such that $(x,E) \tilde{\notin} ((x,E))^C$ and $(y,E) \tilde{\notin} ((y,E))^C$. Thus (X,τ,E) is a Soft JAT_1 space.

Theorem 3.11: A Soft subspace of a Soft JAT_1 space is a Soft JAT_1 space.

Proof: Let (Y,τ,E) be a Soft subspace of a Soft JAT_1 space (X,τ,E) and let $(x,e),(y,e)$ be two distinct Soft points in $\tilde{Y}$. Since those Soft points are also in $\tilde{X}$, by hypothesis there exists two Soft JA open sets (A,E) and (B,E) such that (A,E) contains (x,e) but not (y,e) and (B,E) contains (y,e) but not (x,e). Then $(A,E) \tilde{\cap} \tilde{Y}$ and $(B,E) \tilde{\cap} \tilde{Y}$ are two Soft JA open sets which contains only $(x,e),(y,e)$ respectively but not the other. Hence (Y,τ,E) is also a Soft JAT_1 space.

4. Soft JA Regular Space

Definition 4.1: A Soft topological space (X,τ,E) is said to be **Soft JA Regular (Soft JAT_3) space** if for a Soft point $(x,e) \tilde{\in} \tilde{X}$ and a Soft Closed set (G,E) such that $(x,e) \tilde{\notin} (G,E)$ there exists disjoint Soft JA open sets (A,E) and (B,E) in (X,τ,E) such that $(x,e) \tilde{\in} (A,E)$ and $(G,E) \tilde{\subseteq} (B,E)$.

Proposition 4.2: Let (X,τ,E) be a Soft topological space, (G,E) be a Soft Closed set in $\tilde{X}$ and $(x,e) \tilde{\in} \tilde{X}$ such that $(x,e) \tilde{\notin} (G,E)$ then there exist a Soft JA open set (F,E) such that $(G,E) \tilde{\cap} (F,E) \neq \tilde{\phi}$.

Proof: The argument is evident.

Proposition 4.3: Let (X,τ,E) be a Soft topological space, $(F,E) \tilde{\in} SJAC(X,\tau,E)$ and $(x,e) \tilde{\in} \tilde{X}$, then

1. $(x,e) \tilde{\in} (F,E)$ if the Soft singleton set $(x,E) \tilde{\subseteq} (F,E)$.
2. If $(x,E) \tilde{\cap} (F,E) = \tilde{\phi}$ then $(x,e) \tilde{\notin} (F,E)$.

Proof: The argument is evident.

Theorem 4.4: Let (X,τ,E) be a Soft topological space, (F,E) be a Soft subset of (X,τ,E) and $(x,e) \tilde{\in} \tilde{X}$, if (X,τ,E) is Soft JA regular space then,

1. $(x,e) \tilde{\notin} (F,E)$ if and only if $(x,E) \tilde{\cap} (F,E) = \tilde{\phi}$ for every Soft JA Closed set (F,E).

2. $(x,e) \tilde{\notin} (G,E)$ if and only if $(x,E) \tilde{\cap} (G,E) = \tilde{\phi}$ for every Soft JA open set (G,E).

Proof:

