

MOTION AND CONFORMAL MOTION IN A FOUR DIMENSIONAL FINSLER SPACE

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Abstract:

The aim of the present paper is to discuss the motion and conformal motion in a four dimensional Finsler space. In this paper, we find the conditions on the main scalars and the connection vectors for an infinitesimal transformation to be a motion and a conformal motion.

Keywords: Motion, Conformal Motion, Four dimensional Finsler Space

1. Introduction

In a three dimensional Finsler space, there are three main scalars H, I, J and two connection vectors h and v . B. N. Prasad and A. K. Jaiswal discussed motion in two and three dimensional Finsler spaces [1]. In a four dimensional Finsler space, there are eight main scalars A, B, C, D, E, F, G, H and three h -connection vectors h, j, k and three v -connection vectors u, v, w [2].

2. Scalar Components in Orthonormal Miron Frame

Let F^4 be a four dimensional Finsler space. The Miron frame for this space is constructed by unit vectors (l^i, m^i, n^i, p^i) where $l^i = \dot{x}^i / L$ is normalized supporting element, $m^i = C^i / \tilde{c}$ is normalized torsion vector, n^i is constructed by

$g_{ij}l^i n^j = g_{ij}m^i n^j = 0$, $g_{ij}n^i n^j = 1$, and p^i satisfies $g_{ij}l^i p^j = g_{ij}m^i p^j = g_{ij}n^i p^j = 0$,

$g_{ij}p^i p^j = 1$ and c is length of C .

In Miron frame, an arbitrary tensor $T = (T_j^i)$ is expressed in terms of scalar components as

$$(2.1) \quad T_j^i = T_{\alpha\beta} e_{\alpha}^i e_{\beta}^j,$$

while the tensor T_{jk}^i is expressed as

$$(2.2) \quad T_{jk}^i = T_{\alpha\beta\gamma} e_{\alpha}^i e_{\beta}^j e_{\gamma}^k,$$

where $e_1^i = l^i$, $e_2^i = m^i$, $e_3^i = n^i$, $e_4^i = p^i$ and the summation convention is applied to the indices α, β, γ .

Since the scalar components of the fundamental metric tensor g_{ij} are $\delta_{\alpha\beta}$, we have

$$(2.3) \quad g_{ij} = \delta_{\alpha\beta} = e_{\alpha}^i e_{\beta}^j = l_i l_j + m_i m_j + n_i n_j + p_i p_j.$$

Let $C_{\lambda\mu\nu}$ be the scalar components of LC_{ijk} with respect to Miron frame. Then

$$(2.4) \quad LC_{ijk} = C_{\lambda\mu\nu} e_{\lambda}^i e_{\mu}^j e_{\nu}^k.$$

M. Matsumoto [3] showed that

- 1) $C_{\lambda\mu\nu}$ are completely symmetric.
- 2) $C_{1\mu\nu} = 0$
- 3) $C_{2\mu\mu} = L\tilde{C}$, $C_{3\mu\mu} = C_{4\mu\mu} = \dots = C_{n\mu\mu} = 0$.

Therefore in a four dimensional Finsler space, we have

$$\begin{aligned}
 C_{222} + C_{233} + C_{244} &= L\tilde{c}, \\
 (2.5a) \quad C_{322} + C_{333} + C_{344} &= 0, \\
 C_{422} + C_{433} + C_{444} &= 0.
 \end{aligned}$$

Putting

$$\begin{aligned}
 (2.5b) \quad C_{222} &= A, & C_{233} &= B, & C_{244} &= C, & C_{322} &= D, \\
 C_{333} &= E, & C_{422} &= F, & C_{433} &= G, & C_{234} &= H.
 \end{aligned}$$

We get $C_{344} = -(D+E)$, $C_{444} = -(F+G)$.

Thus A, B, C, D, E, F, G, H are main scalars of the four dimensional Finsler space.

Equation (2.4) may be written as

$$\begin{aligned}
 LC_{ijk} &= A(m_i m_j m_k) + B(m_i n_j n_k + n_i m_j n_k + n_i n_j m_k) \\
 &\quad + C(m_i p_j p_k + p_i m_j p_k + p_i p_j m_k) \\
 &\quad + D(m_i m_j n_k + m_i n_j m_k + n_i m_j m_k) + E(n_i n_j n_k) \\
 (2.6) \quad &\quad + F(m_i m_j p_k + m_i p_j m_k + p_i m_j m_k) + G(n_i n_j p_k \\
 &\quad + n_i p_j n_k + p_i n_j n_k) + H(m_i n_j p_k + m_i p_j n_k \\
 &\quad + n_i m_j p_k + n_i p_j m_k + p_i m_j n_k + p_i n_j m_k) \\
 &\quad - (D+E)(n_i p_j p_k + p_i n_j p_k + p_i p_j n_k) - (F+G)(p_i p_j p_k).
 \end{aligned}$$

Let $H_{\alpha\beta\gamma}$ and $V_{L\alpha\beta\gamma}$ be scalar components of the h- and v-covariant

derivatives $e_{\alpha\beta\gamma}^i$ and $e_{\alpha\beta\gamma}^i|_j$ of $e_{\alpha\beta\gamma}^i$ respectively. Then, we have

$$(2.7a) \quad e_{\alpha\beta\gamma}^i|_j = H_{\alpha\beta\gamma} e_{\beta\gamma}^i e_{\alpha}^j$$

$$(2.7b) \quad Le_{\alpha\beta\gamma}^i|_j = V_{\alpha\beta\gamma} e_{\beta\gamma}^i e_{\alpha}^j$$

We now define vector fields

$$(2.8) \quad h_i = H_{23\gamma} e_{\gamma}^i, \quad j_i = H_{42\gamma} e_{\gamma}^i, \quad k_i = H_{34\gamma} e_{\gamma}^i,$$

$$(2.9) \quad u_i = V_{2)3\gamma} e_\gamma)_i, \quad v_i = V_{4)2\gamma} e_\gamma)_i, \quad w_i = V_{3)4\gamma} e_\gamma)_i$$

The vector fields h_i, j_i, k_i are called h-connection vectors and u_i, v_i, w_i are called v-connection vectors. The scalars $H_{2)3\gamma}, H_{4)2\gamma}, H_{3)4\gamma}$ are scalar components of h-connection vectors h_i, j_i, k_i and $V_{2)3\gamma}, V_{4)2\gamma}, V_{3)4\gamma}$ are scalar components of v-connection vectors u_i, v_i, w_i .

From (2.7) we get

$$(2.10) \quad \begin{aligned} (a) \quad & e^i_{1)j} = l^i_{|j} = 0, \\ (b) \quad & e^i_{2)j} = m^i_{|j} = n^i h_j - p^i j_j, \\ (c) \quad & e^i_{3)j} = n^i_{|j} = p^i k_j - m^i h_j, \\ (d) \quad & e^i_{4)j} = p^i_{|j} = m^i j_j - n^i k_j. \end{aligned}$$

and

$$(2.11) \quad \begin{aligned} (a) \quad & Le^i_{1)j} = Ll^i_{|j} = m^i m_j + n^i n_j + p^i p_j, \\ (b) \quad & Le^i_{2)j} = Lm^i_{|j} = -l^i m_j + n^i u_j - p^i v_j, \\ (c) \quad & Le^i_{3)j} = Ln^i_{|j} = -l^i n_j - m^i u_j + p^i w_j, \\ (d) \quad & Le^i_{4)j} = Lp^i_{|j} = -l^i p_j + m^i v_j - n^i w_j. \end{aligned}$$

3. Lie Derivatives of g_{ij} and LC^i_{jk} in Terms of Scalar Components

Let us consider an infinitesimal transformation

$$(3.1) \quad \bar{x}^i = x^i + X^i dt$$

generated by a tangent vector field $X^i(x)$ of M^n .

The corresponding

variation in y^i is $\bar{y}^i = y^i + (\partial_j X^i) y^j dt$

given by

(3.2)

This transformation gives a process of differentiation called Lie Differentiation.

The Lie derivative \mathfrak{L}_x of mixed tensor T^i_j with respect to above infinitesimal transformation is given by [4]

$$(3.3) \quad \mathfrak{L}_x T^i_j = T^i_{j|r} X^r + T^i_{j|r} | w^r - T^r_{j|r} A^i_r + T^i_r A^r_j,$$

where A^i_j and w^i are (1,1) and (1,0) type tensor fields defined by

$$(3.4) \quad A^i_j = X^i_{|j} + C^i_{jr} w^r.$$

$$(3.5) \quad w^i = X^i_{|0}.$$

Let X_α , $\alpha = 1, 2, 3, 4$ be scalar components of the tangent vector field $X^i(x)$ of the transformation (3.1) then

$$(3.6) \quad \begin{aligned} X^i &= X_\alpha e_\alpha^i = X_1 e_1^i + X_2 e_2^i + X_3 e_3^i + X_4 e_4^i \\ &= X_1 l^i + X_2 m^i + X_3 n^i + X_4 p^i. \end{aligned}$$

If $X_{\alpha,\beta}$ be scalar components of $X^i_{|j}$ then

$$(3.7) \quad X^i_{|j} = X_{\alpha,\beta} e_\alpha^i e_\beta^j,$$

and

$$(3.8) \quad X^i_{|0} = L X_\alpha e_\alpha^i$$

In view of (3.5), (3.8) may be written as

$$(3.9) \quad w^i = L X_{\alpha,1} e_\alpha^i.$$

From (3.4), (3.7) and (3.9), we have

$$(3.10) \quad A_j^i = (X_{\alpha,\beta} + C_{\alpha\beta\lambda} X_{\lambda,1}) e_\alpha^i e_\beta)_j.$$

If $A_{\alpha\beta}$ are the scalar components of A_j^i , then

$$(3.11) \quad A_{\alpha\beta} = X_{\alpha,\beta} + C_{\alpha\beta\lambda} X_{\lambda,1}$$

Now

$$A_{11} = X_{1,1} + C_{11\lambda} X_{\lambda,1} = X_{1,1}$$

$$A_{12} = X_{1,2} + C_{12\lambda} X_{\lambda,1} = X_{1,2}$$

$$A_{13} = X_{1,3} + C_{13\lambda} X_{\lambda,1} = X_{1,3}$$

$$A_{14} = X_{1,4} + C_{14\lambda} X_{\lambda,1} = X_{1,4}$$

$$A_{21} = X_{2,1} + C_{21\lambda} X_{\lambda,1} = X_{2,1}$$

for $C_{1\lambda\mu} = 0$.

