Group Difference Cordial Labeling of some Ladder related Graphs

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Abstract

Let $G = (VG),\ E(G))$ be a graph. Let Γ be a group. For $u \epsilon \Gamma$, let $o(u)$ denotes

the order of u in Γ. Let $f : V(G) \to \Gamma$ be a function. For each edge uv assign the label $\left|o\left(f(u)\right) - o\left(f(v)\right)\right|$. Let $v_f(i)$ denote the number of vertices of G having label i under f. Also $e_f(1)$, $e_f(0)$ respectively denote the number of edges labeled with 1 and not with 1.Now f is called a group difference cordial labeling if $\left|v_f(i) - v_f(j)\right| \leq 1$ for every $i, j \epsilon \Gamma$, $i \neq j$ and $\left|e_f(1) - e_f(0)\right| \leq 1$. A graph which admits a group difference cordial labeling is called group difference cordial graph. In this paper we fix the group Γ as the group $\{1, -1, i, -i\}$ which is the group of fourth roots of unity, that is cyclic with generators $i\ and - i$.

We prove that $L_n \odot K_1$,Open ladder, Slanting ladder and further characterized $O(L_n) \odot K_1$.

Keywords: cordial labeling, difference labeling, group difference cordial labeling

AMS subject classification: 05C78

1 Introduction

Graphs considered here are finite, undirected and simple. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Labelled graphs serve as useful models for a broad range of applications such as : astronomy, circuit design, communication network addressing and models for constraint programming over finite domains.

Cahit [2] introduced the concept of cordial labeling.

Definition 1.1. [2] Let $f : V(G) \to \{0,1\}$ be any function. For each edge xy assign the label $|f(x) - f(y)|$. f is called a cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1. Also the number of edges labelled 0 and the number of edges labeled 1 differ by at most 1.

In[5] , Ponraj et al. introduced a new labeling called difference cordial labeling .

Definition 1.2. [5] Let G be a (p,q) graph .Let $f: V(G) \to \{1,2 \dots p\}$ be a bijection. For each edge, assign the label $|f(u) - f(v)|$. f is called a difference cordial labeling if f is $1-1$ and $\left|e_f(0) - e_f(1)\right| = 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

Athisayanathan et al.[1] introduced the concept of group A cordial labeling.

Definition 1.3. [1] Let A be a group. We denote the order of an element $a \epsilon A$ by $o(a)$.Let$f: V(G) \to A$ be a function. For each edge uv assign the label 1 if $\left(o(f(u)), o(f(v))\right) = 1$ or 0 otherwise . f is called a group A Cordial labeling if $\left|v_f(a) - v_f(b)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with n $(n = 0,1)$ A graph which admits a group A cordial labeling is called a group A cordial graph.

Motivated by these, we define group difference cordial labeling of graphs. Terms not defined here are used in the sense of Harary[4] and Gallian [3].The Cartesian product of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ with the vertex set $V_1 \times V_2$ and two vertices $u = (u_{1,} u_2)$ and $v = (v_{1,} v_2)$ are adjacent whenever$[u_1 = v_1$ and u_2 adj $v_{2,}]$ or $[u_2 = v_2$ and u_1 adj $v_{1,}]$. The graph $L_n = P_n \times P_2$ is called a ladder. Let G_1 , G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 $with$ G_2, $G_1 \odot G_2$ is the $graph$ $obtained$ by $taking$ one $copy$ of $G_1 and$ p_1 copies of G_2 and joining the i th vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2 . An $O(L_n)$ is a ladder graph with 2n vertices and is got from two paths of length n-1 with $V(G) = \{u_i, v_i \,/\, 1 \leq i \leq n\}$ and $E(G) = \{u_i, u_{i+1}, v_i, v_{i+1} \,/\, 1 \leq i \leq n-1\} \cup \{u_i v_i \,/\, 2 \leq i \leq n-1\}$. SL_n is a ladder graph with 2n vertices and is got from two paths of length n-1 with $V(G) = \{u_i, v_i \,/\, 1 \leq i \leq n\}$ and $E(G) = \{u_i u_{i+1}, v_i v_{i+1} \,/\, 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} \,/\, 1 \leq i \leq n-1\}$.