1. Let (F,E) be a Soft JA Closed set in (X,τ,E) such that the Soft point $(x,e) \tilde{\notin} (F,E)$. By hypothesis, (X,τ,E) is Soft JA regular space. Then, there exists a Soft JA open set (G,E) such that $(x,e) \tilde{\in} (G,E)$ and $(F,E) \tilde{\cap} (G,E) = \tilde{\phi}$. Then $(x,E) \tilde{\subseteq} (G,E)$. Hence $(x,E) \tilde{\cap} (F,E) = \tilde{\phi}$. Conversely, if $(x,E) \tilde{\cap} (F,E) = \tilde{\phi}$ then $(x,e) \tilde{\notin} (F,E)$.
2. Let (G,E) be a Soft JA open set such that $(x,e) \tilde{\notin} (G,E)$. If $(x,e) \tilde{\notin} G(e)$ for all $e \tilde{\in} E$, then we get the proof. If $(x,e) \tilde{\notin} G(e_1)$ and $(x,e) \tilde{\in} G(e_2)$ for some $e_1, e_2 \tilde{\in} E$. Then $(x,e) \tilde{\in} G^C(e_1)$ and $(x,e) \tilde{\notin} G^C(e_2)$ for some $e_1, e_2 \tilde{\in} E$. Hence $(G,E)^C$ is Soft JA Closed set such that $(x,e) \tilde{\notin} (G,E)^C$. By (1), $(x,E) \tilde{\cap} (G,E)^C = \tilde{\phi}$ this implies that $(x,E) \tilde{\subseteq} (G,E)$, and so $(x,e) \tilde{\in} (G,E)$ which is a contradiction with $(x,e) \tilde{\notin} G(e_1)$ for some $e_1 \tilde{\in} E$. Therefore $(x,E) \tilde{\cap} (G,E) = \tilde{\phi}$. Conversely, if $(x,E) \tilde{\cap} (G,E) = \tilde{\phi}$, it is evident that $(x,e) \tilde{\notin} (G,E)$. Hence verified.

Theorem 4.5: Let (X,τ,E) be a Soft topological space and $(x,e) \tilde{\in} \tilde{X}$. Then the following are equivalent.

1. (X,τ,E) is Soft JA regular space.
2. For every Soft JA Closed set (G,E) such that $(x,E) \tilde{\cap} (H,E) = \tilde{\phi}$ there exists disjoint Soft JA open sets $(A,E),(B,E)$ such that $(x,E) \tilde{\subseteq} (A,E), (G,E) \tilde{\subseteq} (B,E)$.

Proof: (1) → (2): Let (G, E) be a Soft JA Closed set such that $(x, E) \tilde{\cap} (G, E) = \tilde{\phi}$. Then $(x, e) \tilde{\notin} (G, E)$. Since (X, τ, E) is Soft JA regular space. There exists disjoint Soft JA open sets $(A, E), (B, E)$ such that $(x, e) \tilde{\in} (A, E), (G, E) \tilde{\subseteq} (B, E)$ this implies that $(x, E) \tilde{\subseteq} (A, E), (G, E) \tilde{\subseteq} (B, E)$.

(2) → (1): Let (G, E) be a Soft JA Closed set such that $(x, e) \tilde{\notin} (G, E)$. Then $(x, E) \tilde{\cap} (G, E) = \tilde{\phi}$. By hypothesis, there exists disjoint Soft JA open sets $(A, E), (B, E)$ such that $(x, E) \tilde{\subseteq} (A, E), (G, E) \tilde{\subseteq} (B, E)$. Then $(x, E) \tilde{\subseteq} (A, E), (G, E) \tilde{\subseteq} (B, E)$. Hence (X, τ, E) is Soft JA regular space.

Theorem 4.6: Let (X, τ, E) be a Soft topological space. If (X, τ, E) is Soft JA regular space then $\forall (x, e) \tilde{\in} \tilde{X}$, the Soft singleton set (x, E) is Soft JA Closed in (X, τ, E).