Again, in view of (2.5b) and $C_{1\lambda\mu} = 0$, we have

$$A_{22} = X_{2,2} + C_{22\lambda} X_{\lambda,1} = X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1}$$

$$A_{23} = X_{2,3} + C_{23\lambda} X_{\lambda,1} = X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1}$$

$$A_{24} = X_{2,4} + C_{24\lambda} X_{\lambda,1} = X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1}$$

$$A_{31} = X_{3,1} + C_{31\lambda} X_{\lambda,1} = X_{3,1}$$

$$A_{32} = X_{3,2} + C_{32\lambda} X_{\lambda,1} = X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1}$$

$$A_{33} = X_{3,3} + C_{33\lambda} X_{\lambda,1} = X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1}$$

$$A_{34} = X_{3,4} + C_{34\lambda} X_{\lambda,1} = X_{3,4} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}$$

$$A_{41} = X_{4,1} + C_{41\lambda} X_{\lambda,1} = X_{4,1}$$

$$A_{42} = X_{4,2} + C_{42\lambda} X_{\lambda,1} = X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1}$$

$$A_{43} = X_{4,3} + C_{43\lambda} X_{\lambda,1} = X_{4,3} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}$$

$$A_{44} = X_{4,4} + C_{44\lambda} X_{\lambda,1} = X_{4,4} + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1}$$

In view of (2.10), (2.11) and (3.9), the Lie derivatives of unit vectors are given by

$$(3.12) \quad \begin{aligned} \mathfrak{L}_X l^i &= -X_{1,1} l^i, \\ \mathfrak{L}_X l_i &= X_{1,1} l_i + (X_{1,2} + X_{2,1})m_i + (X_{1,3} + X_{3,1})n_i + (X_{1,4} + X_{4,1})p_i, \end{aligned}$$

$$\begin{aligned}
 \mathfrak{L}_x m^i &= -(X_{1,2} + X_{2,1})l^i - (X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1})m^i \\
 &\quad + (X_\alpha h_\alpha + X_{\alpha,1}u_\alpha - X_{3,2} - DX_{2,1} - BX_{3,1} - HX_{4,1})n^i \\
 &\quad - (X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1})p^i, \\
 \mathfrak{L}_x m_i &= (X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1})m_i + (X_\alpha h_\alpha + X_{\alpha,1}u_\alpha \\
 &\quad + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1})n_i + (-X_\alpha j_\alpha \\
 &\quad - X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1})p_i, \\
 \mathfrak{L}_x n_i &= (-X_\alpha h_\alpha - X_{\alpha,1}u_\alpha + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1})m_i \\
 &\quad + (X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1})n_i + (X_\alpha k_\alpha + X_{\alpha,1}w_\alpha \\
 &\quad + X_{3,4} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1})p_i, \\
 \mathfrak{L}_x n^i &= -(X_{1,3} + X_{3,1})l^i - (X_{\alpha,1}h_\alpha + X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} \\
 &\quad + HX_{4,1})m^i - (X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1})n^i + (X_{\alpha,1}k_\alpha \\
 &\quad + X_{\alpha,1}w_\alpha - X_{4,3} - HX_{2,1} - GX_{3,1} + (D+E)X_{4,1})p^i, \\
 \mathfrak{L}_x p_i &= (X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1})m_i + (X_{4,4} \\
 &\quad + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1})p_i + (-X_\alpha k_\alpha - X_{\alpha,1}w_\alpha \\
 &\quad + X_{4,3} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1})n_i, \\
 \mathfrak{L}_x p^i &= -(X_{1,4} + X_{4,1})l^i - (X_{4,4} + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1})p^i \\
 &\quad + (X_\alpha j_\alpha + X_{\alpha,1}v_\alpha - X_{2,4} - FX_{2,1} - HX_{3,1} - CX_{4,1})m^i \\
 &\quad - (X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + X_{3,4} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1})n^i.
 \end{aligned}$$

The Lie derivative of the metric tensor g_{ij} is given by

$$\begin{aligned}
 \mathfrak{L}_x g_{ij} &= L_X(l_l l_j + m_i m_j + n_i n_j + p_i p_j) \\
 (3.13) \quad &= l_i L_X(l_j) + l_j L_X(l_i) + m_i L_X(m_j) + m_j L_X(m_i) + n_i L_X(n_j) \\
 &\quad + n_j L_X(n_i) + p_i L_X(p_j) + p_j L_X(p_i).
 \end{aligned}$$

$$\begin{aligned}
 &= l_i[X_{1,1}l_j + (X_{1,2} + X_{2,1})m_j + (X_{1,3} + X_{3,1})n_j + (X_{1,4} + X_{4,1})p_j] + l_j[X_{1,1}l_i \\
 &\quad + (X_{1,2} + X_{2,1})m_i + (X_{1,3} + X_{3,1})n_i + (X_{1,4} + X_{4,1})p_i] + m_i[(X_{2,2} + AX_{2,1} \\
 &\quad + DX_{3,1} + FX_{4,1})m_j + (X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1})n_j \\
 &\quad + (-X_\alpha j_\alpha - X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1})p_j] + m_j[(X_{2,2} + AX_{2,1} \\
 &\quad + DX_{3,1} + FX_{4,1})m_i + (X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1})n_i \\
 &\quad + (-X_\alpha j_\alpha - X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1})p_i] + n_i[(-X_\alpha h_\alpha \\
 &\quad - X_{\alpha,1}u_\alpha + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1})m_j + (X_{3,3} + BX_{2,1} + EX_{3,1} \\
 &\quad + GX_{4,1})n_j + (X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + X_{3,4} + HX_{2,1} + GX_{3,1} - (D + E)X_{4,1})p_j \\
 &\quad + n_j[(-X_\alpha h_\alpha - X_{\alpha,1}u_\alpha + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1})m_i + (X_{3,3} + BX_{2,1} \\
 &\quad + EX_{3,1} + GX_{4,1})n_i + (X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + X_{3,4} + HX_{2,1} + GX_{3,1} - (D + E)X_{4,1})p_i \\
 &\quad + p_i[(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1})m_j + (X_{4,4} + CX_{2,1} \\
 &\quad - (D + E)X_{3,1} - (F + G)X_{4,1})p_j + (-X_\alpha k_\alpha - X_{\alpha,1}w_\alpha + X_{4,3} + HX_{2,1} + GX_{3,1} \\
 &\quad - (D + E)X_{4,1})n_j] + p_j[(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1})m_i \\
 &\quad + (X_{4,4} + CX_{2,1} - (D + E)X_{3,1} - (F + G)X_{4,1})p_i + (-X_\alpha k_\alpha - X_{\alpha,1}w_\alpha \\
 &\quad + X_{4,3} + HX_{2,1} + GX_{3,1} - (D + E)X_{4,1})n_i] \\
 &= 2X_{1,1}l_il_j + 2(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1})m_im_j + 2(X_{3,3} + BX_{2,1} \\
 &\quad + EX_{3,1} + GX_{4,1})n_in_j + 2(X_{4,4} + CX_{2,1} - (D + E)X_{3,1} \\
 &\quad - (F + G)X_{4,1})p_ip_j + (X_{1,2} + X_{2,1})(l_im_j + l_jm_i) + (X_{1,3} + X_{3,1})(l_in_j \\
 &\quad + l_jn_i) + (X_{1,4} + X_{4,1})(l_ip_j + l_jp_i) + (m_in_j + m_jn_i)(X_{2,3} + X_{3,2})
 \end{aligned}$$

$$\begin{aligned}
 & + 2DX_{2,1} + 2BX_{3,1} + 2HX_{4,1}) + (m_i p_j + m_j p_i)(X_{4,2} + X_{2,4} + 2FX_{2,1} \\
 & + 2HX_{3,1} + 2CX_{4,1}) + (n_i p_j + n_j p_i)(X_{4,3} + X_{3,4} + 2HX_{2,1} \\
 & + 2GX_{3,1} - 2(D+E)X_{4,1}).
 \end{aligned}$$

From $C^i = g^{ih}C$ and (2.6), we have

$$\begin{aligned}
 LC_{jk}^i &= A(m^i m_{j\bar{k}}) + B(m^i n_{j\bar{k}} + n^i m_{j\bar{k}} + n^i n_{j\bar{k}}) \\
 &+ C(m^i p_{j\bar{k}} + p^i m_{j\bar{k}} + p^i p_{j\bar{k}}) + D(m^i m_{j\bar{k}} + m^i n_{j\bar{k}} + n^i m_{j\bar{k}}) \\
 (3.14) \quad &+ E(n^i n_{j\bar{k}}) + F(m^i m_{j\bar{k}} + m^i p_{j\bar{k}} + p^i m_{j\bar{k}}) + G(n^i n_{j\bar{k}} + n^i p_{j\bar{k}} \\
 &+ p^i n_{j\bar{k}}) + H(m^i n_{j\bar{k}} + m^i p_{j\bar{k}} + n^i m_{j\bar{k}} + n^i p_{j\bar{k}} + p^i m_{j\bar{k}} \\
 &+ p^i n_{j\bar{k}}) - (D+E)(n^i p_{j\bar{k}} + p^i n_{j\bar{k}} + p^i p_{j\bar{k}}) - (F+G)(p^i p_{j\bar{k}}).
 \end{aligned}$$

Lie derivatives of main scalars are given by

$$\begin{aligned}
 \mathfrak{L}_x A &= A|_r X^r + A|_r w^r = A|_r X^r + A|_r LX^r = A|_{,\alpha} X_\alpha + A|_{;\alpha} X_\alpha, \\
 \mathfrak{L}_x B &= B_{,\alpha} X_\alpha + B_{;\alpha} X_{\alpha,1}, \quad \mathfrak{L}_x C = C_{,\alpha} X_\alpha + C_{;\alpha} X_{\alpha,1}, \\
 (3.15) \quad \mathfrak{L}_x D &= D_{,\alpha} X_\alpha + D_{;\alpha} X_{\alpha,1}, \quad \mathfrak{L}_x E = E_{,\alpha} X_\alpha + E_{;\alpha} X_{\alpha,1}, \\
 \mathfrak{L}_x F &= F_{,\alpha} X_\alpha + F_{;\alpha} X_{\alpha,1}, \quad \mathfrak{L}_x G = G_{,\alpha} X_\alpha + G_{;\alpha} X_{\alpha,1}, \\
 \mathfrak{L}_x H &= H_{,\alpha} X_\alpha + H_{;\alpha} X_{\alpha,1},
 \end{aligned}$$

where $A_{,\alpha}$ and $A_{;\alpha}$ are the scalar components of $A|_r$ and $LA|_r$ respectively.