2. Group Difference cordial Graphs

Definition 2.1. Let $G = (V(G), E(G))$ be a graph. Let Γ be a group. For $u \epsilon \Gamma$, let $o(u)$ denote the order of u in Γ. Let $f: V(G) \rightarrow \Gamma$ be a function .For each edge uv assign the label $|o(f(u)) - o(fv))|$. Let $v_f(i)$ denote the number of vertices of G having label i under f. Also e_f (1) , e_f (0) respectively denote the number of edges labeled with 1 and not with 1. Now f is called a group difference cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ for every $i, j \epsilon \Gamma, i \neq j$ and $|e_f(1) - e_f(0)| \leq 1$. A graph which admits a group difference cordial labeling is called group difference cordial graph .

In this paper we take the group Γ as the group $\{1, -1, i, -i\}$ which is the group of fourth roots of unity, that is cyclic with generators i and $-i$.

Example 2.2 The following is a simple example of a $group\ difference\ cordial\ graph$

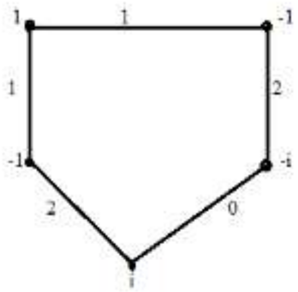

Fig .2.1

Theorem 2.3. The $L_n \odot K_1$ is $a\ group\ difference\ cordial\ graph\ for\ all$ 'n'.

Proof: $Let\ G = L_n \odot K_1$ has $4n$ vertices and $5n - 2\ edges$, f be a group difference cordial labeling of G .$Let\ V(G) = \{u_1, u_2, \ldots u_{4n}\}$. Clearly $L_n \odot K_1$ is a group difference cordial graph for $n \leq 3$. Assume $n \geq 4$ and define $f : V(G) \rightarrow \{1, -1, i, -i\}$ $as\ follows$

Case (i) $n \equiv 0\ (mod\ 4)$. Let $n = 4k, k \geq 1$.

$$f(u_{2i-1}) = \{ \begin{array}{l} i \quad if\ 1 \leq i \leq 2k\ and\ 5k+2 \leq i \leq 7k+1 \\ 1\ if\ 2k+1 \leq i \leq 5k+1\ and\ 7k+2 \leq i \leq 8k\ for\ k \geq 2 \end{array}$$

$$f(u_{2i}) = \{ \begin{array}{l} -i\ if\ 1 \leq i \leq 2k\ and\ 5k+1 \leq i \leq 7k \\ -1\ if\ 2k+1 \leq i \leq 5k\ and\ 7k+1 \leq i \leq 8k \end{array}$$

Clearly $V_f(1) = V_f(-1) = V_f(i) = V_f(-i) = 4k$.Also $e_f(1) = e_f(0) = 10k - 1$.

Therefore f is the group difference cordial labeling of G .

Case(ii) $n \equiv 1\ (mod\ 4)$. Let $n = 4k + 1, k \geq 1. For\ k = 1$,

$$f(u_{2i-1}) = \begin{cases} i \ if \ 1 \le i \le 2k+1 \ and \ 5k+3 \le i \le 7k+2 \\ 1 \ if \ 2k+2 \le i \le 5k+2 \ and \ 7k+3 \le i \le 8k+2 \end{cases}$$

For $k \geq 2$,

$$f(u_{2i-1}) = \begin{cases} i \ if \ 1 \le i \le 2k+1 \ and \ 5k+3 \le i \le 7k+3 \\ 1 \ if \ 2k+2 \le i \le 5k+2 \ and \ 7k+4 \le i \le 8k+2 \end{cases}$$

$$f(u_{2i}) = \begin{cases} -i \ if \ 1 \le i \le 2k, 5k+3 \le i \le 7k+2 \ and \ 7k+3 \le i \le 8k+2 \ for \ k=1 \\ -1 \ if \ 2k+1 \le i \le 5k+2 \ and \ 7k+3 \le i \le 8k+2 \ for \ k \ge 2 \end{cases}$$

Clearly $V_f(1) = V_f(-1) = V_f(i) = V_f(-i) = 4k + 1$.Also $e_f(1) = 10k + 2$ and $e_f(0) = 10k + 1$. Therefore f is the group difference cordial labeling of G .