Proof: Since (X, τ, E) is Soft JA regular space then there exists disjoint Soft JA open sets $(F, E)_y$ and (G, E) such that $(y, E) \tilde{\subseteq} (F, E)_y$ and $(x, E) \tilde{\cap} (F, E)_y = \tilde{\phi}$ and $(x, E) \tilde{\subseteq} (G, E)_y$ and $(y, E) \tilde{\cap} (G, E) = \tilde{\phi}$ where $(y, e) \tilde{\in} ((x, e))^C$. It follows that the Soft union of all $(F, E)_y$ is a Soft subset of $(x, E)^C$. Now to prove the equality in between this we have to prove the vice versa. Now let $\tilde{\cup}\, (y, e) \tilde{\in} ((x, e))^C (F, E)_y = (H, E)$ where $H(e) = \tilde{\cup}\, (y, e) \tilde{\in} ((x, e))^C\, F((y, e))$ for all $e \tilde{\in} E$. By the definition of Soft singleton set, for all $(y, e) \tilde{\in} ((x, e))^C, e \tilde{\in} E, (x, E)^C = ((x, e))^C$ for all $e \tilde{\in} E \tilde{\subseteq} \tilde{\cup}\, (y, e) \tilde{\in} \left((x, e)\right)^C$. $F(e)_y = H(e)$. Thus $\tilde{\cup}\, (y, e) \tilde{\in} ((x, e))^C (F, E)_y = (x, E)^C$ .Therefore, $(x, E)^C$ is Soft JA open set for all $(y, e) \tilde{\in} ((x, e))^C$. Therefore (x, E) is Soft JA Closed set.

Theorem 4.7: A Soft subspace of a Soft JA regular space is Soft regular.

Proof: Let (X, τ, E) be a Soft JA regular space over the universe X and (Y, τ, E) be a Soft sub topological space over Y which is a subset of X. Let $(x, e), (y, e) \tilde{\in} \tilde{Y}$ such that $x \neq y$ and (G, E) be a Soft JA Closed set in (Y, τ, E), we conClude that (Y, τ, E) be a Soft JA Hausdorff space. Then $(G, E) = \tilde{Y} \tilde{\cap} (F, E)$ for some Soft JA Closed set (F, E) in (X, τ, E). Hence $(y, e) = \tilde{Y} \tilde{\cap} (F, E)$. But $(y, e) \tilde{\notin} \tilde{Y}$ so $(y, e) \tilde{\notin} (F, E)$. Since (X, τ, E) is a Soft JA regular space, then there exists disjoint Soft JA open sets (A, E), (B, E) such that $(y, e) \tilde{\in} (A, E)$ and $(F, E) \tilde{\subseteq} (B, E)$. Take $(G_1, E) = \tilde{Y} \tilde{\cap} (A, E), (G_2, E) = \tilde{Y} \tilde{\cap} (B, E)$. Then $(G_1, E), (G_2, E)$ are two Soft JA open sets in $\tilde{Y}$ such that $(y, e) \tilde{\in} (G_1, E), (G, E) \tilde{\subseteq} \tilde{Y} \tilde{\cap} (B, E) = (G_2, E)$ and $(G_1, E) \tilde{\cap} (G_2, E) = (A, E) \tilde{\cap} (B, E) = \tilde{\phi}$. Thus, (Y, τ, E) is also Soft JA regular space.

5. Soft JA Normal Space

Definition 5.1: A Soft topological space (X, τ, E) is known to be **Soft JA Normal (Soft JAT_4) space** if for two disjoint Soft JA Closed sets (F, E) and (G, E) there exists two disjoint Soft JA open sets (A, E) and (B, E) such that $(F, E) \tilde{\in} (A, E)$ and $(G, E) \tilde{\subseteq} (B, E)$.

Theorem 5.2: Let (X, τ, E) be a Soft topological space and $(x, e) \tilde{\in} X$. Then the following are equivalent.

1. (X, τ, E) is Soft JA normal space.
2. For every Soft JA Closed set (F, E) and Soft JA open set (G, E) such that $(F, E) \tilde{\subseteq} (G, E)$, there exists a Soft JA open set (A, E) such that $(F, E) \tilde{\subseteq} (A, E)$, $SJACl(A, E) \tilde{\subseteq} (B, E)$, where $SJACl(A, E)$ is the Soft JA Closure of (A, E).