Now the Lie derivative of LC_{jk}^i is given by

$$\begin{aligned}
 \mathfrak{L}_x (LC_{jk}^i) &= \mathfrak{L}_x [A(m^i m_{j\bar{k}}) + B(m^i n_{j\bar{k}} + n^i m_{j\bar{k}} + n^i n_{j\bar{k}}) \\
 &+ C(m^i p_{j\bar{k}} + p^i m_{j\bar{k}} + p^i p_{j\bar{k}}) + D(m^i m_{j\bar{k}} + m^i n_{j\bar{k}} \\
 &+ m^i n_{j\bar{k}} + n^i m_{j\bar{k}}) + E(n^i n_{j\bar{k}}) + F(m^i m_{j\bar{k}} + m^i p_{j\bar{k}} \\
 (3.16) \quad &+ p^i m_{j\bar{k}}) + G(n^i n_{j\bar{k}} + n^i p_{j\bar{k}} + p^i n_{j\bar{k}}) + H(m^i n_{j\bar{k}} \\
 &+ m^i p_{j\bar{k}} + n^i m_{j\bar{k}} + n^i p_{j\bar{k}} + p^i m_{j\bar{k}} + p^i n_{j\bar{k}}) \\
 &- (D+E)(n^i p_{j\bar{k}} + p^i n_{j\bar{k}} + p^i p_{j\bar{k}}) - (F+G)(p^i p_{j\bar{k}})]
 \end{aligned}$$

$$\begin{aligned}
 &= (A_{,\alpha} X_\alpha + A_{;\alpha} X_{\alpha,1}) m^i m_j m_k - A(X_{1,2} + X_{2,1}) l^i m_j m_k + A(X_\alpha h_\alpha \\
 &+ X_{\alpha,1} \mu_\alpha - X_{3,2} - D X_{2,1} - B X_{3,1} - H X_{4,1}) n^i m_j m_k - A(X_\alpha j_\alpha + X_{\alpha,1} v_\alpha \\
 &+ X_{4,2} + F X_{2,1} + H X_{3,1} + C X_{4,1}) p^i m_j m_k + A(X_{2,2} + A X_{2,1} + D X_{3,1} \\
 &+ F X_{4,1}) m^i m_j m_k + A(X_{\alpha,1} h_\alpha + X_{\alpha,1} u_\alpha + X_{2,3} + D X_{2,1} + B X_{3,1} \\
 &+ H X_{4,1}) m^i n_j m_k + A(-X_{\alpha,1} j_\alpha - X_{\alpha,1} v_\alpha + X_{2,4} + F X_{2,1} + H X_{3,1} \\
 &+ C X_{4,1}) m^i p_j m_k + A(X_{\alpha,1} h_\alpha + X_{\alpha,1} u_\alpha + X_{2,3} + D X_{2,1} + B X_{3,1} \\
 &+ H X_{4,1}) m^i m_j n_k + A(-X_{\alpha,1} j_\alpha - X_{\alpha,1} v_\alpha + X_{2,4} + F X_{2,1} + H X_{3,1} \\
 &+ C X_{4,1}) m^i m_j p_k + (B_{,\alpha} X_\alpha + B_{;\alpha} X_{\alpha,1}) m^i n_j n_k - B(X_{1,2} + X_{2,1}) l^i n_j n_k \\
 &- B(X_{2,2} + A X_{2,1} + D X_{3,1} + F X_{4,1}) m^i n_j n_k + B(X_{\alpha,1} h_\alpha + X_{\alpha,1} u_\alpha \\
 &- X_{3,2} - D X_{2,1} - B X_{3,1} - H X_{4,1}) n^i n_j n_k - B(X_{\alpha,1} j_\alpha + X_{\alpha,1} v_\alpha + X_{4,2} \\
 &+ F X_{2,1} + H X_{3,1} + C X_{4,1}) p^i n_j n_k + B(-X_{\alpha,1} h_\alpha - X_{\alpha,1} u_\alpha + X_{3,2} + D X_{2,1} \\
 &+ B X_{3,1} + H X_{4,1}) m^i m_j n_k + 2B(X_{3,3} + B X_{2,1} + E X_{3,1} + G X_{4,1}) m^i n_j n_k \\
 &+ B(X_\alpha k_\alpha + X_{\alpha,1} \mu_\alpha + X_{3,4} + H X_{2,1} + G X_{3,1} - (D + E) X_{4,1}) m^i p_j n_k \\
 &+ B(-X_\alpha h_\alpha - X_{\alpha,1} \mu_\alpha + X_{3,2} + D X_{2,1} + B X_{3,1} + H X_{4,1}) m^i n_j m_k \\
 &+ B(X_\alpha k_\alpha + X_{\alpha,1} \mu_\alpha + X_{3,4} + H X_{2,1} + G X_{3,1} - (D + E) X_{4,1}) m^i n_j p_k \\
 &+ (B_{,\alpha} X_\alpha + B_{;\alpha} X_{\alpha,1}) n^i m_j n_k - B(X_{1,3} + X_{3,1}) l^i m_j n_k - B(X_\alpha h_\alpha \\
 &+ X_{\alpha,1} \mu_\alpha + X_{2,3} + D X_{2,1} + B X_{3,1} + H X_{4,1}) m^i m_j n_k + B(X_\alpha k_\alpha \\
 &+ X_{\alpha,1} \mu_\alpha - X_{4,3} - H X_{2,1} - G X_{3,1} + (D + E) X_{4,1}) p^i m_j n_k \\
 &+ B(X_{2,2} + A X_{2,1} + D X_{3,1} + F X_{4,1}) n^i m_j n_k + B(X_{\alpha,1} h_\alpha + X_{\alpha,1} u_\alpha \\
 &+ X_{2,3} + D X_{2,1} + B X_{3,1} + H X_{4,1}) n^i n_j n_k + B(-X_{\alpha,1} j_\alpha - X_{\alpha,1} v_\alpha \\
 &+ X_{2,4} + F X_{2,1} + H X_{3,1} + C X_{4,1}) n^i p_j n_k + B(-X_{\alpha,1} h_\alpha - X_{\alpha,1} u_\alpha \\
 &+ X_{3,2} + D X_{2,1} + B X_{3,1} + H X_{4,1}) n^i m_j m_k + B(X_{\alpha,1} k_\alpha + X_{\alpha,1} w_\alpha \\
 &+ X_{3,4} + H X_{2,1} + G X_{3,1} - (D + E) X_{4,1}) n^i m_j p_k + (B_{,\alpha} X_\alpha
 \end{aligned}$$

$$\begin{aligned}
 & +B_{;\alpha} X_{\alpha,1} n^i n_j m_k - B(X_{1,3} + X_{3,1}) l^i n_j m_k - B(X_{\alpha,\alpha} h_{\alpha} + X_{\alpha,\alpha} u_{\alpha}) \\
 & + X_{2,3} + D X_{2,1} + B X_{3,1} + H X_{4,1}) m^i n_j m_k + B(X_{\alpha,\alpha} k_{\alpha} + X_{\alpha,\alpha} w_{\alpha}) \\
 & - X_{4,3} - H X_{2,1} - G X_{3,1} + (D+E) X_{4,1}) p^i n_j m_k + B(-X_{\alpha,\alpha} h_{\alpha} \\
 & - X_{\alpha,\alpha} u_{\alpha} + X_{3,2} + D X_{2,1} + B X_{3,1} + H X_{4,1}) n^i m_j m_k + B(X_{\alpha,\alpha} k_{\alpha}) \\
 & + X_{\alpha,1} w_{\alpha} + X_{3,4} + H X_{2,1} + G X_{3,1} - (D+E) X_{4,1}) n^i p_j m_k \\
 & + B(X_{2,2} + A X_{2,1} + D X_{3,1} + F X_{4,1}) n^i n_j m_k + B(X_{\alpha,\alpha} h_{\alpha} + X_{\alpha,\alpha} u_{\alpha}) \\
 & + X_{2,3} + D X_{2,1} + B X_{3,1} + H X_{4,1}) n^i n_j n_k + B(-X_{\alpha,\alpha} j_{\alpha} - X_{\alpha,\alpha} v_{\alpha}) \\
 & + X_{2,4} + F X_{2,1} + H X_{3,1} + C X_{4,1}) n^i n_j p_k + (C_{;\alpha} X_{\alpha,\alpha} + C_{;\alpha} X_{\alpha,1}) m^i p_j p_k \\
 & - C(X_{1,2} + X_{2,1}) l^i p_j p_k - C(X_{2,2} + A X_{2,1} + D X_{3,1} + F X_{4,1}) m^i p_j p_k \\
 & + C(X_{\alpha} h_{\alpha} + X_{\alpha,1} u_{\alpha} - X_{3,2} - D X_{2,1} - B X_{3,1} - H X_{4,1}) n^i p_j p_k \\
 & - C(X_{\alpha} j_{\alpha} + X_{\alpha,1} v_{\alpha} + X_{4,2} + F X_{2,1} + H X_{3,1} + C X_{4,1}) p^i p_j p_k \\
 & + C(X_{\alpha} j_{\alpha} + X_{\alpha,1} v_{\alpha} + X_{4,2} + F X_{2,1} + H X_{3,1} + C X_{4,1}) m^i m_j p_k \\
 & + 2C(X_{4,4} + C X_{2,1} - (D+E) X_{3,1} - (F+G) X_{4,1}) m^i p_j p_k + C(-X_{\alpha} k_{\alpha}) \\
 & - X_{\alpha,1} w_{\alpha} + X_{4,3} + H X_{2,1} + G X_{3,1} - (D+E) X_{4,1}) m^i n_j p_k + C(X_{\alpha} j_{\alpha}) \\
 & + X_{\alpha,1} v_{\alpha} + X_{4,2} + F X_{2,1} + H X_{3,1} + C X_{4,1}) m^i p_j m_k + C(-X_{\alpha} k_{\alpha}) \\
 & - X_{\alpha,1} w_{\alpha} + X_{4,3} + H X_{2,1} + G X_{3,1} - (D+E) X_{4,1}) m^i p_j n_k \\
 & + (C_{;\alpha} X_{\alpha} + C_{;\alpha} X_{\alpha,1}) p^i m_j p_k - C(X_{1,4} + X_{4,1}) l^i m_j p_k + C(X_{\alpha} j_{\alpha}) \\
 & + X_{\alpha,1} v_{\alpha} - X_{2,4} - F X_{2,1} - H X_{3,1} - C X_{4,1}) m^i m_j p_k - C(X_{\alpha} k_{\alpha}) \\
 & + X_{\alpha,1} w_{\alpha} + X_{3,4} + H X_{2,1} + G X_{3,1} - (D+E) X_{4,1}) n^i m_j p_k + C(X_{2,2}) \\
 & + A X_{2,1} + D X_{3,1} + F X_{4,1}) p^i m_j p_k + C(X_{\alpha,\alpha} h_{\alpha} + X_{\alpha,\alpha} u_{\alpha} + X_{2,3}) \\
 & + D X_{2,1} + B X_{3,1} + H X_{4,1}) p^i n_j p_k + C(-X_{\alpha,\alpha} j_{\alpha} - X_{\alpha,\alpha} v_{\alpha} + X_{2,4}) \\
 & + F X_{2,1} + H X_{3,1} + C X_{4,1}) p^i p_j p_k + C(X_{\alpha,\alpha} j_{\alpha} + X_{\alpha,\alpha} v_{\alpha})
 \end{aligned}$$