Case (iii): $n \equiv 2 \ (mod\ 4)$, Let $n = 4k + 2, k \geq 1$.

$$f(u_{2i-1}) = \begin{cases} i \ if \ 1 \le i \le 2k+1 \ and \ 5k+4 \le i \le 7k+4 \\ 1 \ if \ 2k+2 \le i \le 5k+3 \ and \ 7k+5 \le i \le 8k+4 \end{cases}$$

$$f(u_{2i}) = \begin{cases} -i \ if \ 1 \le i \le 2k+1 \ and \ 5k+4 \le i \le 7k+4 \\ -1 \ if \ 2k+2 \le i \le 5k+3 and \ 7k+5 \le i \le 8k+4 \end{cases}$$

Clearly $V_f(1) = V_f(-1) = V_f(i) = V_f(-I) = 4k + 2$ and $e_f(1) = 10k + 4 = e_f(0)$. Therefore f is the group difference cordial labeling of G .

Case (iv) $n \equiv 3 \ (mod\ 4)$, Let $n = 4k + 3, k \geq 1$.

$$f(u_{2i-1}) = \begin{cases} i \ if \ 1 \le i \le 2k+2 \ and \ 5k+6 \le i \le 7k+6 \\ 1 \ if \ 2k+3 \le i \le 5k+5 \ and \ 7k+7 \le i \le 8k+6 \end{cases}$$

$$f(u_{2i}) = \begin{cases} -i \ if \ 1 \le i \le 2k+1 \ and \ 5k+5 \le i \le 7k+6 \\ -1 \ if \ 2k+2 \le i \le 5k+4 \ and \ 7k+7 \le i \le 8k+6 \end{cases}$$

Clearly $V_f(1) = V_f(-1) = V_f(i) = V_f(-i) = 4k + 3$.Also $e_f(1) = 10k + 7$ and $e_f(0) = 10k + 6$. Therefore f is the group difference cordial labeling of G .

The labeling of the graph $L_5 \odot K_1$ is given in the figure:

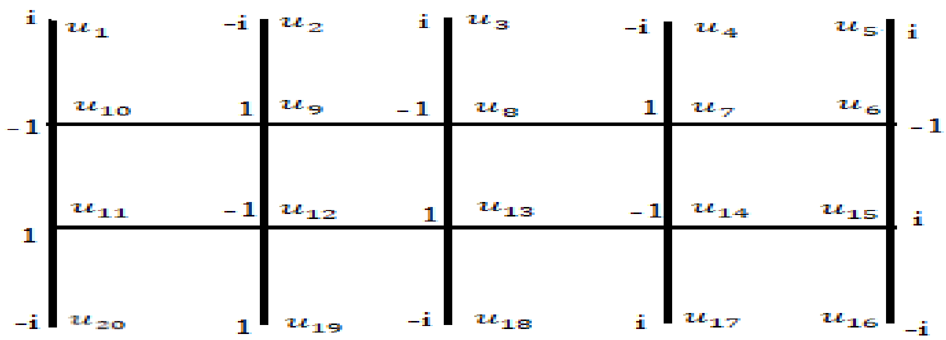

Fig .2.2

Theorem 2.4. The $O(L_n)$ is a group difference cordial graph for all 'n'.

Proof : $Let\ G = O(L_n)$ has $2n$ vertices and $3n-4\ edges, f$ be a group difference cordial labeling of G . $Let\ V\ (G) = \{u_1, u_2, \dots u_{2n}\}$. Clearly $O(L_n)$ is a group difference cordial graph for $n \le 3$. Assume $n \ge 4$ and define $f : V(G) \to \{1, -1, i, -i\}\ as\ follows.$

Case (i) If n is even, fix the labeling of u_1 as "i" for all the open ladder

Sub case (i): $n \equiv 0\ (mod\ 4)$, Let $n = 4k, k \ge 1$.

$f(u_{2i-1}) = \{-1\ if\ 2 \le i \le k+1\ -i\ if\ k+2 \le i \le 2k+1\ 1\ if\ 2k+2 \le i \le 3k+1\ i\ if\ 3k+2 \le i \le 4k\ for\ k \ge 2$

$f(u_{2i}) = \{1\ if\ 1 \le i \le k\ i\ if\ k+1 \le i \le 2k\ -1\ if\ 2k+1 \le i \le 3k\ -i\ if\ 3k+1 \le i \le 4k$

Clearly $V_f(1) = V_f(-1) = V_f(i) = V_f(-i) = k$ and $e_f(1) = 6k-2 = e_f(0)$

Sub case (ii) : $n \equiv 2\ (mod\ 4)$, Let $n = 4k+2,\ k \ge 1$.