Proof:

(1) → (2): Let (F, E) be a Soft JA Closed set and (G, E) be Soft JA open set such that $(F, E) \tilde{\subseteq} (G, E)$. Then (F, E) and (G, E) are Soft JA Closed sets such that $(F, E) \tilde{\cap} (G, E)^C = \tilde{\phi}$. It follows by (1), there exists Soft JA open sets (A, E) and (B, E) such that $(F, E) \tilde{\subseteq} (A, E)$ $(G, E)^C \tilde{\subseteq} (B, E)$ and $(A, E) \tilde{\cap} (B, E) = \tilde{\phi}$. $(A, E) \tilde{\subseteq} (B, E)^C$, so $SJACl(A, E) \tilde{\subseteq} SJACl(B, E)^C$ where (G, E) is Soft JA open set. Also, $SJACl(B, E)^C \tilde{\subseteq} (G, E)$. Hence $SJACl(A, E) \tilde{\subseteq} SJACl(B, E)^C \tilde{\subseteq} (G, E)$. Thus, $(F, E) \tilde{\subseteq} (A, E), SJACl(A, E) \tilde{\subseteq} (G, E)$.

(2) → (1): Let $(A,E),(B,E)$ be Soft JA Closed sets such that $(A,E)\,\tilde{\cap}\,(B,E)=\tilde{\phi}$. Then $(A,E)\,\tilde{\subseteq}\,(B,E)^C$. Then by hypothesis, there exist a Soft JA open set (F,E) such that $(A,E)\,\tilde{\subseteq}\,(F,E), SJACl(F,E)\,\tilde{\subseteq}\,(A,E)^C$. So $(B,E)^C\,\tilde{\subseteq}\,SJACl(F,E)^C$, $(A,E)\,\tilde{\subseteq}\,(F,E)$ and $SJACl((F,E)^C)\,\tilde{\cap}\,(F,E)=\tilde{\phi}$, where (F,E) and $SJACl((F,E)^C)$ are Soft JA open sets. Thus (X,τ,E) is Soft JA normal space.

Theorem 5.3: A Soft JA Closed subspace of a Soft JA normal space is Soft JA normal.

Proof: Let (Y,τ,E) be a Soft JA Closed subspace of a Soft JA normal space (X,τ,E). Let $(A,E),(B,E)$ be Soft JA Closed sets in $\tilde{Y}$ such that $(A,E)\,\tilde{\cap}\,(B,E)=\tilde{\phi}$. Then $(A,E)=\tilde{Y}\,\tilde{\cap}\,(F,E),(B,E)=\tilde{Y}\,\tilde{\cap}\,(G,E)$ for some Soft JA Closed sets $(F,E),(G,E)$ in $\tilde{X}$. Since $\tilde{Y}$ is a Soft JA Closed subset of $\tilde{X}$. Then $(A,E),(B,E)$ are Soft JA Closed sets in $\tilde{X}$ such that $(A,E)\,\tilde{\cap}\,(B,E)=\tilde{\phi}$. Hence by Soft JA normality there exist Soft JA open sets (U,E) and (V,E) such that, $(A,E)\,\tilde{\subseteq}\,(U,E),(B,E)\,\tilde{\subseteq}\,(V,E)$ and $(U,E)\,\tilde{\cap}\,(V,E)=\tilde{\phi}$.

Since $(A,E),(B,E)$ are Soft JA Closed subsets in $\tilde{Y}$ then $(A,E)\,\tilde{\subseteq}\,\tilde{Y}\,\tilde{\cap}\,(U,E),(B,E)\,\tilde{\subseteq}\,\tilde{Y}\,\tilde{\cap}\,(V,E)$ and $[\tilde{Y}\,\tilde{\cap}\,(U,E)]\,\tilde{\cap}\,[\tilde{Y}\,\tilde{\cap}\,(V,E)]=\tilde{\phi}$ where $\tilde{Y}\,\tilde{\cap}\,(U,E),\tilde{Y}\,\tilde{\cap}\,(V,E)$ are Soft JA open sets in $\tilde{Y}$. Hence (Y,τ,E) is Soft JA normal.

Theorem 5.4: If $f:(X,\tau,E)\to(Y,\sigma,K)$ is Soft JA continuous, injection and (Y,σ,K) is Soft JAT_0 space then (X,τ,E) is also Soft JAT_0 space.