$$\begin{aligned}
 & + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1}) p^i m_j m_k + C(-X_{\alpha, \alpha} - X_{\alpha, 1} w_{\alpha} \\
 & + X_{4,3} + HX_{2,1} + GX_{3,1} - (D + E) X_{4,1}) p^i m_j n_k + (C, \alpha X_{\alpha} \\
 & + C X_{\alpha, 1}) p^i p_j m_k - C(X_{1,4} + X_{4,1}) l^i p_j m_k + C(X_{\alpha, \alpha} j + X_{\alpha, 1} v \\
 & - X_{2,4} - FX_{2,1} - HX_{3,1} - CX_{4,1}) m^i p_j m_k - C(X_{\alpha, \alpha} k + X_{\alpha, 1} w_{\alpha} \\
 & + X_{3,4} + HX_{2,1} + GX_{3,1} - (D + E) X_{4,1}) n^i p_j m_k + C(X_{\alpha, \alpha} j \\
 & + X_{\alpha, \alpha} y_{\alpha} + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1}) p^i m_j m_k + C(-X_{\alpha, \alpha} k_{\alpha} \\
 & - X_{\alpha, 1} w_{\alpha} + X_{4,3} + HX_{2,1} + GX_{3,1} - (D + E) X_{4,1}) p^i n_j m_k + C(X_{2,2} \\
 & + AX_{2,1} + DX_{3,1} + FX_{4,1}) p^i p_j m_k + C(X_{\alpha, \alpha} h + X_{\alpha, 1} u + X_{2,3} \\
 & + DX_{2,1} + BX_{3,1} + HX_{4,1}) p^i p_j n_k + C(-X_{\alpha, \alpha} j - X_{\alpha, 1} v + X_{2,4} \\
 & + FX_{2,1} + HX_{3,1} + CX_{4,1}) p^i p_j p_k + (D X_{\alpha, \alpha} + D X_{\alpha, 1}) m^i m_j n_k \\
 & - D(X_{1,2} + X_{2,1}) l^i m_j n_k + D(X_{\alpha, \alpha} h + X_{\alpha, 1} u - X_{3,2} - DX_{2,1} \\
 & - BX_{3,1} - HX_{4,1}) n^i m_j n_k - D(X_{\alpha, \alpha} j + X_{\alpha, 1} v + X_{4,2} + FX_{2,1} \\
 & + HX_{3,1} + CX_{4,1}) p^i m_j n_k + D(X_{\alpha, \alpha} h + X_{\alpha, 1} u + X_{2,3} + DX_{2,1} \\
 & + BX_{3,1} + HX_{4,1}) m^i n_j n_k + D(-X_{\alpha, \alpha} j - X_{\alpha, 1} v + X_{2,4} + FX_{2,1} \\
 & + HX_{3,1} + CX_{4,1}) m^i p_j n_k + D(-X_{\alpha, \alpha} h - X_{\alpha, 1} u + X_{3,2} + DX_{2,1} \\
 & + BX_{3,1} + HX_{4,1}) m^i m_j m_k + D(X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1}) m^i m_j n_k \\
 & + D(X_{\alpha, \alpha} k_{\alpha} + X_{\alpha, \alpha} y_{\alpha} + X_{3,4} + HX_{2,1} + GX_{3,1} - (D + E) X_{4,1}) m^i m_j p_k \\
 & + (D, \alpha X_{\alpha} + D, \alpha X_{\alpha, 1}) m^i n_j m_k - D(X_{1,2} + X_{2,1}) l^i n_j m_k + D(X_{\alpha, \alpha} h_{\alpha} \\
 & + X_{\alpha, \alpha} \mu_{\alpha} - X_{3,2} - DX_{2,1} - BX_{3,1} - HX_{4,1}) n^i n_j m_k - D(X_{\alpha, \alpha} j_{\alpha} \\
 & + X_{\alpha, \alpha} y_{\alpha} + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1} p^i n_j m_k + D(-X_{\alpha, \alpha} h_{\alpha} \\
 & - X_{\alpha, \alpha} \mu_{\alpha} + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1}) m^i m_j m_k + D(X_{3,3} \\
 & + BX_{2,1} + EX_{3,1} + GX_{4,1}) m^i n_j m_k + D(X_{\alpha, \alpha} k_{\alpha} + X_{\alpha, 1} w_{\alpha} + X_{3,4} \\
 & + HX_{2,1} + GX_{3,1} - (D + E) X_{4,1}) m^i p_j m_k + D(X_{\alpha, \alpha} h_{\alpha} + X_{\alpha, 1} u \\
 & + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1}) m^i n_j n_k + D(-X_{\alpha, \alpha} j_{\alpha} - X_{\alpha, 1} v)
 \end{aligned}$$

$$\begin{aligned}
 & + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1})m^i n_j p_k + (D_{,\alpha} X_{\alpha} + D_{;\alpha} X_{\alpha,1})n^i m_j m_k \\
 & - D(X_{1,3} + X_{3,1})l^i m_j m_k - D(X_{\alpha} h_{\alpha} + X_{\alpha,1} u_{\alpha} + X_{2,3} v_{\alpha}) + DX_{2,1} \\
 & + BX_{3,1} + HX_{4,1})m^i m_j m_k - D(X_{3,3} BX_{2,1} + EX_{3,1} + GX_{4,1})n^i m_j m_k \\
 & + D(X_{\alpha} k_{\alpha} + X_{\alpha} w_{\alpha} - X_{4,3} - HX_{2,1} - GX_{3,1} + (D+E)X_{4,1})p^i m_j m_k \\
 & + 2D(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1})n^i m_j m_k + D(X_{\alpha} h_{\alpha} + X_{\alpha,1} u_{\alpha} \\
 & + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1})n^i n_j m_k + D(-X_{\alpha} j_{\alpha} - X_{\alpha,1} v_{\alpha}) \\
 & + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1})n^i p_j m_k + D(X_{\alpha} h_{\alpha} + X_{\alpha,1} u_{\alpha} \\
 & + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1})n^i m_j n_k + D(-X_{\alpha} j_{\alpha} - X_{\alpha,1} v_{\alpha}) \\
 & + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1})n^i m_j p_k + (E_{,\alpha} X_{\alpha} + E_{;\alpha} X_{\alpha,1})n^i n_j n_k \\
 & - E(X_{1,3} + X_{3,1})l^i n_j n_k - E(X_{\alpha} h_{\alpha} + X_{\alpha,1} u_{\alpha} + X_{2,3} v_{\alpha}) + DX_{2,1} \\
 & + BX_{3,1} + HX_{4,1})m^i n_j n_k + E(X_{\alpha} k_{\alpha} + X_{\alpha,1} w_{\alpha} - X_{4,3} - HX_{2,1} \\
 & - GX_{3,1} + (D+E)X_{4,1})p^i n_j n_k + E(-X_{\alpha} h_{\alpha} - X_{\alpha,1} u_{\alpha} + X_{3,2} v_{\alpha}) \\
 & + DX_{2,1} + BX_{3,1} + HX_{4,1})n^i m_j n_k + E(X_{\alpha} k_{\alpha} + X_{\alpha,1} w_{\alpha} + X_{3,4} v_{\alpha}) \\
 & + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1})n^i p_j n_k + E(-X_{\alpha} h_{\alpha} - X_{\alpha,1} u_{\alpha} \\
 & + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1})n^i n_j m_k + E(X_{3,3} + BX_{2,1} \\
 & + EX_{3,1} + GX_{4,1})n^i n_j n_k + E(X_{\alpha} k_{\alpha} + X_{\alpha,1} w_{\alpha} + X_{3,4} v_{\alpha}) + HX_{2,1} \\
 & + GX_{3,1} - (D+E)X_{4,1})n^i n_j p_k + (F_{,\alpha} X_{\alpha} + F_{;\alpha} X_{\alpha,1})m^i m_j p_k \\
 & - F(X_{1,2} + X_{2,1})l^i m_j p_k + F(X_{\alpha} h_{\alpha} + X_{\alpha,1} u_{\alpha} - X_{3,2} v_{\alpha}) - DX_{2,1} \\
 & - BX_{3,1} - HX_{4,1})n^i m_j p_k - F(X_{\alpha} j_{\alpha} - X_{\alpha,1} v_{\alpha} + X_{4,2} w_{\alpha}) + FX_{2,1} \\
 & + HX_{3,1} + CX_{4,1})p^i m_j p_k + F(X_{\alpha} h_{\alpha} + X_{\alpha,1} u_{\alpha} + X_{2,3} v_{\alpha}) + DX_{2,1} \\
 & + BX_{3,1} + HX_{4,1})m^i n_j p_k + F(-X_{\alpha} j_{\alpha} - X_{\alpha,1} v_{\alpha} + X_{2,4} w_{\alpha}) + FX_{2,1} \\
 & + HX_{3,1} + CX_{4,1})m^i p_j p_k + F(X_{\alpha} j_{\alpha} + X_{\alpha,1} v_{\alpha} + X_{4,2} w_{\alpha}) + FX_{2,1}
 \end{aligned}$$

$$\begin{aligned}
& + HX_{3,1} + CX_{4,1})m^i m_j m_k + F(X_{4,4} + CX_{2,1} - (D + E)X_{3,1} \\
& - (F + G)X_{4,1})m^i m_j p_k + F(-X_{\alpha\alpha} - X_{\alpha,1}w + X_{4,3} + HX_{2,1} \\
& + GX_{3,1} - (D + E)X_{4,1})m^i m_j n_k + (F_{,\alpha} X_{\alpha} + F_{;\alpha} X_{\alpha,1})m^i p_j m_k \\
& - F(X_{1,2} + X_{2,1})l^i p_j m_k + F(X_{\alpha\alpha} h + X_{\alpha,1}u - X_{3,2} - DX_{2,1} \\
& - BX_{3,1} - HX_{4,1})n^i p_j m_k - F(X_{\alpha} j_{\alpha} + X_{\alpha,1}v_{\alpha} + X_{4,2} + FX_{2,1} \\
& + HX_{3,1} + CX_{4,1})p^i p_j m_k + F(X_{\alpha\alpha} j + X_{\alpha,1}v + X_{4,2} + FX_{2,1} \\
& + HX_{3,1} + CX_{4,1})m^i m_j m_k + F(X_{4,4} + CX_{2,1} - (D + E)X_{3,1} \\
& - (F + G)X_{4,1})m^i p_j m_k + F(-X_{\alpha\alpha} - X_{\alpha,1}w + X_{4,3} + HX_{2,1} \\
& + GX_{3,1} - (D + E)X_{4,1})m^i n_j m_k + F(X_{\alpha\alpha} h + X_{\alpha,1}u + X_{2,3} \\
& + DX_{2,1} + BX_{3,1} + HX_{4,1})m^i p_j n_k + F(-X_{\alpha\alpha} j - X_{\alpha,1}v + X_{2,4} \\
& + FX_{2,1} + HX_{3,1} + CX_{4,1})m^i p_j p_k + (F_{,\alpha} X_{\alpha} + F_{;\alpha} X_{\alpha,1})p^i m_j m_k \\
& - F(X_{1,4} + X_{4,1})l^i m_j m_k - F(X_{4,4} + CX_{2,1} - (D + E)X_{3,1} \\
& - (F + G)X_{4,1})p^i m_j m_k + F(X_{\alpha\alpha} j + X_{\alpha,1}v - X_{2,4} - FX_{2,1} \\
& - HX_{3,1} - CX_{4,1})m^i m_j m_k - F(X_{\alpha\alpha} k + X_{\alpha,1}w + X_{3,4} + HX_{2,1} \\
& + GX_{3,1} - (D + E)X_{4,1})n^i m_j m_k + 2F(X_{2,2} + AX_{2,1} + DX_{3,1} \\
& + FX_{4,1})p^i m_j m_k + F(X_{\alpha\alpha} h + X_{\alpha,1}u + X_{2,3} + DX_{2,1} + BX_{3,1} \\
& + HX_{4,1})p^i n_j m_k + F(-X_{\alpha\alpha} j - X_{\alpha,1}v + X_{2,4} + FX_{2,1} + HX_{3,1} \\
& + CX_{4,1})p^i p_j m_k + F(X_{\alpha\alpha} h + X_{\alpha,1}u + X_{2,3} + DX_{2,1} + BX_{3,1} \\
& + HX_{4,1})p^i m_j n_k + F(-X_{\alpha\alpha} j - X_{\alpha,1}v + X_{2,4} + FX_{2,1} + HX_{3,1} \\
& + CX_{4,1})p^i m_j p_k + (G_{,\alpha} X_{\alpha} + G_{;\alpha} X_{\alpha,1})n^i n_j p_k - G(X_{1,3} \\
& + X_{3,1})l^i n_j p_k - G(X_{\alpha\alpha} h + X_{\alpha,1}u + X_{2,3} + DX_{2,1} + BX_{3,1}
\end{aligned}$$