$f(u_{2i-1}) = \{-1\ if\ 2 \le i \le k+1\ -i\ if\ k+2 \le i \le 2k+2\ 1\ if\ 2k+3 \le i \le 3k+2\ i\ if\ 3k+3 \le i \le 4k+2$

$f(u_{2i}) = \{1\ if\ 1 \le i \le k+1\ i\ if\ k+2 \le i \le 2k+1\ -1\ if\ 2k+2 \le i \le 3k+2\ -i\ if 3k+3 \le i \le 4k+2$

Clearly $V_f(1) = V_f(-1) = V_f(i) = V_f(-i) = 2k+1$ and $e_f(1) = 6k+1 = e_f(0)$.

Therefore f is the group difference cordial labeling of G .

Case (ii) If n is odd,

Subcase (i) $n \equiv 1\ (mod\ 4)$, Let $n = 4k+1, k \ge 1$.

$$f(u_{2i-1}) = \begin{cases} 1\ if\ 1 \le i \le k+1\ and\ 2k+2 \le i \le 3k+1 \\ i\ if\ k+2 \le i \le 2k+1\ and\ 3k+2 \le i \le 4k+1 \end{cases}$$

$$f(u_{2i}) = \begin{cases} -1\ if\ 1 \le i \le k\ and\ 2k+1 \le i \le 3k+1 \\ -i\ if\ k+1 \le i \le 2k\ and\ 3k+2 \le i \le 4k+1 \end{cases}$$

Clearly $V_f(1) = V_f(-1) = 2k+1$, $V_f(i) = V_f(-i) = 2k$ and $e_f(1) = 6k$, $e_f(0) = 6k-1$.

Sub case (ii) : $n \equiv 3\ (mod\ 4)$, Let $n = 4k+3,\ k \ge 1$.

$$f(u_{2i-1}) = \begin{cases} 1\ if\ 1 \le i \le k+1 and\ 2\ k+3 \le i \le 3k+3 \\ i\ if\ k+2 \le i \le 2k+2\ and\ 3k+4 \le i \le 4k+3 \end{cases}$$

$$f(u_{2i}) = \begin{cases} -1\ if\ 1 \le i \le k+1\ and\ 2k+2 \le i \le 3k+2 \\ -i\ if\ k+2 \le i \le 2k+1 and\ 3k+3 \le i \le 4k+3 \end{cases}$$

Clearly $V_f(1) = V_f(-1) = 2k+2,\ V_f(i) = V_f(-i) = 2k+1$ and $e_f(1) = 6k+3$, $e_f(0) = 6k+2$. Therefore f is the group difference cordial labeling of G .

The labeling of the graph $O(L_7)$ is given in the figure:

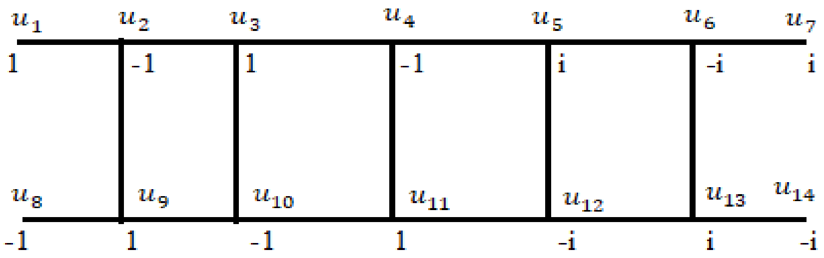

Fig .2.3

Theorem 2.5.

The graph $O(L_n) \odot K_1$ *is a group difference cordial graph for all* 'n'..

Proof: *Let* $G = O(L_n) \odot K_1$ has $4n$ vertices and $5n-4$ *edges* . f be a group difference cordial labeling of G . *Let* $V(G) = \{u_1, u_2, \ldots u_{4n}\}$. *Clearly* $O(L_n) \odot K_1$ *is a group difference cordial graph for* $n \le 3$. Assume $n \ge 4$ and define $f: V(G) \rightarrow \{1, -1, i, -i\}$

Case (i) If n is even, fix the labeling of $f(u_1) = f(u_3) = i$ and $f(u_2) = f(u_4) = -i$ for all graphs.

Sub case(i) $n \equiv 0\ (mod\ 4)$, Let $n = 4k,\ k \ge 1$.

$$f(u_{2i-1}) = \begin{cases} 1\ if\ \ 3 \le i \le 4k+2 \\ i\ if\ \ 4k+3 \le i \le 8k \end{cases}$$

$$f(u_{2i}) = \begin{cases} -1\ if\ 3 \le i \le 4k+2 \\ -i\ if\ \ 4k+3 \le i \le 8k \end{cases}$$

Clearly $V_f(1) = V_f(-1) = V_f(i) = V_f(-i) = 4k$ and $e_f(1) = 10k-2 = e_f(0)$.