Proof: Let $f:(X,\tau,E)\to(Y,\sigma,K)$ be Soft JA continuous, Soft injection map. Then for any two distinct Soft points $(x,e),(y,e)$ in $\tilde{X}$, there exists two distinct Soft points in $\tilde{Y}$ such that $f((x,e))=(u,k),f((y,e))\ =\ (v,k)$. Since (Y,σ,K) is Soft JAT_0 space, there exists atleast one Soft set either (A,E) or (B,E) such that $(u,k)\,\tilde{\in}\,(A,E)$ but $(v,k)\,\tilde{\notin}\,(A,E)$ and $(v,k)\,\tilde{\in}\,(B,E)$ but $(u,k)\,\tilde{\notin}\,(B,E)$. Then $(x,e)\,\tilde{\in}\,f^{-1}(A,E),(y,e)\,\tilde{\notin}\,f^{-1}(A,E)$ and $(y,e)\,\tilde{\in}\,f^{-1}(B,E),(x,e)\,\tilde{\notin}\,f^{-1}(B,E)$. Since f is Soft JA continuous, $f^{-1}(A,E),f^{-1}(B,E)$ are Soft JA open sets in (X,τ,E). Then (X,τ,E) is also Soft JAT_0 space.

Theorem 5.5: If $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ is Soft JA continuous, Soft injection and (Y,σ,K) is Soft JAT_1 space then (X,τ,E) is also Soft JAT_1 space.

Proof: Let $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ be Soft JA continuous, injection map. Then for any two distinct Soft points $(x,e),(y,e)$ in $\tilde{X}$, there exists two distinct Soft points in $\tilde{Y}$ such that $f((x,e))=((u,k)),f((y,e))=((v,k))$. Since (Y,σ,K) is Soft JAT_1 space, there exists two Soft open sets $(A,E),(B,E)$ such that $(u,k)\tilde{\in}(A,E)$ but $(v,k)\tilde{\notin}(A,E)$ and $(v,k)\tilde{\in}(B,E)$ but $(u,k)\tilde{\notin}(B,E)$. Then $(x,e)\tilde{\in}f^{-1}(A,E),(y,e)\tilde{\notin}f^{-1}(A,E)$ and $(y,e)\tilde{\in}f^{-1}(B,E),(x,e)\tilde{\notin}f^{-1}(B,E)$. Since f is Soft JA continuous, $f^{-1}(A,E),f^{-1}(B,E)$ are Soft JA open sets in (X,τ,E). Then (X,τ,E) is also Soft JAT_1 space.

Theorem 5.6: If $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ is Soft strongly JA open, bijective and (X,τ,E) is Soft JAT_1 space then (Y,σ,K) is also Soft JAT_1 space.

Proof: Let $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ be Soft strongly JA open, bijective map. Let (X,τ,E) be Soft JA T_1 space and $(u,k),(v,k)$ be any two distinct Soft points in $\tilde{Y}$. Since f is Soft bijective, there exists distinct Soft points $(x,e),(y,e)$ in $\tilde{X}$ such that $f((x,e))=(u,k),f((y,e))=(v,k)$. Now being (X,τ,E) is Soft JA T_1 space, there exists two Soft JA open sets $(A,E),(B,E)$ such that $(x,e)\tilde{\in}(A,E),(y,e)\tilde{\notin}(A,E)$ and $(y,e)\tilde{\in}(B,E),(x,e)\tilde{\notin}(B,E)$. Since $f((x,e))=(u,k),f((y,e))=(v,k)$, then $(u,k)\tilde{\in}f(A,E),(v,k)\tilde{\notin}f(A,E)$ and $(v,k)\tilde{\in}f(B,E),(u,k)\tilde{\notin}f(B,E)$. Since f is Soft strongly JA open, $f(A,E),f(B,E)$ are Soft JA open sets in (Y,σ,K) such that $(u,k)\tilde{\in}f(A,E),(v,k)\tilde{\notin}f(A,E)$ and $(v,k)\tilde{\in}f(B,E),(u,k)\tilde{\notin}f(B,E)$. Then (Y,σ,K) is Soft JAT_1 space.