$$\begin{aligned}
 & + HX_{4,1} m^i n_j p_k + G(X_{\alpha, \alpha} k + X_{\alpha, 1} w - X_{4,3}) - HX_{2,1} - GX_{3,1} \\
 & + (D + E) X_{4,1} p^i n_j p_k + G(-X_{\alpha, \alpha} h - X_{\alpha, 1} u + X_{3,2}) + DX_{2,1} \\
 & + BX_{3,1} + HX_{4,1} n^i m_j p_k + G(X_{\alpha, \alpha} k + X_{\alpha, 1} w + X_{3,4}) + HX_{2,1} \\
 & + GX_{3,1} - (D + E) X_{4,1} n^i p_j p_k + G(X_{\alpha, \alpha} j + X_{\alpha, 1} v + X_{4,2}) \\
 & + FX_{2,1} + HX_{3,1} + CX_{4,1} n^i n_j m_k + G(X_{4,4} + CX_{2,1} - (D + E) X_{3,1}) \\
 & - (F + G) X_{4,1} n^i n_j p_k + G(-X_{\alpha, \alpha} k - X_{\alpha, 1} w + X_{4,3}) + HX_{2,1} \\
 & + GX_{3,1} - (D + E) X_{4,1} n^i n_j n_k + (G_{,\alpha} X_{\alpha} + G_{;\alpha} X_{\alpha,1}) n^i p_j n_k \\
 & - G(X_{1,3} + X_{3,1}) l^i p_j n_k - G(X_{\alpha, \alpha} h + X_{\alpha, 1} u + X_{2,3}) + DX_{2,1} \\
 & + BX_{3,1} + HX_{4,1} m^i p_j n_k + G(X_{\alpha, \alpha} k + X_{\alpha, 1} w - X_{4,3}) - HX_{2,1} \\
 & - GX_{3,1} - (D + E) X_{4,1} p^i p_j n_k + G(X_{\alpha, \alpha} j + X_{\alpha, 1} v + X_{4,2}) \\
 & + FX_{2,1} + HX_{3,1} + CX_{4,1} n^i m_j n_k + G(X_{4,4} + CX_{2,1} - (D + E) X_{3,1}) \\
 & - (F + G) X_{4,1} n^i p_j n_k + G(-X_{\alpha, \alpha} k - X_{\alpha, 1} w + X_{4,3}) + HX_{2,1} \\
 & + GX_{3,1} - (D + E) X_{4,1} n^i n_j n_k + G(-X_{\alpha, \alpha} h - X_{\alpha, 1} u + X_{3,2}) \\
 & + DX_{2,1} + BX_{3,1} + HX_{4,1} n^i p_j m_k + G(X_{\alpha, \alpha} k + X_{\alpha, 1} w + X_{3,4}) \\
 & + HX_{2,1} + GX_{3,1} - (D + E) X_{4,1} n^i p_j p_k + (G_{,\alpha} X_{\alpha} + G_{;\alpha} X_{\alpha,1}) p^i n_j n_k \\
 & - G(X_{1,4} + X_{4,1}) l^i n_j n_k - G(X_{4,4} + CX_{2,1} - (D + E) X_{3,1}) \\
 & - (F + G) X_{4,1} p^i n_j n_k + G(X_{\alpha, \alpha} j + X_{\alpha, 1} v - X_{2,4}) - FX_{2,1} \\
 & - HX_{3,1} - CX_{4,1} m^i n_j n_k - G(X_{\alpha, \alpha} k + X_{\alpha, 1} w + X_{3,4}) + HX_{2,1} \\
 & + GX_{3,1} - (D + E) X_{4,1} n^i n_j n_k + G(-X_{\alpha, \alpha} h - X_{\alpha, 1} u + X_{3,2}) \\
 & + DX_{2,1} + BX_{3,1} + HX_{4,1} p^i m_j n_k + 2G(X_{3,3} + BX_{2,1} + EX_{3,1}) \\
 & + GX_{4,1} p^i n_j n_k + G(X_{\alpha, \alpha} k + X_{\alpha, 1} w + X_{3,4}) + HX_{2,1} + GX_{3,1} \\
 & - (D + E) X_{4,1} p^i p_j n_k + G(-X_{\alpha, \alpha} h - X_{\alpha, 1} u + X_{3,2}) + DX_{2,1}
 \end{aligned}$$

$$\begin{aligned}
 & +BX_{3,1} + HX_{4,1})p^i n_j m_k + G(X_{\alpha\alpha} k + X_{\alpha,1} w + X_{3,4} + HX_{2,1} \\
 & +GX_{3,1} - (D+E)X_{4,1})p^i n_j p_k (H_{,\alpha} X_\alpha + H_{;\alpha} X_{\alpha,1})m^i n_j p_k - H(X_{1,2} \\
 & +X_{2,1})l^i n_j p_k - H(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1})m^i n_j p_k + H(X_{\alpha\alpha} h \\
 & +X_{\alpha,\mu} - X_{3,2} - DX_{2,1} - BX_{3,1} - HX_{4,1})n^i n_j p_k - H(X_{\alpha\alpha} j_\alpha \\
 & +X_{\alpha,\nu} + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1})p^i n_j p_k + H(-X_d h_\alpha \\
 & -X_{\alpha,\mu} + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1})m^i m_j p_k + H(X_{3,3} \\
 & +BX_{2,1} + EX_{3,1} + GX_{4,1})m^i n_j p_k + H[X_{\alpha\alpha} k + X_{\alpha,1} w + X_{3,4} \\
 & +HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}]m^i p_j p_k + H(X_{\alpha\alpha} j + X_{\alpha,1} v \\
 & +X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1})m^i n_j m_k + H(X_{4,4} + CX_{2,1} \\
 & -(D+E)X_{3,1} - (F+G)X_{4,1})m^i n_j p_k + H(-X_{\alpha\alpha} k - X_{\alpha,1} w \\
 & +X_{4,3} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1})m^i n_j n_k + (H_{,\alpha} X_\alpha \\
 & +H_{;\alpha} X_{\alpha,1})m^i p_j n_k - H(X_{1,2} + X_{2,1})l^i p_j n_k - H(X_{2,2} + AX_{2,1} \\
 & +DX_{3,1} + FX_{4,1})m^i p_j n_k + H(X_{\alpha\alpha} h + X_{\alpha,1} u - X_{3,2} - DX_{2,1} \\
 & -BX_{3,1} - HX_{4,1})n^i p_j n_k - H(X_{\alpha\alpha} j + X_{\alpha,1} v + X_{4,2} + FX_{2,1} \\
 & +HX_{3,1} + CX_{4,1})p^i p_j n_k + H(X_{\alpha\alpha} j + X_{\alpha,1} v + X_{4,2} + FX_{2,1} \\
 & +HX_{3,1} + CX_{4,1})m^i m_j n_k + H(X_{4,4} + CX_{2,1} - (D+E)X_{3,1} \\
 & -(F+G)X_{4,1})m^i p_j n_k + H[-X_{\alpha\alpha} k - X_{\alpha,1} w + X_{4,3} + HX_{2,1} \\
 & +GX_{3,1} - (D+E)X_{4,1}]m^i n_j n_k + H(-X_{\alpha\alpha} h - X_{\alpha,1} u + X_{3,2} \\
 & +DX_{2,1} + BX_{3,1} + HX_{4,1})m^i p_j m_k + H(X_{3,3} + BX_{2,1} + EX_{3,1} \\
 & +GX_{4,1})m^i p_j n_k + H(X_{\alpha\alpha} k + X_{\alpha,1} w + X_{3,4} + HX_{2,1} + GX_{3,1} \\
 & -(D+E)X_{4,1})m^i p_j p_k + (H_{,\alpha} X_\alpha + H_{;\alpha} X_{\alpha,1})n^i m_j p_k - H(X_{1,3} \\
 & +X_{3,1})l^i m_j p_k - H(X_{\alpha\alpha} h + X_{\alpha,1} u + X_{2,3} + DX_{2,1} + BX_{3,1} \\
 & +HX_{4,1})m^i m_j p_k - H(X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1})n^i m_j p_k \\
 & +H(X_\alpha k_\alpha + X_{\alpha,1} w_\alpha - X_{4,3} - HX_{2,1} - GX_{3,1} + (D+E)X_{4,1})p^i m_j p_k
 \end{aligned}$$

$$\begin{aligned}
& +H(X_{2,2}+AX_{2,1}+DX_{3,1}+FX_{4,1})n^i m_j p_k + H(X_{\alpha,\alpha}+X_{\alpha,1} u_{\alpha}) \\
& +X_{2,3}+DX_{2,1}+BX_{3,1}+HX_{4,1})n^i n_j p_k + H(-X_{\alpha,\alpha} j_{\alpha}-X_{\alpha,1} v_{\alpha}) \\
& +X_{2,4}+FX_{2,1}+HX_{3,1}+CX_{4,1})n^i p_j p_k + H(X_{\alpha,\alpha} j_{\alpha}+X_{\alpha,1} v_{\alpha}) \\
& +X_{4,2}+FX_{2,1}+HX_{3,1}+CX_{4,1})n^i m_j m_k + H(X_{4,4}+CX_{2,1}) \\
& -(D+E)X_{3,1}-(F+G)X_{4,1})n^i m_j p_k + H(-X_{\alpha,\alpha} k_{\alpha}-X_{\alpha,1} w_{\alpha}) \\
& +X_{4,3}+HX_{2,1}+GX_{3,1}-(D+E)X_{4,1})n^i m_j p_k + (H_{,\alpha} X_{\alpha}) \\
& +H_{;\alpha} X_{\alpha,1})n^i p_j m_k - H(X_{1,3}+X_{3,1})l^i p_j m_k - H(X_{\alpha,\alpha} h_{\alpha}+X_{\alpha,1} u_{\alpha}) \\
& +X_{2,3}+DX_{2,1}+BX_{3,1}+HX_{4,1})m^i p_j m_k - H(X_{3,3}+BX_{2,1}) \\
& +EX_{3,1}+GX_{4,1})n^i p_j m_k + H(X_{\alpha,\alpha} k_{\alpha}+X_{\alpha,1} w_{\alpha}-X_{4,3}+HX_{2,1}) \\
& -GX_{3,1}+(D+E)X_{4,1})p^i p_j m_k + H(X_{\alpha,\alpha} j_{\alpha}+X_{\alpha,1} v_{\alpha}+X_{4,2}) \\
& +FX_{2,1}+HX_{3,1}+CX_{4,1})n^i m_j m_k + H(X_{4,4}+CX_{2,1})-(D+E)X_{3,1}) \\
& -(F+G)X_{4,1})n^i p_j m_k + H(-X_{\alpha,\alpha} k_{\alpha}-X_{\alpha,1} w_{\alpha}+X_{4,3}+HX_{2,1}) \\
& +GX_{3,1}-(D+E)X_{4,1})n^i n_j m_k + H(X_{2,2}+AX_{2,1}+DX_{3,1}) \\
& +FX_{4,1})n^i p_j m_k + H(X_{\alpha,\alpha} h_{\alpha}+X_{\alpha,1} u_{\alpha}+X_{2,3}+DX_{2,1}+BX_{3,1}) \\
& +HX_{4,1})n_i p_j n_k + H(-X_{\alpha,\alpha} j_{\alpha}-X_{\alpha,1} v_{\alpha}+X_{2,4}+FX_{2,1}+HX_{3,1}) \\
& +CX_{4,1})n_i p_j p_k + (H_{,\alpha} X_{\alpha}+H_{;\alpha} X_{\alpha,1})p^i m_j n_k - H(X_{1,4}) \\
& +X_{4,1})l^i m_j n_k + H(X_{4,4}+CX_{2,1})-(D+E)X_{3,1}-(F+G)X_{4,1})p^i m_j n_k \\
& +H(X_{\alpha,\alpha} j_{\alpha}+X_{\alpha,1} v_{\alpha}-X_{2,4}-FX_{2,1}-HX_{3,1}-CX_{4,1})m^i m_j n_k \\
& -H(X_{\alpha,\alpha} k_{\alpha}+X_{\alpha,1} w_{\alpha}+X_{3,4}+HX_{2,1}+GX_{3,1})-(D+E)X_{4,1})n^i m_j n_k \\
& +H(X_{2,2}+AX_{2,1}+DX_{3,1}+FX_{4,1})p^i m_j n_k + H(X_{\alpha,\alpha} h_{\alpha}+X_{\alpha,1} u_{\alpha}) \\
& +X_{2,3}+DX_{2,1}+BX_{3,1}+HX_{4,1})p^i n_j n_k + H(-X_{\alpha,\alpha} j_{\alpha}-X_{\alpha,1} v_{\alpha}) \\
& +X_{2,4}+FX_{2,1}+HX_{3,1}+CX_{4,1})p^i p_j n_k + H(-X_{\alpha,\alpha} h_{\alpha}-X_{\alpha,1} u_{\alpha}+E X_{\alpha,\alpha})
\end{aligned}$$