Sub case (ii) : $n \equiv 2 (mod\ 4)$, Let $n = 4k+2,\ k \ge 1$.

$$f(u_{2i-1}) = \begin{cases} 1\ if\ \ 3 \le i \le 4k+4 \\ i\ \ if\ \ 4\,k+5 \le i \le 8k+4 \end{cases}$$

$$f(u_{2i}) = \begin{cases} -1\ if\ \ 3 \le i \le 4k+4 \\ -i\ if 4\,k+5 \le i \le 8k+4 \end{cases}$$

Clearly $V_f(1) = V_f(-1) = V_f(i) = V_f(-i) = 4k+2$ and $e_f(1) = 10k+3 = e_f(0)$.

Therefore f is the group difference cordial labeling of G .

Case (ii) If n is odd , fix the labeling of $f(u_1) = i$ and $f(u_2) = -i$ for all graphs.

Sub case (i)$n \equiv 1\ (mod\ 4)$, Let $n = 4k+1,\ k \geq 1$.

$$f(u_{2i-1}) = \{ \begin{smallmatrix} 1\ if\ \ 2\leq i\leq 4k+2 \\ i\ if\ \ 4k+3\leq i\leq 8k+2 \end{smallmatrix}$$

$$f(u_{2i}) = \{ \begin{smallmatrix} -1\ if\ \ 2\leq i\leq 4k+2 \\ -i\ if 4k+3\leq i\leq 8k+2 \end{smallmatrix}$$

Clearly $V_f(1) = V_f(-1) = V_f(i) = V_f(-i) = 4k+1$and $e_f(1) = 10k, e_f(0) = 10k+1$.

Sub case (ii) : $n \equiv 3(mod\ 4)$, Let $n = 4k+3,\ k \geq 1$.

$$f(u_{2i-1}) = \{ \begin{smallmatrix} 1\ if\ \ 2\leq i\leq 4k+4 \\ i\ if\ \ 4\ k+5\leq i\leq 8k+6 \end{smallmatrix}$$

$$f(u_{2i}) = \{ \begin{smallmatrix} -1\ if\ \ 2\leq i\leq 4k+4 \\ -i\ if 4\ k+5\leq i\leq 8k+6 \end{smallmatrix}$$

Clearly $V_f(1) = V_f(-1) = V_f(I) = V_f(-i) = 4k+3$ and $e_f(1) = 10k+5$, $e_f(0) = 10k+6$. Therefore f is the group difference cordial labeling of G .

Theorem 2.6. The Slanting ladder is a group difference cordial graph for all $'n'$.

Proof: $Let\ G = SL_n$ has $2n$ vertices and $3n-3\ edges$, f be a group difference cordial labeling of G .$Let\ V(G) = \{u_1, u_2 \dots u_{2n}\}$. Clearly SL_n is a group difference cordial graph for $n \leq 3$. Assume $n \geq 4$ and define $f : V(G) \to \{1, -1, i, -i\}\ as\ follows$

Case (i) $n \equiv 0\ (mod\ 4)$. Let $n = 4k, k \geq 1$

$$f(u_{2i-1}) = \{ \begin{smallmatrix} 1\ if\ \ 1\leq i\leq k\ and\ 2k+2\leq i\leq 3k+1 \\ i\ if\ \ k+1\leq i\leq 2k+1\ for\ k\geq 1\ and\ 3k+2\leq i\leq 4k\ for\ k\geq 2 \end{smallmatrix}$$

$$f(u_{2i}) = \{ \begin{smallmatrix} -1\ if\ \ 1\leq i\leq k\ and\ 2k+1\leq i\leq 3k \\ -i\ if k+1\leq i\leq 2k\ and\ 3k+1\leq i\leq 4k \end{smallmatrix}$$

Clearly$V_f(1) = V_f(-1) = V_f(i) = V_f(-i) = 2k$.Also $e_f(1) = 6k-2, e_f(0) = 6k-1$.

Therefore f is the group difference cordial labeling of G .