Theorem 5.7: If $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ is Soft JA irresolute, Soft injection and (Y,σ,K) is Soft JAT_1 space then (X,τ,E) is also Soft JAT_1 space.

Proof: Let $f:(X,\tau,E)\rightarrow(Y,\sigma,K)$ be Soft JA irresolute, Soft injection map. Then for any two distinct Soft points $(x,e),(y,e)$ in $\tilde{X}$, there exists two distinct Soft points in $\tilde{Y}$ such that

$f\big((x,e)\big) = (u,k), f\big((y,e)\big) = (v,k)$. Since (Y,σ,K) is Soft JAT_1 space, there exists two Soft JA open sets $(A,E),(B,E)$ such that $(u,k) \tilde{\in} (A,E)$ but $(v,k) \tilde{\notin} (A,E)$ and $(v,k) \tilde{\in} (B,E)$ but $(u,k) \tilde{\notin} (B,E)$. Then $(x,e) \tilde{\in} f^{-1}(A,E),(y,e) \tilde{\notin} f^{-1}(A,E)$ and $(y,e) \tilde{\in} f^{-1}(B,E),(x,e) \tilde{\notin} f^{-1}(B,E)$. Since f is Soft JA Irresolute, $f^{-1}(A,E), f^{-1}(B,E)$ are Soft JA open sets in (X,τ,E). Then (X,τ,E) is also Soft JAT_1 space.

Theorem 5.8: If $f:(X,\tau,E) \to (Y,\sigma,K)$ is Soft JA continuous, Soft Closed injection and (Y,σ,K) is Soft Regular space then (X,τ,E) is also Soft JA regular space.

Proof: Let (F,E) be a Soft Closed set in (X,τ,E) and $(x,e) \tilde{\notin} (F,E)$. Now (Y,σ,K) is Soft regular space then there exists disjoint Soft open sets (A,E) and (B,E) such that $f((x,e)) \tilde{\in} (A,E)$ and $f(F,E) \tilde{\subset} (B,E)$.Then $(x,e) \tilde{\in} f^{-1}(A,E)$ and $(F,E) \tilde{\subset} f^{-1}(B,E)$. Since f is Soft JA continuous, $f^{-1}(A,E), f^{-1}(B,E)$ are Soft JA open sets in (X,τ,E). Since f is Soft injection, $f^{-1}(A,E) \tilde{\cap} f^{-1}(B,E) = f^{-1}[(A,E) \tilde{\cap} (B,E)] = f^{-1}\big(\tilde{\phi}\big) = \tilde{\phi}$. Hence (X,τ,E) is Soft JA regular space.

Theorem 5.9: If $f:(X,\tau,E) \to (Y,\sigma,K)$ is Soft JA irresolute, Soft Closed injection and (Y,σ,K) is Soft JA regular space then (X,τ,E) is also Soft JA regular space.

Proof: Let (F,E) be a Soft JA Closed set in (X,τ,E) and $(x,e) \tilde{\notin} (F,E)$. Now (Y,σ,K) is Soft JA regular space then there exists disjoint Soft JA open sets (A,E) and (B,E) such that $f((x,e)) \tilde{\in} (A,E)$ and $f(F,E) \tilde{\subset} (B,E)$. Then $(x,e) \tilde{\in} f^{-1}(A,E)$ and $(F,E) \tilde{\subset} f^{-1}(B,E)$. Since f is Soft JA irresolute then $f^{-1}(A,E), f^{-1}(B,E)$ are Soft JA open sets in (X,τ,E). Since f is Soft injection, $f^{-1}(A,E) \tilde{\cap} f^{-1}(B,E) = f^{-1}[(A,E) \tilde{\cap} (B,E)] = f^{-1}\big(\tilde{\phi}\big) = \tilde{\phi}$. Hence (X,τ,E) is Soft JA regular space.

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