$$\begin{aligned}
 & +E_{;\alpha} X_{\alpha,1}) p^i n_j p_k + (D+E)(X_{1,4} + X_{4,1}) l^i n_j p_k - (D+E)(X_\alpha j_\alpha \\
 & + X_{\alpha,1} v_\alpha - X_{2,4} - F X_{2,1} - H X_{3,1} - C X_{4,1}) m^i n_j p_k + (D+E)(X_\alpha k_\alpha \\
 & + X_{\alpha,1} w_\alpha + X_{3,4} + H X_{2,1} + G X_{3,1} - (D+E) X_{4,1}) n^i n_j p_k \\
 & - (D+E)(-X_\alpha h_\alpha - X_{\alpha,1} u_\alpha + X_{3,2} + D X_{2,1} + B X_{3,1} \\
 & + H X_{4,1}) p^i m_j p_k - (D+E)(X_{3,3} + B X_{2,1} + E X_{3,1} \\
 & + G X_{4,1}) p^i n_j p_k - (D+E)(X_\alpha k_\alpha + X_{\alpha,1} w_\alpha + X_{3,4} + H X_{2,1} \\
 & + G X_{3,1} - (D+E) X_{4,1}) p^i p_j p_k - (D+E)(X_\alpha j_\alpha + X_{\alpha,1} v_\alpha \\
 & + X_{4,2} + F X_{2,1} + H X_{3,1} + C X_{4,1}) p^i n_j m_k - (D+E)(-X_\alpha k_\alpha \\
 & - X_{\alpha,1} w_\alpha + X_{4,3} + H X_{2,1} + G X_{3,1} - (D+E) X_{4,1}) p^i n_j n_k \\
 & - (D_{,\alpha} X_{\alpha,1} + D_{;\alpha} X_{\alpha,1} + E_{,\alpha} X_{\alpha,1} + E_{;\alpha} X_{\alpha,1}) p^i p_j n_k \\
 & + (D+E)(X_{1,4} + X_{4,1}) l^i p_j n_k - (D+E)(X_\alpha j_\alpha + X_{\alpha,1} v_\alpha \\
 & - X_{2,4} - F X_{2,1} - H X_{3,1} - C X_{4,1}) m^i p_j n_k + (D+E)(X_\alpha k_\alpha \\
 & + X_{\alpha,1} w_\alpha + X_{3,4} + H X_{2,1} + G X_{3,1} - (D+E) X_{4,1}) n^i p_j n_k \\
 & - (D+E)(X_\alpha j_\alpha + X_{\alpha,1} v_\alpha + X_{4,2} + F X_{2,1} + H X_{3,1} + C X_{4,1}) p^i m_j n_k \\
 & - (D+E)(-X_\alpha k_\alpha - X_{\alpha,1} w_\alpha + X_{4,3} + H X_{2,1} + G X_{3,1} \\
 & - (D+E) X_{4,1}) p^i n_j n_k - (D+E)(-X_\alpha h_\alpha - X_{\alpha,1} u_\alpha + X_{3,2} \\
 & + D X_{2,1} + B X_{3,1} + H X_{4,1}) p^i p_j m_k - (D+E)(X_{3,3} + B X_{2,1} \\
 & + E X_{3,1} + G X_{4,1}) p^i p_j n_k - (D+E)(X_\alpha k_\alpha + X_{\alpha,1} w_\alpha + X_{3,4} \\
 & + H X_{2,1} + G X_{3,1} - (D+E) X_{4,1}) p^i p_j p_k - (F_{,\alpha} X_{\alpha,1} + F_{;\alpha} X_{\alpha,1} \\
 & + G_{,\alpha} X_\alpha + G_{;\alpha} X_{\alpha,1}) p^i p_j p_k + (F+G)(X_{1,4} + X_{4,1}) l^i p_j p_k \\
 & - (F+G)(X_\alpha j_\alpha + X_{\alpha,1} v_\alpha - X_{2,4} - F X_{2,1} - H X_{3,1} - C X_{4,1}) m^i p_j p_k \\
 & + (F+G)(X_\alpha k_\alpha + X_{\alpha,1} w_\alpha + X_{3,4} + H X_{2,1} + G X_{3,1})
 \end{aligned}$$

$$\begin{aligned}
 & -(D+E)X_{4,1})n^i p_j p_k - (F+G)(X_{\alpha,\alpha} j_\alpha + X_{\alpha,1} v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} \\
 & + CX_{4,1})p^i m_j p_k - (F+G)(-X_{\alpha,\alpha} k_\alpha - X_{\alpha,1} w_\alpha + X_{4,3} + HX_{2,1} + GX_{3,1} \\
 & -(D+E)X_{4,1})p^i n_j p_k - (F+G)(X_{\alpha,\alpha} j_\alpha + X_{\alpha,1} v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} \\
 & + CX_{4,1})p^i p_j m_k - (F+G)(X_{4,4} + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1})p^i p_j p_k \\
 & -(F+G)(-X_\alpha k_\alpha - X_\alpha w_\alpha + X_{4,3} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1})p^i p_j n_k
 \end{aligned}$$

Let $B_{\alpha\beta\gamma}$ be the scalar components of $\mathbf{f}_x(LC_{jk}^i)$. Then we have

$$\mathbf{f}_x(LC_{jk}^i) = B_{\alpha\beta\gamma} e^i e^\alpha e^\beta e^\gamma.$$

Obviously $B_{\alpha\beta\gamma} = B_{\alpha\gamma\beta}$ and from (3.16), we get

$$B_{\alpha\beta 1} = 0$$

$$B_{122} = -F(X_{1,4} + X_{4,1}) - D(X_{1,3} + X_{3,1}) - A(X_{1,2} + X_{2,1})$$

$$B_{123} = -2H(X_{1,4} + X_{4,1}) - 2D(X_{1,2} + X_{2,1}) - 2B(X_{1,3} + X_{3,1})$$

$$B_{124} = -2H(X_{1,3} + X_{3,1}) - 2F(X_{1,2} + X_{2,1}) - 2C(X_{1,4} + X_{4,1})$$

$$B_{133} = -G(X_{1,4} + X_{4,1}) - B(X_{1,2} + X_{2,1}) - E(X_{1,3} + X_{3,1})$$

$$B_{134} = 2(D+E)(X_{1,4} + X_{4,1}) - 2H(X_{1,2} + X_{2,1}) - 2G(X_{1,3} + X_{3,1})$$

$$B_{144} = (D+E)(X_{1,3} + X_{3,1}) - C(X_{1,2} + X_{2,1}) + (F+G)(X_{1,4} + X_{4,1})$$

$$\begin{aligned}
 B_{222} = & F(3X_\alpha j_\alpha + 3X_{\alpha,1} v_\alpha + 2X_{4,2} - X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1}) \\
 & + D(-3X_\alpha h_\alpha - 3X_{\alpha,1} u_\alpha + 2X_{3,2} - X_{2,3} + DX_{2,1} + BX_{3,1} \\
 & + HX_{4,1}) + (A_{;\alpha} X_\alpha + A_{;\alpha} X_{\alpha,1}) - A(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1})
 \end{aligned}$$

$$\begin{aligned}
 B_{223} = & 2H(2X_\alpha j_\alpha + 2X_{\alpha,1} v_\alpha + X_{4,2} - X_{2,4}) + 2F(-X_\alpha k_\alpha \\
 & - X_{\alpha,1} w_\alpha + X_{4,3} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}) + 2(D_{;\alpha} X_\alpha \\
 & + D_{;\alpha} X_{\alpha,1}) + 2D(X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1}) + 2B(-X_\alpha h_\alpha \\
 & - X_{\alpha,1} u_\alpha + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1}) + (2A - 2B)(X_\alpha h_\alpha \\
 & + X_{\alpha,1} u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1})
 \end{aligned}$$

$$\begin{aligned}
 B_{224} &= 2H(-2X_\alpha h_\alpha - 2X_{\alpha,1}u_\alpha + X_{3,2} - X_{2,3}) + 2F(X_{4,4} + CX_{2,1}) \\
 &\quad - (D+E)X_{3,1} - (F+G)X_{4,1}) + 2(F_{,\alpha}X_\alpha + F_{;\alpha}X_{\alpha,1}) \\
 &\quad + 2D(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + X_{3,4} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}) \\
 &\quad + 2C(2X_\alpha j_\alpha + 2X_{\alpha,1}v_\alpha + X_{4,2} - X_{2,4}) + 2A(-X_\alpha j_\alpha \\
 &\quad - X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1}) \\
 B_{233} &= 2H(-X_\alpha k_\alpha - X_{\alpha,1}w_\alpha + X_{4,3} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}) \\
 &\quad + G(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha - X_{2,4} - FX_{2,1} - HX_{3,1} - CX_{4,1}) + 2D(X_\alpha h_\alpha \\
 &\quad - X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1}) + (B_{,\alpha}X_\alpha + B_{;\alpha}X_{\alpha,1}) \\
 &\quad - B(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1}) + 2B(X_{3,3} + BX_{2,1} + EX_{3,1} \\
 &\quad + GX_{4,1}) - E(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1}) \\
 B_{234} &= -2(D+E)(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha - X_{2,4} - FX_{2,1} - HX_{3,1} - CX_{4,1}) \\
 &\quad + 2(H_{,\alpha}X_\alpha + H_{;\alpha}X_{\alpha,1}) - 2H(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1}) \\
 &\quad + 2H(X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1}) + (B_{,\alpha}X_\alpha + B_{;\alpha}X_{\alpha,1}) \\
 &\quad + 2H(X_{4,4} + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1}) \\
 &\quad + (2F - 2G)(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} \\
 &\quad + HX_{4,1}) + 2D(-X_\alpha j_\alpha - X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1} + HX_{3,1} \\
 &\quad + CX_{4,1}) + 2C(-X_\alpha k_\alpha - X_{\alpha,1}w_\alpha + X_{4,3} + HX_{2,1} \\
 &\quad + GX_{3,1} - (D+E)X_{4,1}) + 2B(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha \\
 &\quad + X_{3,4} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}) \\
 B_{244} &= (D+E)(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1}) \\
 &\quad + 2H(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + X_{3,4} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}) \\
 &\quad + 2F(-X_\alpha j_\alpha - X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1}) \\
 &\quad + (C_{,\alpha}X_\alpha + C_{;\alpha}X_{\alpha,1}) - C(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1}) \\
 &\quad + 2C(X_{4,4} + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1}) - (F+G)(X_\alpha j_\alpha \\
 &\quad + X_{\alpha,1}v_\alpha - X_{2,4} - FX_{2,1} - HX_{3,1} - CX_{4,1})
 \end{aligned}$$