Case(ii)$n \equiv 1(mod\ 4)$. Let $n = 4k+1, k \geq 1$.

$$f(u_{2i-1}) = \{ \begin{smallmatrix} 1\ if\ \ 1\leq i\leq k+1 and\ 2k+2\leq i\leq 3k+1 \\ i\ if\ \ k+2\leq i\leq 2k+1\ and\ 3k+2\leq i\leq 4k+1 \end{smallmatrix}$$

$$f(u_{2i}) = \{ \begin{smallmatrix} -1\ if\ \ 1\leq i\leq k\ and\ \ 2k+1\leq i\leq 3k+1 \\ -i\ if k+1\leq i\leq 2k\ and\ 3k+2\leq i\leq 4k+1 \end{smallmatrix}$$

Clearly $V_f(1) = V_f(-1) = 2k+1, V_f(i) = V_f(-i) = 2k$.Also $e_f(1) = 6k = e_f(0)$.

Therefore f is the group difference cordial labeling of G .

Case(iii): $n \equiv 2\ \ (mod\ 4)$, Let $n = 4k+2, k \geq 1$.

$$f(u_{2i-1}) = \{ \begin{smallmatrix} 1\ if\ \ 1\leq i\leq k+1 and\ 2k+3\leq i\leq 3k+2 \\ i\ if\ \ k+2\leq i\leq 2k+2\ and\ 3k+3\leq i\leq 4k+2 \end{smallmatrix}$$

$$f(u_{2i}) = \begin{cases} -1 \; if \;\; 1 \le i \le k \; and \;\; 2k+2 \le i \le 3k+2 \\ -i \; if k+1 \le i \le 2k+1 \; and \; 3k+3 \le i \le 4k+2 \end{cases}$$

Clearly $V_f\ (1) = V_f\ (-1) = V_f\ (i) = V_f\ (-i) = 2k+1$and $e_f\ (1) = 6k+1$, $e_f(0) = 6k+2$. Therefore f is the group difference cordial labeling of G .

Case (iv) $n \equiv 3\ (mod\ 4)$, Let $n = 4k+3, k \ge 1$.

$$f(u_{2i-1}) = \begin{cases} 1 \; if \;\; 1 \le i \le k+1 and \; 2k+3 \le i \le 3k+3 \\ i \;\; if \;\; k+2 \le i \le 2k+2 \; and \; 3k+4 \le i \le 4k+3 \end{cases}$$

$$f(u_{2i}) = \begin{cases} -1 \; if \;\; 1 \le i \le k+1 \; and \;\; 2k+2 \le i \le 3k+2 \\ -i \; if k+2 \le i \le 2k+1 \; and \; 3k+3 \le i \le 4k+3 \end{cases}$$

Clearly $V_f\ (1) = V_f\ (-1) = 2k+2, V_f\ (i) = V_f\ (-i) = 2k+1$.Also $e_f\ (1) = 6k+3$ $= e_f(0)$. Therefore f is the group difference cordial labeling of G .

The labeling of the graph SL_8 is given in the figure:

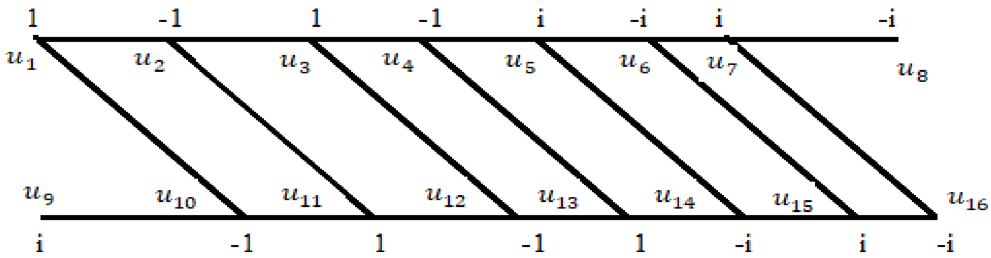

Fig .2.4

Result and Discussion

The investigation reported in this paper is an effort of group difference cordial labeling .As all the graphs are not group difference cordial graph, it is very interesting and challenging to investigate these labeling for the graph which admit these labeling .Here we have contributed some new results by investigating group difference cordial labeling for some graphs such as $L_n \odot K_1$,Open ladder,Slanting ladder, $O(L_n) \odot K_1$.

Conclusion

In this we have found group difference cordial labeling of different types of graphs such as $L_n \odot K_1$,Open ladder,Slanting ladder, $O(L_n) \odot K_1$ are established. Investigating group difference cordial labeling in other classes of graphs for future work.

Conflict of interest

The authors declare that they have no conflict of interest.

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