$$B_{322} = 2H(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1}) - F(X_\alpha k_\alpha$$

$$+ X_{\alpha,1}w_\alpha + X_{3,4} + HX_{2,1} + GX_{3,1} - (D + E)X_{4,1}) + (D_{,\alpha} X_\alpha$$

$$+ D_{;\alpha} X_{\alpha,1}) - D(X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1}) + 2D(X_{2,2}$$

$$+ AX_{2,1} + DX_{3,1} + FX_{4,1}) + (A - 2B)(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha$$

$$- X_{3,2} - DX_{2,1} - BX_{3,1} - HX_{4,1})$$

$$B_{323} = 2H(-2X_\alpha k_\alpha - 2X_{\alpha,1}w_\alpha + X_{4,3} - X_{3,4}) + 2G(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha$$

$$+ X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1}) + 2(B_{,\alpha} X_\alpha + B_{;\alpha} X_{\alpha,1})$$

$$+ 2D(2X_\alpha h_\alpha + 2X_{\alpha,1}u_\alpha + X_{2,3} - X_{3,2}) + 2B(X_{2,2} + AX_{2,1} + DX_{3,1}$$

$$+ FX_{4,1}) + 2E(-X_\alpha h_\alpha - X_{\alpha,1}u_\alpha + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1})$$

$$B_{324} = -2(D + E)(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1})$$

$$+ 2(H_{,\alpha} X_\alpha + H_{;\alpha} X_{\alpha,1}) - 2H(X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1})$$

$$+ 2H(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1}) + 2H(X_{4,4} + CX_{2,1}$$

$$- (D + E)X_{3,1} - (F + G)X_{4,1}) + 2G(-X_\alpha h_\alpha - X_{\alpha,1}u_\alpha$$

$$+ X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1}) + 2F(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha - X_{3,2}$$

$$- DX_{2,1} - BX_{3,1} - HX_{4,1}) + 2D(-X_\alpha j_\alpha - X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1}$$

$$+ HX_{3,1} + CX_{4,1}) + (2B - 2C)(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + X_{3,4} + HX_{2,1}$$

$$+ GX_{3,1} - (D + E)X_{4,1})$$

$$B_{333} = G(-3X_\alpha k_\alpha - 3X_{\alpha,1}w_\alpha + 2X_{4,3} - X_{3,4} + HX_{2,1} + GX_{3,1}$$

$$- (D + E)X_{4,1}) + B(3X_\alpha h_\alpha + 3X_{\alpha,1}u_\alpha + 2X_{2,3} - X_{3,2}$$

$$+ DX_{2,1} + BX_{3,1} + HX_{4,1}) + (E_{,\alpha} X_\alpha + E_{;\alpha} X_{\alpha,1})$$

$$+ E(X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1})$$

$$B_{334} = 2(D + E)(2X_\alpha k_\alpha + 2X_{\alpha,1}w_\alpha + X_{3,4} - X_{4,3}) + 2H(2X_\alpha h_\alpha$$

$$+ 2X_{\alpha,1}u_\alpha + X_{2,3} - X_{3,2}) + 2(G_{,\alpha} X_\alpha + G_{;\alpha} X_{\alpha,1}) - 2H(X_{3,3}$$

$$+ BX_{2,1} + EX_{3,1} + GX_{4,1}) + 2H(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1})$$

$$\begin{aligned}
 & + 2G(X_{4,4} + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1}) + 2B(-X_\alpha j_\alpha \\
 & - X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1}) + 2E(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha \\
 & + X_{3,4} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}) \\
 B_{344} = & -(D_{,\alpha}X_\alpha + D_{,\alpha}X_{\alpha,1} + E_{,\alpha}X_\alpha + E_{,\alpha}X_{\alpha,1}) + (D+E)(X_{3,3} \\
 & + BX_{2,1} + EX_{3,1} + GX_{4,1}) - 2(D+E)(X_{4,4} + CX_{2,1} \\
 & - (D+E)X_{3,1} - (F+G)X_{4,1}) + 2H(-X_\alpha j_\alpha - X_{\alpha,1}v_\alpha \\
 & + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1}) + C(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha \\
 & - X_{3,2} - DX_{2,1} - BX_{3,1} - HX_{4,1}) + (F+3G)(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha \\
 & + X_{3,4} + HX_{2,1} + GX_{3,1} - (D+E)X_{4,1}) \\
 B_{422} = & 2H(-X_\alpha h_\alpha - X_{\alpha,1}u_\alpha + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1}) \\
 & + (F_{,\alpha}X_\alpha + F_{,\alpha}X_{\alpha,1}) - F(X_{4,4} + CX_{2,1} - (D+E)X_{3,1} \\
 & - (F+G)X_{4,1}) + 2F(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1}) \\
 & + D(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha - X_{4,3} - HX_{2,1} - GX_{3,1} + (D+E)X_{4,1}) \\
 & + (2C-A)(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1}) \\
 B_{423} = & -2(2D+E)(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1}) \\
 & + 2(H_{,\alpha}X_\alpha + H_{,\alpha}X_{\alpha,1}) + 2H(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1}) \\
 & - 2H(X_{4,4} + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1}) + 2H(X_{3,3} \\
 & + BX_{2,1} + EX_{3,1} + GX_{4,1}) + 2G(-X_\alpha h_\alpha - X_{\alpha,1}u_\alpha + X_{3,2} + DX_{2,1} \\
 & + BX_{3,1} + HX_{4,1}) + 2F(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} \\
 & + HX_{4,1}) + 2C(-X_\alpha k_\alpha - X_{\alpha,1}w_\alpha + X_{4,3} + HX_{2,1} + GX_{3,1} \\
 & - (D+E)X_{4,1}) + 2B(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha - X_{4,3} - HX_{2,1} \\
 & - GX_{3,1} + (D+E)X_{4,1}) \\
 B_{424} = & -2(D+E)(-X_\alpha h_\alpha - X_{\alpha,1}u_\alpha + X_{3,2} + DX_{2,1} + BX_{3,1} + HX_{4,1}) \\
 & + 2H(2X_\alpha k_\alpha + 2X_{\alpha,1}w_\alpha - X_{4,3} + X_{3,4}) - 2(2F+G)(X_\alpha j_\alpha
 \end{aligned}$$

$$+ X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1}) + 2F(-X_\alpha j_\alpha$$

$$- X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1}) + 2(C_{,\alpha} X_\alpha$$

$$+ C_{;\alpha} X_{\alpha,1}) + 2C(X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1})$$

$$B_{433} = -2(D + E)(-X_\alpha k_\alpha - X_{\alpha,1}w_\alpha + X_{4,3} + HX_{2,1} + GX_{3,1}$$

$$- (D + E)X_{4,1}) + 2H(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1}$$

$$+ BX_{3,1} + HX_{4,1}) + (G_{,\alpha} X_\alpha + G_{;\alpha} X_{\alpha,1}) - G(X_{4,4} + CX_{2,1}$$

$$- (D + E)X_{3,1} - (F + G)X_{4,1}) + 2G(X_{3,3} + BX_{2,1} + EX_{3,1}$$

$$+ GX_{4,1}) - B(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2} + FX_{2,1} + HX_{3,1} + CX_{4,1})$$

$$+ E(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha - X_{4,3} - HX_{2,1} - GX_{3,1} + (D + E)X_{4,1})$$

$$B_{434} = -2(D_{,\alpha} X_\alpha + D_{;\alpha} X_{\alpha,1} + E_{,\alpha} X_\alpha + E_{;\alpha} X_{\alpha,1}) - 2(D + E)(X_{3,3}$$

$$+ BX_{2,1} + EX_{3,1} + GX_{4,1}) + 2H(-2X_\alpha j_\alpha - 2X_{\alpha,1}v_\alpha$$

$$+ X_{2,4} - X_{4,2}) + 2G(2X_\alpha k_\alpha + 2X_{\alpha,1}w_\alpha - X_{4,3} + X_{3,4})$$

$$+ 2C(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + X_{2,3} + DX_{2,1} + BX_{3,1} + HX_{4,1})$$

$$- 2(F + G)(-X_\alpha k_\alpha - X_{\alpha,1}w_\alpha + X_{4,3} + HX_{2,1} + GX_{3,1}$$

$$- (D + E)X_{4,1})$$

$$B_{444} = -(D + E)(3X_\alpha k_\alpha + 3X_{\alpha,1}w_\alpha - X_{4,3} + 2X_{3,4} + HX_{2,1}$$

$$+ GX_{3,1} - (D + E)X_{4,1}) - C(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + X_{4,2}$$

$$+ FX_{2,1} + HX_{3,1} + CX_{4,1}) - (F_{,\alpha} X_\alpha + F_{;\alpha} X_{\alpha,1} + G_{,\alpha} X_\alpha$$

$$+ G_{;\alpha} X_{\alpha,1}) - (F + G)(X_{4,4} + CX_{2,1} - (D + E)X_{3,1} - (F + G)X_{4,1})$$

$$+ 2C(-X_\alpha j_\alpha - X_{\alpha,1}v_\alpha + X_{2,4} + FX_{2,1} + HX_{3,1} + CX_{4,1})$$

4. Motion in Terms of Scalar Components

An infinitesimal transformation is said to be a motion if and only if [5]

$$\mathfrak{L}_x g_{ij} = 0.$$

Thus, the infinitesimal transformation (3.1) is a motion if and only if

$$\begin{aligned}
 (4.1) \quad & X_{1,1} = 0, \\
 & X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1} = 0, \\
 & X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1} = 0, \\
 & X_{4,4} + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1} = 0, \\
 & X_{1,2} + X_{2,1} = 0, \\
 & X_{1,3} + X_{3,1} = 0, \\
 & X_{1,4} + X_{4,1} = 0, \\
 & X_{2,3} + X_{3,2} + 2DX_{2,1} + 2BX_{3,1} + 2HX_{4,1} = 0, \\
 & X_{4,2} + X_{2,4} + 2FX_{2,1} + 2HX_{3,1} + 2CX_{4,1} = 0, \\
 & X_{4,3} + X_{3,4} + 2HX_{2,1} + 2GX_{3,1} - 2(D+E)X_{4,1} = 0.
 \end{aligned}$$

Thus, we have

Theorem 4.1: The infinitesimal transformation (3.1) is a motion in four dimensional Finsler space if and only if (4.1) holds.

In view of Theorem 4.1, if the infinitesimal transformation (3.1) is a motion then the scalar components of the Lie derivative of LC_{jk}^i are

$$\begin{aligned}
 B_{122} &= 0, & B_{123} &= 0, \\
 B_{124} &= 0, & B_{133} &= 0, \\
 B_{134} &= 0, & B_{144} &= 0, \\
 B_{222} &= 3F(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) \\
 &\quad - 3D(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) + (A_{,\alpha}X_\alpha + A_{;\alpha}X_{\alpha,1}), \\
 B_{223} &= 4H(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) \\
 &\quad - 2F(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + 2(D_{,\alpha}X_\alpha \\
 &\quad + D_{;\alpha}X_{\alpha,1}) - 4B(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) \\
 &\quad + 2A(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}),
 \end{aligned}$$

$$\begin{aligned}
 B_{224} &= -4H(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) + 2(F_{,\alpha}X_\alpha + F_{;\alpha}X_{\alpha,1}) \\
 &\quad + 2D(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + 4C(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha \\
 &\quad + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) - 2A(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}), \\
 B_{233} &= -2H(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + G(X_\alpha j_\alpha \\
 &\quad + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) + (2D - E)(X_\alpha h_\alpha - X_{\alpha,1}u_\alpha \\
 &\quad + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) + (B_{,\alpha}X_\alpha + B_{;\alpha}X_{\alpha,1}), \\
 B_{234} &= -2(2D + E)(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) + 2(H_{,\alpha}X_\alpha \\
 &\quad + H_{;\alpha}X_{\alpha,1}) + (B_{,\alpha}X_\alpha + B_{;\alpha}X_{\alpha,1}) + (2F - 2G)(X_\alpha h_\alpha \\
 &\quad + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) - 2C(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} \\
 &\quad - \frac{1}{2}X_{4,3}) + 2B(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}), \\
 B_{244} &= (D + E)(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) + 2H(X_\alpha k_\alpha \\
 &\quad + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + (C_{,\alpha}X_\alpha + C_{;\alpha}X_{\alpha,1}) \\
 &\quad - (3F + G)(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha - \frac{1}{2}X_{2,4} + \frac{1}{2}X_{4,2}), \\
 B_{322} &= 2H(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) - F(X_\alpha k_\alpha \\
 &\quad + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + (D_{,\alpha}X_\alpha + D_{;\alpha}X_{\alpha,1}) \\
 &\quad + (A - 2B)(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) \\
 B_{323} &= -4H(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + 2G(X_\alpha j_\alpha \\
 &\quad + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) + 2(B_{,\alpha}X_\alpha + B_{;\alpha}X_{\alpha,1}) \\
 &\quad - 2(2D - E)(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) \\
 B_{324} &= -2(2D + E)(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) + 2(H_{,\alpha}X_\alpha \\
 &\quad + H_{;\alpha}X_{\alpha,1}) + 2(F - G)(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) \\
 &\quad + 2(B - C)(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) \\
 B_{333} &= -3G(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + 3B(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha \\
 &\quad + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) + (E_{,\alpha}X_\alpha + E_{;\alpha}X_{\alpha,1})
 \end{aligned}$$

$$B_{334} = 2(2D + 3E)(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + 2(G_{,\alpha}X_\alpha$$

$$+ G_{;\alpha}X_{\alpha,1}) + 4H(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3}$$

$$- \frac{1}{2}X_{3,2}) - 2B(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4})$$

$$B_{344} = -(D_{,\alpha}X_\alpha + D_{;\alpha}X_{\alpha,1} + E_{,\alpha}X_\alpha + E_{;\alpha}X_{\alpha,1}) - 2H(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha$$

$$+ \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) + C(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2})$$

$$B_{422} = -2H(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) + (F_{,\alpha}X_\alpha + F_{;\alpha}X_{\alpha,1})$$

$$+ D(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + (2C - A)(X_\alpha j_\alpha$$

$$+ X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4})$$

$$B_{423} = -2(2D + E)(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4}) + 2(H_{,\alpha}X_\alpha$$

$$+ H_{;\alpha}X_{\alpha,1}) + 2(F - G)(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3}$$

$$- \frac{1}{2}X_{3,2}) + 2(B - C)(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3})$$

$$B_{424} = 2(D + E)(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2}) + 4H(X_\alpha k_\alpha$$

$$+ X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + 2(C_{,\alpha}X_\alpha + C_{;\alpha}X_{\alpha,1})$$

$$- 2(3F + G)(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4})$$

$$B_{433} = (2D + 3E)(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) + (G_{,\alpha}X_\alpha$$

$$+ G_{;\alpha}X_{\alpha,1}) + 2H(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2})$$

$$- B(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4})$$

$$B_{434} = -2(D_{,\alpha}X_\alpha + D_{;\alpha}X_{\alpha,1} + E_{,\alpha}X_\alpha + E_{;\alpha}X_{\alpha,1}) + 2(F + 3G)(X_\alpha k_\alpha$$

$$+ X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) - 4H(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha + \frac{1}{2}X_{4,2}$$

$$- \frac{1}{2}X_{2,4}) + 2C(X_\alpha h_\alpha + X_{\alpha,1}u_\alpha + \frac{1}{2}X_{2,3} - \frac{1}{2}X_{3,2})$$

$$B_{444} = -3(D + E)(X_\alpha k_\alpha + X_{\alpha,1}w_\alpha + \frac{1}{2}X_{3,4} - \frac{1}{2}X_{4,3}) - (F_{,\alpha}X_\alpha$$

$$+ F_{;\alpha}X_{\alpha,1} + G_{,\alpha}X_\alpha + G_{;\alpha}X_{\alpha,1}) - C(X_\alpha j_\alpha + X_{\alpha,1}v_\alpha$$

$$+ \frac{1}{2}X_{4,2} - \frac{1}{2}X_{2,4})$$

Since the Lie-derivative of LC_{jk}^i with respect to a motion vanishes identically, the scalar components $B_{\alpha\beta\gamma}$ of $\mathfrak{L}_x(LC_{jk}^i)$ are zero. Therefore we must have

$$\begin{aligned}
 0 &= 3FP - 3DQ + (A_{,\alpha} X_\alpha + A_{;\alpha} X_{\alpha,1}), \\
 0 &= 4HP - 2FR + 2(D_{,\alpha} X_\alpha + D_{;\alpha} X_{\alpha,1}) + (2A - 4B)Q, \\
 0 &= -4HQ + 2(F_{,\alpha} X_\alpha + F_{;\alpha} X_{\alpha,1}) + 2DR + (4C - 2A)P, \\
 0 &= -2HR + GP + (2D - E)Q + (B_{,\alpha} X_\alpha + B_{;\alpha} X_{\alpha,1}), \\
 0 &= -2(2D + E)P + 2(H_{,\alpha} X_\alpha + H_{;\alpha} X_{\alpha,1}) + (B_{,\alpha} X_\alpha + B_{;\alpha} X_{\alpha,1}) \\
 &\quad + (2F - 2G)Q - 2CR + 2BR, \\
 0 &= (D + E)Q + 2HR + (C_{,\alpha} X_\alpha + C_{;\alpha} X_{\alpha,1}) - (3F + G)P, \\
 0 &= 2HP - FR + (D_{,\alpha} X_\alpha + D_{;\alpha} X_{\alpha,1}) + (A - 2B)Q \\
 0 &= -4HR + 2GP + 2(B_{,\alpha} X_\alpha + B_{;\alpha} X_{\alpha,1}) - 2(2D - E)Q \\
 0 &= -2(2D + E)P + 2(H_{,\alpha} X_\alpha + H_{;\alpha} X_{\alpha,1}) + 2(F - G)Q + 2(B - C)R \\
 0 &= -3GR + 3BQ + (E_{,\alpha} X_\alpha + E_{;\alpha} X_{\alpha,1}) \\
 0 &= 2(2D + 3E)R + 2(G_{,\alpha} X_\alpha + G_{;\alpha} X_{\alpha,1}) + 4HQ - 2BP \\
 0 &= -(D_{,\alpha} X_\alpha + D_{;\alpha} X_{\alpha,1} + E_{,\alpha} X_\alpha + E_{;\alpha} X_{\alpha,1}) - 2HP + CQ \\
 0 &= -2HQ + (F_{,\alpha} X_\alpha + F_{;\alpha} X_{\alpha,1}) + DR + (2C - A)P \\
 0 &= -2(2D + E)P + 2(H_{,\alpha} X_\alpha + H_{;\alpha} X_{\alpha,1}) + 2(F - G)Q + 2(B - C)R \\
 0 &= 2(D + E)Q + 4HR + 2(C_{,\alpha} X_\alpha + C_{;\alpha} X_{\alpha,1}) - 2(3F + G)P \\
 0 &= (2D + 3E)R + (G_{,\alpha} X_\alpha + G_{;\alpha} X_{\alpha,1}) + 2HQ - BP \\
 0 &= -2(D_{,\alpha} X_\alpha + D_{;\alpha} X_{\alpha,1} + E_{,\alpha} X_\alpha + E_{;\alpha} X_{\alpha,1}) + 2(F + 3G)R \\
 &\quad - 4HP + 2CQ \\
 0 &= -3(D + E)R - (F_{,\alpha} X_\alpha + F_{;\alpha} X_{\alpha,1} + G_{,\alpha} X_\alpha + G_{;\alpha} X_{\alpha,1}) - CP,
 \end{aligned}$$

where

$$P = X_\alpha j_\alpha + X_{\alpha,1} v_\alpha + \frac{1}{2} X_{4,2} - \frac{1}{2} X_{2,4},$$

$$Q = X_\alpha h_\alpha + X_{\alpha,1} u_\alpha + \frac{1}{2} X_{2,3} - \frac{1}{2} X_{3,2},$$

$$R = X_\alpha k_\alpha + X_{\alpha,1} w_\alpha + \frac{1}{2} X_{3,4} - \frac{1}{2} X_{4,3}.$$

5. Conformal Motion in Terms of Scalar Components

An infinitesimal transformation is said to be a conformal motion if and only if there exists a scalar function $\Phi(x)$ such that

$$\mathfrak{L}_x g_{ij} = 2\Phi g_{ij}.$$

Thus, from (2.3) and (3.13) the infinitesimal transformation (3.1) is conformal motion if and only if

$$\begin{aligned}
 &X_{1,1} = \Phi, \\
 &X_{2,2} + AX_{2,1} + DX_{3,1} + FX_{4,1} = \Phi, \\
 &X_{3,3} + BX_{2,1} + EX_{3,1} + GX_{4,1} = \Phi, \\
 &X_{4,4} + CX_{2,1} - (D+E)X_{3,1} - (F+G)X_{4,1} = \Phi, \\
 (5.1) \quad &X_{1,2} + X_{2,1} = 0, \\
 &X_{1,3} + X_{3,1} = 0, X_{1,4} + X_{4,1} = 0, \\
 &X_{2,3} + X_{3,2} + 2DX_{2,1} + 2BX_{3,1} + 2HX_{4,1} = 0, \\
 &X_{4,2} + X_{2,4} + 2FX_{2,1} + 2HX_{3,1} + 2CX_{4,1} = 0, \\
 &X_{4,3} + X_{3,4} + 2HX_{2,1} + 2GX_{3,1} - 2(D+E)X_{4,1} = 0.
 \end{aligned}$$

Thus, we conclude:

Theorem 5.1: The infinitesimal transformation (3.1) is a conformal motion in a four dimensional Finsler space if and only if there exists a scalar function $\Phi(x)$ such that (5.1) holds.

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